Novel Numerical Solution of Non-linear Heat Transfer of Nanofluid over a Porous Cylinder: Buongiorno-Forchheimer Model

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Abstract
The study aims to investigate numerically a two dimensional, steady, heat transfer over a cylinder in porous medium with suspending nanoparticles. Buongiorno model is adopted for nanofluid transport on free convection flow taking the slip mechanism of Brownian motion and thermophoresis into account. Boussinesq approximation is considered to account for buoyancy. The boundary layer conservation equations are transformed into dimensionless, and then elucidated using robust Keller-box implicit code numerically. The numerical results are displayed graphically and deliberated quantitatively for various values of thermo-physical parameters. Our results shows that, increasing Forchheimer parameter, $\Lambda$, clearly swamps the nanofluid momentum development, decreasing the flow for some distance near the cylinder viscous region, later its reverse the trend and asymptotically reaches the far field flow velocity. Furthermore, as increases thermophoresis, heat transfer and nanoparticle volume concentration increased in the boundary layer. The present results are validated with the available results of similar study and is found to be in good coincident. The study finds applications in heat exchangers technology, materials processing and geothermal energy storage etc.

Keywords:

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$a$</td>
<td>Radius of the cylinder</td>
</tr>
<tr>
<td>$C$</td>
<td>Nanoparticle volume concentration</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Coefficient of Skin friction value</td>
</tr>
<tr>
<td>$Da$</td>
<td>Darcy parameter</td>
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<td>$D_B$</td>
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<td>$D_T$</td>
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<td>Non-dimensional stream function</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
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<td>$Gr$</td>
<td>Grashof number</td>
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<tr>
<td>$K$</td>
<td>Permeability</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Thermal conductivity</td>
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<tr>
<td>$Nb$</td>
<td>Brownian motion parameter</td>
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</table>

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$Nu$ Nusselt number
$Pr$ Prandtl number
$Sc$ Schmidt number
$Sh$ Sherwood number
$Nu$ Nusselt number
$Gr$ Grashof number
$K$ Permeability
$k_m$ Thermal conductivity
$Nb$ Brownian motion parameter

Nomenclature

$\alpha$ Thermal diffusivity

$T$ Temperature
$t$ Time
$u, v$ $x$ and $y$—velocity components
$x$ Stream wise coordinate
$y$ Transverse coordinate

Greek symbols
\( \beta \) The coefficients of thermal expansion
\( \Gamma \) Inertial drag coefficient
\( \varepsilon \) Porosity
\( \eta \) Dimensionless radial coordinate
\( \theta \) Non-dimensional temperature
\( \Lambda \) Forchheimer parameter
\( \mu \) Dynamic viscosity
\( \zeta \) Non-dimensional tangential coordinate
\( \Phi \) Azimuthal coordinate
\( \rho_f \) Density of base fluid
\( (\rho c)_m \) Effective heat capacity
\( \nu \) Kinematic viscosity
\( \phi \) Non-dimensional volume fraction
\( \psi \) Non-dimensional stream function

**Subscripts**
- \( w \) wall conditions
- \( \infty \) Ambient conditions

**Superscripts**
- \( ^t \) Derivative with respect to \( \eta \)

### 1. Introduction

Enhancement of heat transfer in engineering applicants are very important because conventional fluid such as oil, water that are used in engineering applications possess less thermal conductivity. Thermal enhancement of convective fluids can be upgrade by dispersion of nano-sized particle into the convective fluids. The resultant combination of convective fluid (base fluid) and nano-sized particle (nanoparticles) is known as Nanofluid (Choi [1]). Recently, the studies of nanofluid flows have reported significantly in thermal sciences due to their thermal performance relative to the regular fluids as Wang and Wu [2-3]. Numerous models such as the single-phase model (Choi [1]), the dispersion model (Xuan and Roetzel [4]), the non-homogeneous two component model [5] have been developed to study the transport of Nanofluid. Specifically, the Buongiorno nanofluid transport model is developed on the basis of slip mechanism of the nanoparticle with respect to the relative velocity. By means of this model Beg et al. [6] presented a mathematical model for nanofluid transport past vertical wall with oxtactic microorganisms. Das et al. [7] investigated the heat source/sink on transient laminar magnetic field with nanoparticle flow using conventional single-phase (homogeneous) model. Sheikholeslami and Ganji [8], presents a review on the transport of nanofluid and heat transfer. Gorla et al. [9] have been studied the MHD flow of dusty fluid with nanoparticles saturated in a porous medium. Murthy et al. [10] considered the transport of nanofluid embedding non-Darcy porous medium. Besthapu et al. [11] studied the mixed convection flow with nanoparticles taking thermal stratification and viscous dissipation into account numerically. Aly [12] examined the free convection of nanofluid flow over circular cylinder where the walls are passively controlled in a porous enclosure using finite volume method. Most recently, Ahmadi et al. [13] studied thermal conductivity of CuO/EG nanofluid by employing group method of data handling and genetic algorithm approaches. More works of nanofluids flow and heat transfer analysis can be found in literatures ([14-15]).

Transport phenomena in porous media constitute numerous important flow regimes in many branches of engineering and applied physics. The vast majority of models have considered isotropic, homogenous porous media, usually employing the Darcy law, which is effective for low velocity, viscous-dominated flows. It is known that the porous media are heterogeneous and yields variable porosity. Initially, Roblee et al. [16] studied the flow through media of variable radial porosity in chemical engineering system. Later Vafai [17] studied a theoretical study in porous region with inertial forces (Forchheimer drag), also presented experimental results in detail. Zueco et al. [18] Employed network simulation method to study the MHD effect on a porous micro-structural liquid stream with Darcy-Forchheimer forces. An interesting investigation on the natural convection in Darcian porous media was given by Minkowycz and Cheng [19]. Hamzeh et al. [20] studied the heat transfer and fluid flow past of a sphere. Kumari and Gorla [21] have investigated for the Magneto-convection flow
with suspending nanoparticles past a wedge in a non-Newtonian fluid. Kameswaran et al. [22] shown mixed convection flow of nanofluid over porous wavy surface. Beg et al. [23] studied numerically the flow in orthotropic Darcian porous media from a rotating cone. Very recently, Vasu et al. [24] investigated the entropy generation analysis in porous medium taking thermally stratification into account. Bég et al. [25] have investigated heat transfer and fluid flow over an inclined plate numerically taking Soret/Dufour effects into account. Munawar et al. [26] and Yih [27] have discussed the laminar heat transfer flow over a cylinder embedding porous regime. Prasad et al. [28] presented a numerical study for multiphysical flow of fluid over a cylinder saturating in a variable porosity. Vasu et al. [29] analyzed the influence of Soret and Dufour on magnetic heat transfer flow over a sphere in a porous medium. Satya Narayana and Venkateswarlu [30] have presented a numerical solution for a transient MHD natural convection of a nanofluid past a porous plate in a rotating system. Satya Narayana et al. [31] have been presented a MHD heat transfer with thermal radiation of nanofluid in porous rotating domain numerically. Harish Babu et al. [32] considered a steady magnetic flow of a Jeffery nanofluid using non-homogeneous model.

Motivating the above studies and vital application of nanofluid flow in porous regime. The main purpose of the current study is to analyze the steady viscous incompressible flow of nanofluid in a non-Darcy porous medium over a horizontal cylinder. The finite difference results through Keller-box scheme are presented for highly influential thermophysical parameters. The study has wide applications in heat exchangers, materials processing and geothermal energy storage etc.

2. Mathematical Formulation

Consider a 2-D incompressible free convection laminar flow of nanofluid over a non-Darcy porous horizontal cylinder. Figure 1 shows graphical flow configuration. $a$ denoting the radius of the cylinder. The coordinates $x$ and $y$ are determined along perimeter of circular cylinder and normal to the surface respectively. $\Phi = x/a$ is an angle between $y$-axis and the vertically downward line from center of cylinder ($0 \leq \Phi \leq \pi$) shown in Figure 1. $g$, acts downwards. $T_w (> T_\infty)$ and $C_w (> C_\infty)$ are wall temperature and concentration of the horizontal cylinder respectively, they are more than the far field temperature and concentration.

Governing conservation equations as below ([5], [14], [15], [27]):

Continuity Equation:

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

Momentum Equation:

$$\frac{\rho_v}{\epsilon} \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \frac{\mu}{K} \mathbf{v} - \Gamma \mathbf{v}^2 
+ \left[ C \rho_p + (1-C) \left( \mu \left( T-T_\infty \right) \right) \sin \left( x/a \right) \right] g \quad (2)$$

Energy Equation:

$$\left( \rho c \right)_m \frac{\partial T}{\partial t} + \left( \rho c \right)_f \mathbf{v} \cdot \nabla T = k_m \nabla^2 T 
+ \varepsilon \left( \rho c \right)_f \left[ D_f \nabla C \cdot \nabla T + \left( D_f/T_w \right) \nabla T \cdot \nabla T \right] \quad (3)$$

Concentration Equation:

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon} \mathbf{v} \cdot \nabla C = D_f \nabla^2 C + \left( D_f/T_w \right) \nabla^2 T \quad (4)$$

Boundary conditions:

At $y = 0, \ u = 0, \ v = 0, \ T = T_\infty, \ C = C_w \quad (5)$

As $y \to \infty, \ u = 0, \ T \to T_\infty, \ C \to C_\infty \quad (6)$
We write \( \mathbf{v} = (u, v) \).

Here \( \mu \) is dynamic viscosity, \( \rho_f \) and \( \rho_p \) are fluid density and density of particle respectively, \( \beta \) is fluid’s volume expansion coefficient, \( (\rho c)_m \) is heat capacity, \( D_B \) and \( D_T \) are coefficient of Brownian diffusion and coefficient of thermophoretic diffusion, \( k_m \) is thermal conductivity, \( g \) is gravity.

The momentum equation Eq. (2) can be written, by using proper value of reference pressure, as
\[
\nabla p + \frac{\mu}{K} \mathbf{v} + \Gamma \nabla^2 \mathbf{v} = \left[ (1 - C_e) \rho_{fe} \beta (T - T_e) \sin (x/a) \right] + \left( \rho_p - \rho_{fe} \right) (C - C_e) \sin (x/a) \right] \] (7)

By means of the boundary-layer approximation and Boussinesq approximation, the equations (1) - (4) which govern the flow are reduced to
\[
\frac{\partial u}{\partial x} + \frac{\mu}{K} \frac{\partial v}{\partial y} = 0 \tag{8}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left[(1 - C_e) \rho_{fe} \beta (T - T_e) \sin (x/a) \right] + \left( \rho_p - \rho_{fe} \right) (C - C_e) \sin (x/a) \right] + v \frac{\partial^2 u}{\partial y^2} - \frac{v}{K} u - \Gamma u \tag{9}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \tau \left[ D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_e} \frac{\partial T}{\partial y} \right] \tag{10}
\]

\[
\frac{1}{c} \left( \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_n \frac{\partial C}{\partial y} + \frac{D_T}{T_e} \frac{\partial T}{\partial y} \tag{11}
\]

Where
\[
\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \tau = \frac{(\rho c)_m}{(\rho c)_f}
\]

Introduced a stream function \( \psi \) defined by
\[
u \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x},
\]

So that Eq. (8) is satisfied identically.

Introduce the suitable dimensionless variables:
\[
\xi = \frac{x}{a}, \quad \eta = \frac{y}{a} \sqrt{\frac{Gr}{\nu^2}}, \quad f(\xi, \eta) = \frac{\psi}{\nu x \sqrt{\frac{Gr}{\nu^2}}},
\]

\[
\theta(\xi, \eta) = \frac{T - T_w}{T_w - T_e}, \quad \phi(\xi, \eta) = \frac{C - C_e}{C_w - C_e},
\]

\[
Gr = \left(1 - \rho_{fe}\right) \frac{\rho_{fe} \beta (T_w - T_e) a^3}{\nu^2}
\]

Using dimensionless variables, Eqs. (8) to (11), obtained in the dimensionless forms as follows:
\[
f' + f'' - (1 + \xi \Lambda) f' + \sin \xi (\theta - N \phi) \tag{13}
\]

\[
\theta' + \phi' + Nb \theta' \phi' + Nt (\phi')^2 = \xi (f' \frac{\partial f}{\partial \xi} - f'' \frac{\partial \phi}{\partial \xi}),
\]

\[
\phi' + Sc \phi' + \left(\frac{Nt}{Nb}\right) \phi'' = \xi (f' \frac{\partial \phi}{\partial \xi} - \phi' \frac{\partial f}{\partial \xi}).
\]

The transformed dimensionless boundary conditions are:
\[
\eta = 0: \quad f' = 0, \quad f = 0, \quad \theta = 1, \quad \phi = 1 \tag{16a}
\]
\[
\eta \rightarrow \infty: \quad f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \tag{16b}
\]

where \( \Phi \) is the azimuthal coordinate, \( \xi \) is the non-dimensional tangential coordinate, \( \Lambda = \Gamma a \) is the non-Darcy parameter, \( Da = \frac{K}{\alpha_m} \) is a Darcy parameter, \( Pr = \frac{\nu}{\alpha_m} \) is the Prandtl number, \( Nr = \frac{(\rho_p - \rho_{fe})(C_w - C_e)}{\rho_{fe} (1 - C_e) \beta (T_w - T_e)} \) is buoyancy ratio parameter, \( Sc = \frac{v}{D_m c} \), \( Nb = \frac{\tau D_m (C_w - C_e)}{v} \) and \( Nt = \frac{\tau D_m (T_w - T_e)}{v T_e} \) are respectively the Schmidt number. Brownian motion parameter and thermophoresis parameter.

The quantities of physical choice, we calculated the coefficient of skin-friction, local Nusselt and Sherwood number as:
\[
\frac{1}{2} C_f \sqrt{Gr} = \xi f' \tag{17a}
\]
\[ \frac{Nu}{\sqrt{Gr}} = -\theta'(\xi, 0) \] (17b)  
\[ \frac{Sh}{\sqrt{Gr}} = -\phi'(\xi, 0) \] (17c)

3. Solution using Keller-Box Finite Difference Code

The numerical analysis integrates the non-dimensional equations (13)-(15) subject to the boundary conditions (16) by an implicit finite-difference approximation with an efficient finite difference scheme. Keller-box method [33] described in the book of Cebeci and Bradshaw [34]. The scheme is an unconditionally stable. This method has been employed for various complex domains like aerodynamic problems (Chiam [35], Rees, and Pop [36]), heat transfer in porous regime (Bég et al. [37], Prasad et al. [38]), heat and mass transfer in micropolar regime (Bég et al. [39]). Recently, Gorla and Vasu [40] and Gorla et al. [41] have employed to study an unsteady heat transfer of a non-Newtonian nanofluid. The numerical method is not described for the sake of conciseness. The solution process of Keller box method is given in many references including Gorla and Vasu [40]. Because of conservation of space, the detailed solution is omitted here. Considered a uniform grid of size 1501 x 31 in the \( \eta, \xi \) region. Computations have been carried out with \( \Delta \xi = 0.1 \), and \( \Delta \eta = 0.002 \). For the desired accuracy, convergence criterion has been fixed at \( 10^{-3} \) as the change between any two successive iterations. Fig. 2 shows the representation of computational cell for the Keller box method after meshing. The results have also been shown to be grid-independent.

4. Numerical Validations

In order to judge the validation of numerical outcomes, the current results of the local heat transfer coefficient \( -\theta'(\xi, 0) \) are compared with results of Merkin [42] and Yih [27] for various values of \( \xi \) for \( Da \to \infty \), \( \Lambda = 0 \), \( Pr = Gr = 1 \), \( Nt = Nb = Nr = 0 \), \( f_w = 0 \), \( Sc = 0 \)

Table 1 shows the validation of present result by assuming that the porous field and nanofluid effects are negated in the models. It is found that the current numerical solution is in good compliance. Further, the local heat transfer coefficient \( -\theta'(\xi, 0) \) are decreased along the edge of cylinder. Hence a very strong thermal enhancement (0) is achieved past a circular cylinder.

<table>
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<tr>
<th>( \xi )</th>
<th>Merkin [47]</th>
<th>Yih [27]</th>
<th>Present results</th>
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<tr>
<td>0.0</td>
<td>0.4212</td>
<td>0.4214</td>
<td>0.42145</td>
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<td>0.4</td>
<td>0.4182</td>
<td>0.4184</td>
<td>0.41835</td>
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<tr>
<td>0.8</td>
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<td>0.4096</td>
<td>0.40897</td>
</tr>
<tr>
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<td>0.3950</td>
<td>0.39532</td>
</tr>
<tr>
<td>1.6</td>
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<td>2.0</td>
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<td>0.34660</td>
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<tr>
<td>2.4</td>
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</tr>
<tr>
<td>2.8</td>
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<td>0.25917</td>
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<tr>
<td>( \pi )</td>
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<td>0.1962</td>
<td>0.19654</td>
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</table>

5. Results and Discussions

This section is focused to the physical insight through numerical results of nanofluid transport over a cylinder in a non-Darcy porous regime taking Buongiorno-Forchheimer model into account. The numerically computed results for velocity, coefficient of skin-friction, temperature, local Nusselt number, nanoparticle volume concentration and local Sherwood number for different values of dimensionless thermophysical parameters viz., \( Nb, Nt, Nr, \Lambda, Da, Sc, Pr \), tangential coordinate \( (\xi) \) are presented along the radial coordinate \( (\eta) \) in form of tables and figures. The numerical results are validated and verified through a comparison made with previously reported.
work. The comparisons are found to be in an excellent compliance (see section 4).

**Table 2:** Values of $f''(\xi,0)$, $-\theta'(\xi,0)$ and $-\phi'(\xi,0)$ for different Pr, $\Lambda$ and $\xi$ when $Gr = 1$, $Nt = Nr = Nt = 10^{-5}$, $Da = 0.1$, $Sc = 0.6$.

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$Nt$</th>
<th>$Gr$</th>
<th>$\rho(\xi,0)$</th>
<th>$\rho(\xi,0)$</th>
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<tr>
<td>1</td>
<td>0</td>
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<td>0.200</td>
<td>0.200</td>
<td>0.200</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0.1</td>
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<td>0.200</td>
<td>0.200</td>
<td>0.200</td>
<td>0.200</td>
</tr>
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</table>

The effect of varying $\Lambda$ and Pr on the coefficient of skin friction, heat transfer and nanoparticle volume fraction coefficient are presented in tabular form in Table 2. It is found that skin friction and coefficients of nanoparticle volume fraction are increased whereas heat transfer coefficients reduced along the tangential coordinate $(\xi)$ for $\Lambda$ and Pr. It is worth mention that the thermal enhancement occurs due to nanoparticles. Increasing Pr, the values of $-\theta'(\xi,0)$ are significantly increased whereas $f''(\xi,0)$ and $-\phi'(\xi,0)$ values are decreased. The same tendency has been observed and in good coincidence with the earlier results by Prasad et al. [38]. Also it is found that reduction of skin friction values and heat transfer coefficients due to increasing Forchheimer parameter. The reverse trend is observed for local mass transfer coefficients.

Figs. 3-5 describes the influence of $Nb$ on $f$', $\theta$ and $\phi$ for water based nanofluids over the horizontal circular cylinder regime. Enlarging $Nb$ leads to rise in velocity as well as temperature whereas opposite trend is observed for nanoparticle volume fraction concentration $(\phi)$. Larger values of $Nb$ approach to smaller nano-particles [5] and this boosts acceleration of the hydrodynamics. Also $\phi(\eta)$ decreases with the increase $Nb$.

Fig. 3. Influence of Nb on velocity profile

Fig. 4. Influence of Nb on temperature distribution

Fig. 5. Influence of Nb on nanofluid volume fraction

Fig. 6–8 shows the numerical result of the thermophoresis on $f'$, $\theta$ and $\phi$. Moving of the particles in the way of shrinking temperature due to a termal gradient forces, the phenomenon is called thermophoresis. Thermophoretic parameter (Nt) involves in equations (14) and
(15) and it plays a significantly influential in the thermal diffusion and nanoparticle diffusion in the domain. As $Nt$ increases, the velocity of nanofluid decreases.

Fig. 6. Influence of $Nt$ on velocity profile

Fig. 7. Influence of $Nt$ on temperature distribution

Fig. 8. Influence of $Nt$ on nanofluid volume fraction

It is also from Fig 7 and Fig 8, witnessed that an rise in $Nt$ leads to thermal enhancement and raise nanoparticle concentration i.e. thermal and nanofluid volume fraction volume fraction boundary layer increased so that the thermal layer raised. Larger thermophoresis indicates durable movement of nano-particles with respect to rate of heat temperature away from the cylinder surface.

Fig. 9. Influence of $Nr$ on velocity profile

Fig. 10. Influence of $Nr$ on temperature distribution

Fig. 11. Influence of $Nr$ on nanofluid volume fraction
Figs. 9-11 demonstrate the impact of buoyancy ratio parameter on $f'$, $\theta$ and $\phi$ for water based nanofluid. With increasing $Nr$ values, the velocity profile is strongly decreases in the boundary layer; we can see the tendency of velocity in Fig 9. However from Fig 10 and Fig 11, the opposite behavior is seen in profiles of temperature $\theta$ and nanoparticle concentration $\phi$.

Figures 12 to 14 display the influence of Forchheimer inertial parameter ($\Lambda$) on the flow variables $f'$, $\theta$ and $\phi$ in the boundary layer regime of nanofluid flow past a cylinder. Quadratic Forchheimer drag appear in equation (13) and is directly proportional to $\Lambda$. Form Fig. 12 it is evident that increasing $\Lambda$ clearly floods the nanofluid momentum development, decreasing the flow for some distance near the cylinder viscous region, later its reverse the trend and asymptotically reaches the far field flow velocity. Also, an increase $\Lambda$ it is found that enhancement in thermal and nanoparticle concentration.

Figs. 15 – 17 depict the hydrodynamics, heat transfer and nanofluid volume fraction behaviors past the cylinder for different values of $\xi$. Velocity is clearly slow down with increasing $\xi$ values (Fig. 15) for some distance. Conversely a large increase in ($\theta$) and ($\phi$) occurs with increasing $\xi$ values, shown in Figs 16 and 17. Temperature and nanofluid volume fraction are both enhanced.
Fig. 16. Impact of \( \xi \) on temperature distribution

Fig. 17. Impact of \( \xi \) on nanofluid volume fraction

Fig. 18. Behaviour of local skin friction coefficient for various Nb.

The influence of Nb and Nt on \( f'(\xi,0) \), \( -\theta'(\xi,0) \) and \( -\phi'(\xi,0) \) over cylinder surface are presented in Figs. 18–20 and Figs. 21–23 respectively. With an increasing influential nanofluid slip parameters Nb, Nt corresponding to increasingly developed contributions in \( f'(\xi,0) \), \( -\theta'(\xi,0) \) and \( -\phi'(\xi,0) \) are consistently enhanced i.e. the gradients of flows considerably accelerated along the cylinder surface. It is worth mention that the enhancement occurs due to nanoparticles in the boundary layer regime.

Fig. 19. Behaviour of local Nusselt number for different Nb

Fig. 20. Behaviour of \(-\phi'(\xi,0)\) results for different Nb

Fig. 21. Behaviour of local skin friction coefficient for different Nt
Figs. 22–26 depicts the distribution of \( f'(\xi,0) \), \(-\theta'(\xi,0)\) and \(-\phi'(\xi,0)\) along the cylinder periphery (\( \xi \) coordinate) for various values buoyancy ration parameter (\( Nr \)). For increasing \( Nr \), corresponding to lesser influences of flow gradient, wall shear stress is steadily condensed. With growing \( Nr \), local Nusselt number and local Sherwood number is considerably decreased and increased respectively.

6. Conclusions

In this paper, the numerical investigation of free convection of nanofluid flow past a horizontal circular cylinder embedded in a non-Darcy porous medium. Buongiorno-Forchheimer
model has been employed for nanofluid flow modeling in porous medium. The transformed nonlinear system is solved by using Keller’s box method. Furthermore, validation of current solutions has been done by comparing with the existing solution in the literature. The important outcomes can be concise as:

i. Velocity and temperature are increasing function of $Nb$ while nanoparticle volume fraction concentration is decreasing function of $Nb$.

ii. As increases thermophoresis heat transfer increased in the boundary layer and simultaneously intensifies particle deposition for from the fluid region, in that way increases nanoparticle volume fraction.

iii. With increasing buoyancy ratio parameter value, the velocity profile is strongly increases the boundary layer regime.

iv. It is evident that increasing $\Lambda$ clearly swamps the nanofluid momentum development, decreasing the flow for some distance near the cylinder viscous region, later its reverse the trend and asymptotically reaches the far field flow velocity.

v. With an increasing influential nanofluid slip parameters $Nb$, $Nt$ corresponding to increasingly developed contributions in $f'(\xi, 0)$, $-\theta'(\xi, 0)$ and $-\phi'(\xi, 0)$ i.e. the gradients of flows considerably accelerated along the cylinder surface.

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**References**


