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MHD CASSON FLUID FLOW THROUGH A VERTICAL PLATE

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Abstract

In this study, effects of numerous physical quantities like dissipation, thermal radiation, and induced magnetic field on magnetohydrodynamic Casson fluid flow through a vertical plate is addressed. The non-dimensional multivariable governing equations are solved numerically by by means of Runge- Kutta method along with shooting technique. The behavior of velocity, temperature and induced magnetic fields for different physical aspects is discussed through graphical illustrations. The influence of physical constants like Casson fluid (β), Magnetic parameter *M*, Soret number *Sc*, Prandtl number *Pr*, Magnetic Prandtl number etc., are analyzed on induced magnetic field, temperature and velocity. Interesting observation of this study is that, the effect of velocity distribution obeys the physical nature of well-known Newtonian and all other Non-Newtonian fluids.

1. Introduction

At the maximum temperature, the impact of thermal radiation weighty on the flow of viscous fluid field. These impacts are very important in many industrial areas, such as solar power technology, electrical power generation and aeronautical engineering. Several investigators are also investigated about in this field like, the impact of induced magnetic field on heat and mass transfer of the flow of stagnation-point towards the surface of the stretching sheet was done by Ali et al. [1]. Raptis and Massalas [2] and Aziz and Afify [3] studied the effect of induced magnetic field near a stagnation-point of Casson fluid flow across stretching surface with velocity slip. The effects of radiation parameter on the vertical porous plate conducting with viscous and incompressible fluid of an optically thin gray studied Raptis [4]. The numerical grades for the temperature and velocity distributions are exposed for dissimilar non-dimensional components under the physical quantities. The concept of convective heat transfer is major role in the behavior and processing of viscous and non-viscous fluid flows. In present the mechanics of non-viscous fluid flows a different task to mathematicians,

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physicists and engineers. The simplest subclass of the rate type on non-viscous fluids is known as Casson fluid. This rheological model was introduced originally by Casson [5] in his research the flow of equation for pigment oil-suspensions is taken as printing ink. Oka [18] was the introduced and developed the generalized non-viscous Casson fluid model as a special situation for the study of fluid flow characteristics in an elastic tube. Jayaraman et al. [19] have been premeditated the work of Oka's and suggested that the Casson fluid is more practically applicable for blood Oxygenators.

The non-viscous boundary layer fluid flow convoyed by the Nusselt number with the surface of stretching was deliberated by Pragmatic [6] and observed that the impact of growing the Casson parameter is realized to defeat the velocity. But the temperature is improved with growing Casson parameter. Radiation of thermal boosts the efficacy thermal diffusivity and rises the temperature. It is noticed that the velocity-friction coefficient rises by the growth in a suction parameter. MHD mixed Casson fluid in a vertical flat surface with chemical and porous medium has been studied Arthur et al. [7], using Fourth-order Runge-Kutta algorithm with Newton Raphson shooting method. Hayat et al. [8] and Mahanta and Shaw [9] studied the physical appearance of Casson fluid with heat transfer of different channels. Khalid et al. [10] studied the impact of oscillating and MHD on non-Newtonian fluid in vertical porous plate. Pal and Mandal [11] examined induced magnetic field impact on a nanofluid at a stagnation point flow for the case of non-isothermal surface of stretched sheet. Vajravelu et al. [17] studied the peristaltic transport of a non-Newtonian incompressible fluid in an elastic tube. Anki Reddy [12] studied the steady two-dimensional MHD Casson fluid flow over a convective boundary layer with an exponentially stretching surface with inclined permeable in the presence of thermal radiation and chemical reaction. The exponentially stretching sheet of MHD Casson fluid flow with permeable bed and heat transfer physical appearance was considered by Raju et al. [13]. The influence of slip conditions on non-viscous conducting fluid flow over a non-linearly stretching sheet with viscous heating in the porous medium is investigated Imran et al. [14]. It is observed that the enhances of slip parameters are boosts the velocity of Casson fluid. It is also noticed that the impact of slip is much effect on temperature distribution in assessment with velocity distribution. The impact of MHD on free convective incompressible non-viscous flow of vertical porous flat plate with heat source and slip has been studied Raju et al. [20]. The influence of radiation, heat generation, thermophoresis on MHD mixed convection Jeffrey fluid flow with inclined permeable moving plate has been elaborated by Raju et al. [21].

Motivated by aforementioned studies, the massive possible industrial applications, it is of paramount interest to consider the effect of induced magnetic field on Casson fluid flow of conducting past a vertical plate. Motivation of the current study is to inspect the simultaneous impacts of induced magnetic field, Casson (β) fluid on velocity, temperature and induced magnetic distributions over a vertical plate. Here we use numerical technique for obtaining the graphs of velocity, temperature and induced magnetic distributions.

2. Mathematical analysis

We consider the two-dimensional steady free convection flow of incompressible electrically conducting Casson fluid flow along an infinite vertical plate under the influence of magnetic field. The $y'-$axis is normal to upward direction plate of $x'-$axis. We assume that the applied magnetic field is perpendicular to the plate. The magnetic field is of the form $H' = (H'_{x'}, H_0, 0)$, here H_0 is known to the strength of transverse magnetic field. Assuming u', v' as the velocity apparatuses of parallel and its normal to the plate respectively. Based on the above-mentioned conditions, the rheological equations are given by (Bhattacharyya, [15]; Sharada and Shankar [16]).

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c, \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c, \end{cases}$$

For the present problem, the governing and boundary layer equations are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$v'\frac{\partial u'}{\partial y'} = g\beta_1(T' - T'_\infty) + \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_0}{\rho}H_0\frac{\partial H'_{x'}}{\partial y'} \tag{2}$$

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho Cp}\frac{\partial q_r}{\partial y'} + \frac{\nu}{\rho Cp}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\rho Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{3}$$

$$v'\frac{\partial H'_{x'}}{\partial y'} = H_0\frac{\partial^2 u'}{\partial y'^2} + \frac{1}{\sigma\,\mu_0}\frac{\partial^2 H'_{x'}}{\partial y'^2} \tag{4}$$

Where β, g, β_1, T', T'_∞, ν, μ_0, ρ, κ, Cp, q_r and σ are parameter of the Casson fluid, acceleration due to gravity, coefficient of volume expansion, fluid temperature, temperature of fluid at infinity, kinematic viscosity, magnetic permeability, fluid density, thermal conductivity, specific heat constant pressure, radioactive heat flux and electrical conductivity respectively.

$$\begin{aligned} &u' = 0, v' = -v_0, T' = T'_w, \frac{\partial H'_{x'}}{\partial y'} = 0 \text{ as } y' = 0 \\ &u' \to U_0,\ T' \to T'_\infty,\ H'_{x'} \to 0 \text{ as } y' \to \infty \end{aligned} \tag{5}$$

Here the suction velocity v_0 is assumed to be constant. The temperature at the wall is assumed to be T'_w, and the constant free stream velocity is considered here is U_0. From Eq. (1) it is evident that

$$v' = -v_0 \tag{6}$$

The expansion of local thermal radiation is

$$-\frac{\partial q_r}{\partial y'} = 4\,\alpha\,\sigma\,(T'^4_\infty - T'^4) \tag{7}$$

Where σ and α are Stefan-Boltzmann constant and the absorption coefficient. Reduce the Eq. (3) by using Eq. (7).

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\,Cp}\frac{\partial^2 T'}{\partial y'^2} + \frac{4a\,\sigma}{\rho\,Cp}(T'^4_\infty - T'^4) + \frac{\nu}{\rho\,Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\,\rho\,Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{8}$$

Now T'^4 can be expressed as

$$T'^4 \cong 4T'^3_\infty\,T' - 3T'^4_\infty \tag{9}$$

By using Eq. (9), Eq. (8) becomes

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\,Cp}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\,a\,\sigma\,T'^3_\infty}{\rho\,Cp}(T'_\infty - T') + \frac{\nu}{\rho\,Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\,\rho\,Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{10}$$

Using these transformations

$$y = \frac{y' v_0}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ Pm = \nu\sigma\mu_0, H = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}}\frac{H_{x'}}{U_0},$$

$$u = \frac{u'}{U_0},\ \Pr = \frac{\rho\,\nu\,Cp}{\kappa}, M = \left(\frac{\mu_0}{\rho}\right)^{1/2}\frac{H_0}{v_0},$$

$$G = \frac{\nu\ g\ \beta_1 (T'_w - T'_\infty)}{U_0 \nu^2{}_0},\ S = \frac{16\ a\ \sigma\ T'^3{}_\infty \nu^2}{\kappa\ \nu^2{}_0}$$

$$Ec = \frac{U^2{}_0}{Cp\,(T'_w - T'_\infty)} \tag{11}$$

Eqs (2-4) and Eq (10) reduces to

$$\left(1+\frac{1}{\beta}\right)\frac{d^2u}{dy^2} + \frac{du}{dy} - M\ \frac{dH}{dy} + G\ \theta = 0 \tag{12}$$

$$\frac{d^2H}{dy^2} + Pm\ \frac{dH}{dy} + M\ Pm\ \frac{du}{dy} = 0 \tag{13}$$

$$\frac{d^2\theta}{dy^2} + \Pr\ \frac{d\theta}{dy} - S\ \theta + \Pr\ Ec\left(1+\frac{1}{\beta}\right)\left(\frac{du}{dy}\right)^2 + \frac{\Pr\ Ec}{Pm}\left(\frac{dH}{dy}\right)^2 = 0 \tag{14}$$

The matching conditions at the boundary are

$$u = 0,\quad \frac{dH}{dy} = 0,\quad \theta = 1,\quad at\quad y = 0,$$

$$u \to 1,\ H \to 0,\ \theta \to 0,\ as\ y \to \infty, \tag{15}$$

3. Results and discussion

The numerical calculations are carried out for different values of the parameters of the problem to get the physical insight. Eqs. (12)–(14) solved numerically using the boundary condition Eq.(15) by the technique of shooting and the results are following:

Figs. 1-5, velocity distributions are plotted for viscous and non-viscous (Casson) fluids. This obeys the physical nature of Newtonian and Casson fluid flows. This figure establishes that the Casson fluid velocity decays with an growth of β, M, S, $\Pr$ and Pm, for fixed values of $\beta = 1$, $\mathrm{M} = 0.5$, $\mathrm{G} = 2$, $Pm = 0.06$, $\Pr = 0.71$, $S = 1$ and $Ec = 0.01$. On the other hand, Fig.6 displays the variations in Casson fluid velocity from which it is noticed that Casson fluid velocity increases with growth of Grashoof number G. The temperature distribution θ for numerous physical parameters $\Pr$ and S are exhibited in Figs.7 and 8. Temperature decreases with an rise of $\Pr$ and S. From Figs. 9 and 10, depicted the impact of Eckert number Ec and Grashoof number G on temperature θ for the other fixed parameters. It is found that, temperature increases with an increasing value of Ec and G.

Fig.11 shows that the impact of M on the induced magnetic field H is increased with decreasing of M at the value of $M(=-0.5)$. Figs.12 and 13 we observe that the impact of Grashoof number G and magnetic Prandtl number Pm, the induced magnetic field H is increased with an increasing of Grashoof number G and magnetic Prandtl number Pm at $M(=-0.5)$. Figs.14 and 15 the induced magnetic field is increased with decreasing of Prandtl number $\Pr$ and Soret number S, at $M(=-0.5)$ and the induced magnetic field increases with an increase of Prandtl number $\Pr$ and Soret number S, at $M(=0.5)$. Fig.16 the induced magnetic field H is increased with growing of Casson fluid parameter β, for $M(=-0.5)$ and is increased with decreasing of Casson fluid parameter β, for $M(=0.5)$. And also, we observe from the above figures, the induced magnetic field is obeying the physical nature of Newtonian and Casson fluid flows.

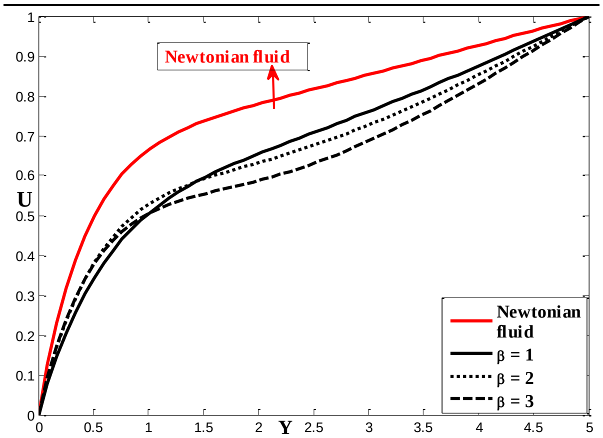

Fig. 1. Effect of **β** on velocity distribution

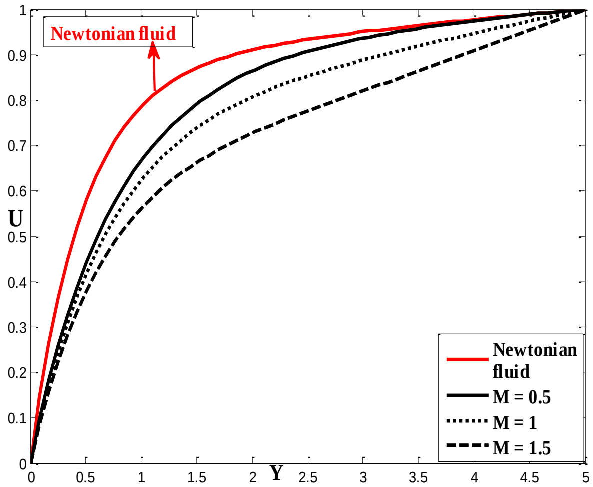

Fig. 2. Effect of Magnetic parameter M on velocity distribution.

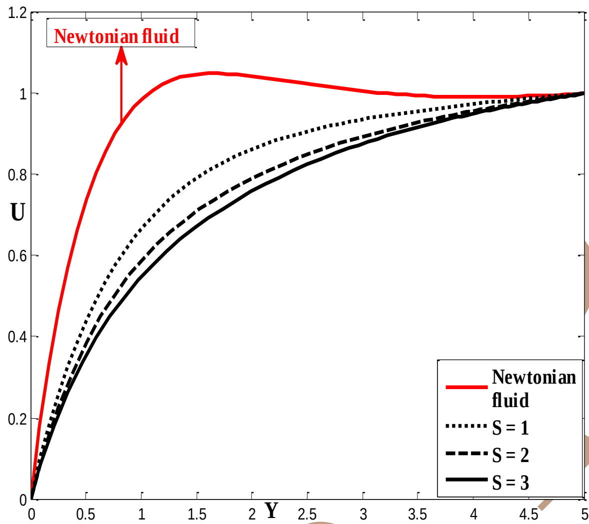

Fig. 3. The effect of Soret number S, on velocity *u*.

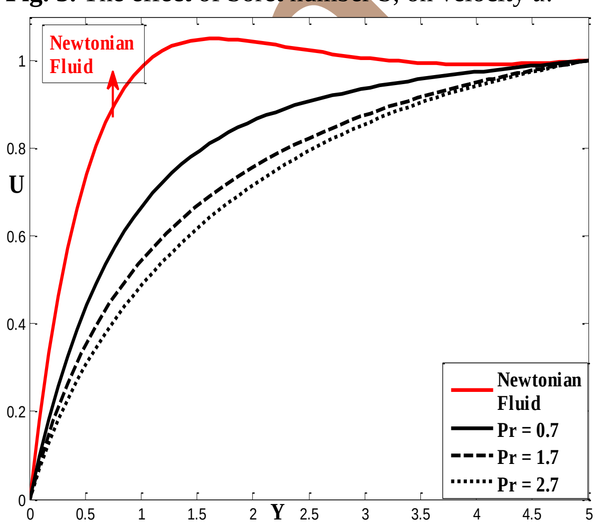

Fig. 4. The velocity distribution for different values of Prandtl number Pr.

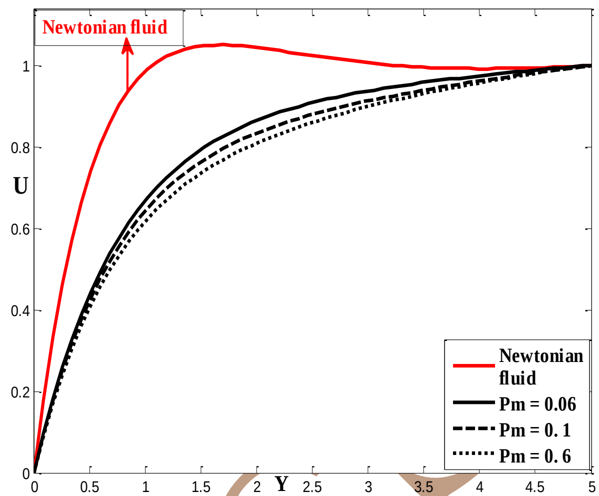

Fig. 5. Effect of Magnetic Prandtl number Pm on velocity.

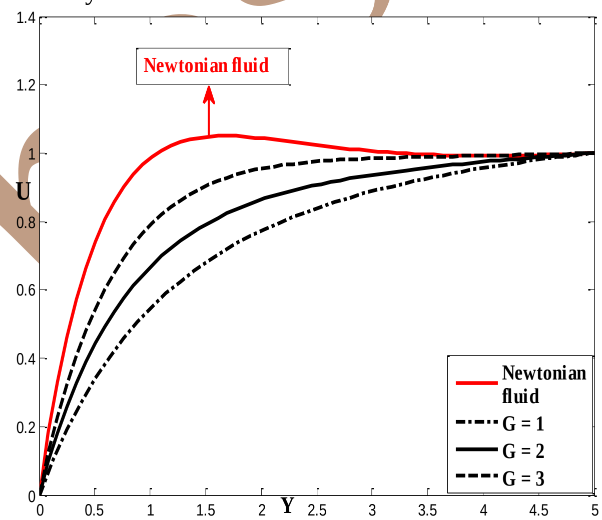

Fig. 6. The impact of Grashoof number G on velocity.

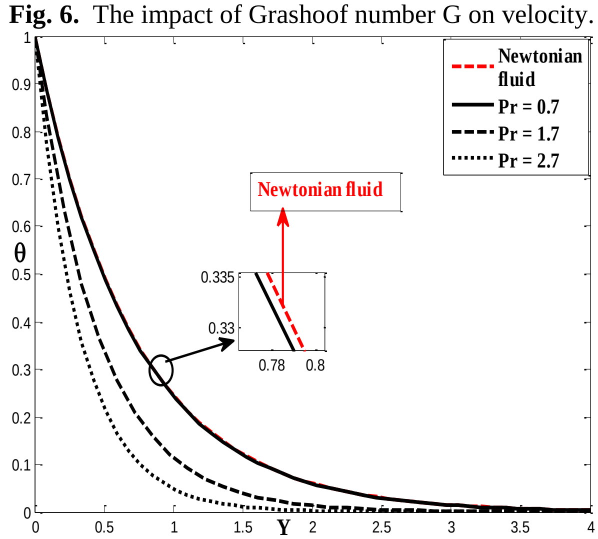

Fig. 7. The effect of Prandtl number Pr on temperature distribution.

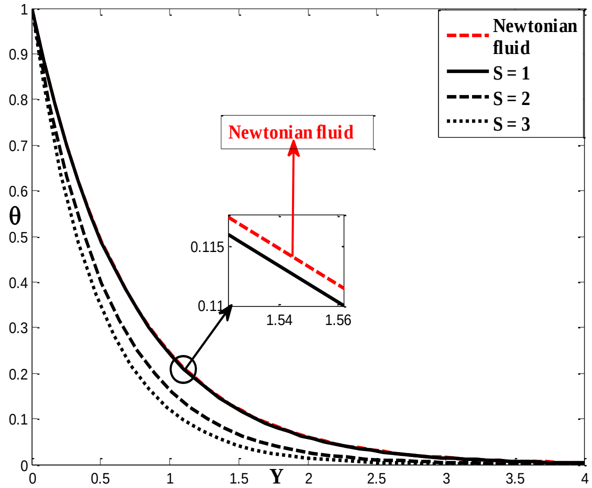

Fig. 8. Impact of Soret number S on temperature distribution.

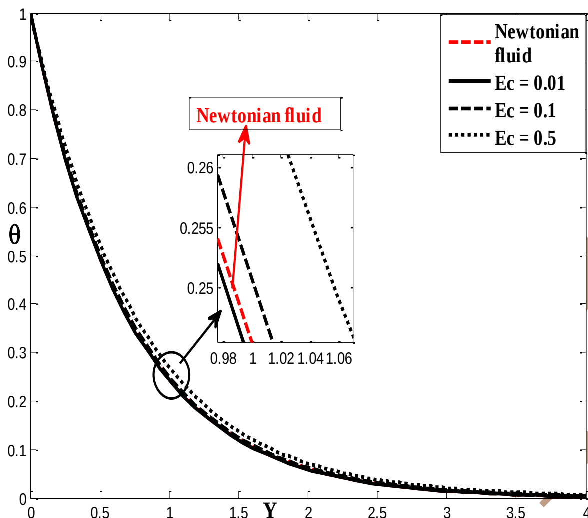

Fig. 9. Temperature distribution for various values of Ec .

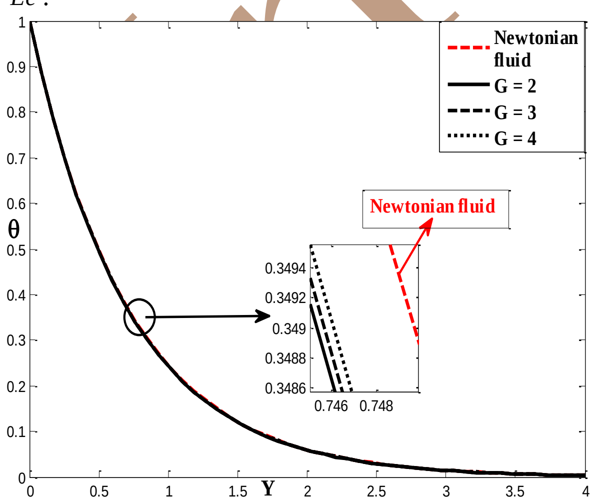

Fig. 10. Effect of Grashoof number G on temperature.

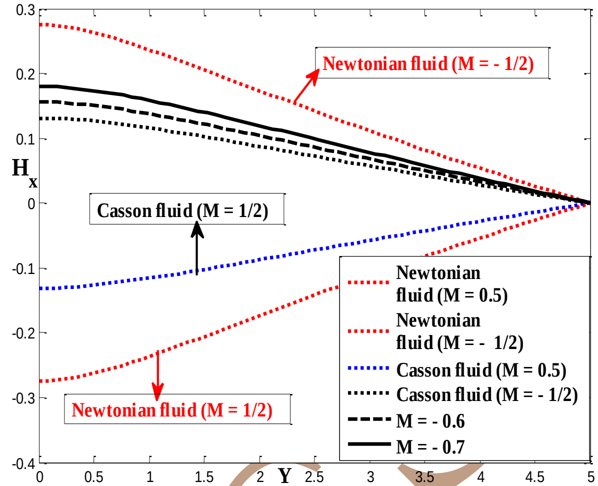

Fig. 11. The effect of M on profile H for different values of M.

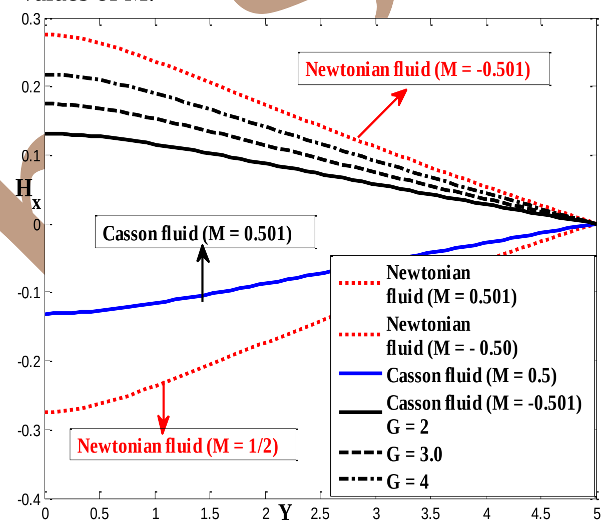

Fig. 12. Effect of Grashoof number G on induced magnetic field.

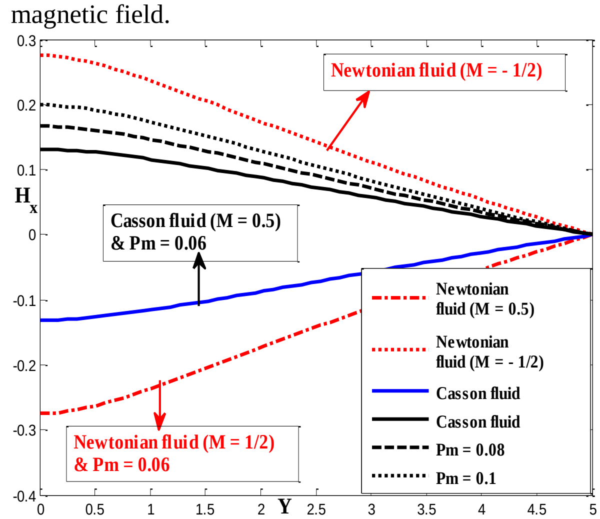

Fig. 13. The impact of various values of Pm on induced magnetic field distribution.

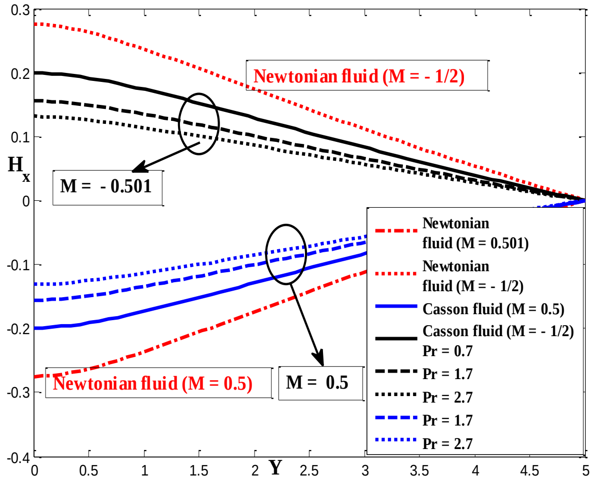

Fig. 14. The influence of numerous values of Pr on induced magnetic field.

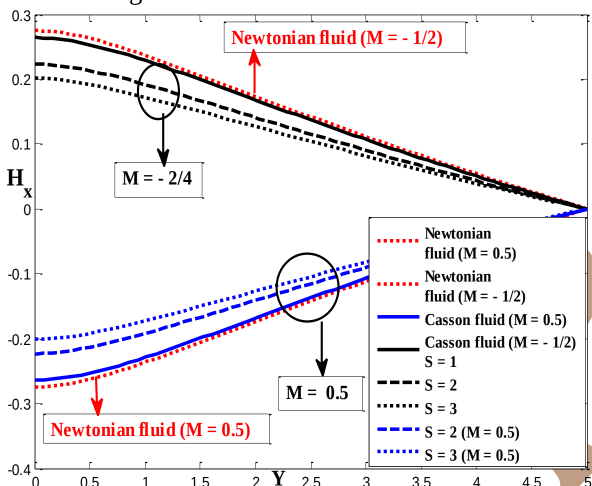

Fig. 15. The induced magnetic field profile for various values of Soret number S.

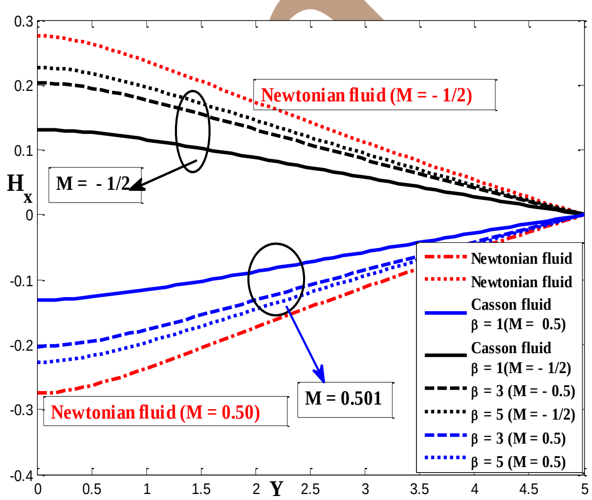

Fig. 16. The effect of Casson fluid parameter β on induced magnetic field.

4. Conclusion:

A numerical study is presented to investigate the impact of induced magnetic field on Casson fluid flow past a vertical plate. The governing equations are solved numerically using the Runge- Kutta method along shooting technique. The numerical results are obtained for a wide range of values of the physical parameters.

(i) The Casson fluid velocity decreases with an increase of β, M, S, Pr and Pm.

(ii) Temperature distribution decrease with an increases of the values of Pr and S.

(iii) The profile H is improved with decreasing of magnetic parameter M.

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