JCARME
ISSN: 2228-7922
Journal of Computational and Applied Research in Mechanical Engineering
Vol. 9, No. 2, pp. 343-350
jcarme.sru.ac.ir

MHD Casson fluid flow through a vertical plate

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Article info:

Type:	Research
Received:	20/08/2018
Revised:	29/06/2019
Accepted:	03/07/2019
Online:	09/07/2019

Keywords:
Casson fluid,
Induced magnetic field,
MHD,
Natural Convection,
Slip.

Abstract

In this study, effects of numerous physical quantities like dissipation, thermal radiation, and induced magnetic field on magnetohydrodynamic Casson fluid flow through a vertical plate is addressed. The non-dimensional multivariable governing equations are solved numerically by by means of Runge-Kutta method along with shooting technique. The behavior of velocity, temperature and induced magnetic fields for different physical aspects is discussed through graphical illustrations. The influence of physical constants like Casson fluid (β), Magnetic parameter *M*, Soret number *Sc*, Prandtl number *Pr*, Magnetic Prandtl number etc., on induced magnetic field, temperature and velocity is analyzed. Interesting observation of this study is that the effect of velocity distribution obeys the physical nature of well-known Newtonian and all other Non-Newtonian fluids.

1. Introduction

At the maximum temperature, the impact of thermal radiation is weighty on the flow of viscous fluid field. These impacts are very important in many industrial areas, such as solar power technology, electrical power generation and aeronautical engineering. Several investigators have also investigated about this field. The impact of induced magnetic field on heat and mass transfer of the flow of stagnation-point towards the surface of the stretching sheet was done by Ali et al. [1]. Raptis and Massalas [2] and Aziz and Afify [3] studied the effect of induced magnetic field near a stagnation-point of Casson fluid flow across stretching surface with velocity slip. The effects of radiation parameter on the vertical porous plate conducting with viscous and incompressible fluid of an optically thin gray were studied by Raptis [4]. The numerical grades for the temperature and velocity distributions are exposed for dissimilar non-dimensional components under the physical quantities. The concept of convective heat transfer has a major role in the behavior and processing of viscous and non-viscous fluid flows. In the present, the mechanics of non-viscous fluid flows performs a different task to mathematicians, physicists and engineers. The simplest subclass of the rate type on non-viscous fluids is known as Casson fluid. This rheological model was introduced originally by Casson [5] in his research, where

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the flow of equation for pigment oil-suspensions is taken as printing ink.
The non-viscous boundary layer fluid flow convoyed by the Nusselt number with the surface of stretching was deliberated by Pragmatic [6]; and it was observed that the impact of growing the Casson parameter is realized to defeat the velocity. But the temperature is improved with growing Casson parameter. The radiation of thermal boosts the efficacy thermal diffusivity and rises the temperature. It is noticed that the velocity-friction coefficient rises by the growth in a suction parameter. MHD mixed Casson fluid in a vertical flat surface with chemical and porous medium has been studied by Arthur et al. [7], using Fourth-order Runge-Kutta algorithm with Newton Raphson shooting method. Hayat et al. [8] and Mahanta and Shaw [9] studied the physical appearance of Casson fluid with heat transfer of different channels. Khalid et al. [10] studied the impact of oscillating and MHD on non-Newtonian fluid in vertical porous plate. Pal and Mandal [11] examined the impact of an induced magnetic field on a nanofluid at a stagnation point flow for the case of non-isothermal surface of stretched sheet. Anki Reddy [12] studied the steady two-dimensional MHD Casson fluid flow over a convective boundary layer with an exponentially stretching surface with inclined permeable in the presence of thermal radiation and chemical reaction. The exponentially stretching sheet of MHD Casson fluid flow with permeable bed and heat transfer physical appearance was considered by Raju et al. [13]. The influence of slip conditions on non-viscous conducting fluid flow over a non-linearly stretching sheet with viscous heating in the porous medium was investigated by Imran et al. [14]. It is observed that the enhancement of slip parameters boosts the velocity of Casson fluid. It is also noticed that the impact of slip is much effective on temperature distribution in assessment with velocity distribution. MHD stagnation point flow of Casson fluid is addressed by Bhattacharya [15]. Sharada and Shankar [16] also addressed the mixed convection flow of a Casson fluid over an exponentially stretching surface. Peristaltic pumping of a Casson fluid in an elastic tube was addressed by Vajravelu et al. [17]. Oka [18] introduced and developed the generalized non-viscous Casson fluid model as a special situation for the study of fluid flow characteristics in an elastic tube. Jayaraman et al. [19] have been premeditated the work of Oka's and suggested that the Casson fluid is more practically applicable for blood Oxygenators. The impact of MHD on free convective incompressible non-viscous flow of vertical porous flat plate with heat source and slip has been studied Raju et al. [20]. The influence of radiation, heat generation, and thermophoresis on MHD mixed convection Jeffrey fluid flow with inclined permeable moving plate has been elaborated by Raju et al. [21].
Motivated by the aforementioned studies, the massive possible industrial applications, it is of paramount interest to consider the effect of induced magnetic field on Casson fluid flow of conducting past a vertical plate. Motivation of the current study is to inspect the simultaneous impacts of induced magnetic field, Casson (β) fluid on velocity, temperature and induced magnetic distributions over a vertical plate. Here we use numerical techniques for obtaining the graphs of velocity, temperature and induced magnetic distributions.

2. Mathematical analysis

We consider the two-dimensional steady free convection flow of incompressible electrically conducting Casson fluid flow along an infinite vertical plate under the influence of magnetic field. The $y'-$axis is normal to upward direction plate of $x'-$axis. We assume that the applied magnetic field is perpendicular to the plate. The magnetic field is of the form $H' = (H'_{x'}, H_0, 0)$, here H_0 is known to the strength of transverse magnetic field. u', v' are assumed as the velocity apparatuses of parallel and its normal to the plate respectively. Based on the above-mentioned conditions, the rheological equations are given by (Bhattacharyya, [15]; Sharada and Shankar [16]).

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij,} & \pi > \pi_c, \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij,} & \pi < \pi_c, \end{cases}$$

For the present problem, the governing and boundary layer equations are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$v'\frac{\partial u'}{\partial y'} = g\beta_1(T' - T'_\infty) + \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_0}{\rho}H_0\frac{\partial H'_{x'}}{\partial y'} \tag{2}$$

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho Cp}\frac{\partial q_r}{\partial y'} + \frac{\nu}{\rho Cp}\left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma \rho Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{3}$$

$$v'\frac{\partial H'_{x'}}{\partial y'} = H_0\frac{\partial^2 u'}{\partial y'^2} + \frac{1}{\sigma\ \mu_0}\frac{\partial^2 H'_{x'}}{\partial y'^2} \tag{4}$$

Where β, g, β_1, T', T'_∞, ν, μ_0, ρ, κ, Cp, q_r and σ are parameter of the Casson fluid, acceleration due to gravity, coefficient of volume expansion, fluid temperature, temperature of fluid at infinity, kinematic viscosity, magnetic permeability, fluid density, thermal conductivity, specific heat constant pressure, radioactive heat flux and electrical conductivity respectively.

$$\begin{aligned} &u' = 0, v' = -v_0, T' = T'_w, \frac{\partial H'_{x'}}{\partial y'} = 0 \text{ as } y' = 0 \\ &u' \to U_0,\ T' \to T'_\infty,\ H'_{x'} \to 0 \text{ as } y' \to \infty \end{aligned} \tag{5}$$

Here the suction velocity v_0 is assumed to be constant. The temperature at the wall is assumed to be T'_w, and the constant free stream velocity considered here is U_0. From Eq. (1) it is evident that

$$v' = -v_0 \tag{6}$$

The expansion of local thermal radiation is

$$-\frac{\partial q_r}{\partial y'} = 4\ \alpha\ \sigma\ (T'^4_\infty - T'^4) \tag{7}$$

Where σ and α are Stefan-Boltzmann constant and the absorption coefficient. Reduce Eq. (3) by using Eq. (7).

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\ Cp}\frac{\partial^2 T'}{\partial y'^2} + \frac{4a\ \sigma}{\rho\ Cp}(T'^4_\infty - T'^4) + \frac{\nu}{\rho\ Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\ \rho\ Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{8}$$

Now T'^4 can be expressed as

$$T'^4 \cong 4T'^3_\infty\ T' - 3T'^4_\infty \tag{9}$$

By using Eq. (9), Eq. (8) becomes

$$v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho\ Cp}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\ a\ \sigma\ T'^3_\infty}{\rho\ Cp}(T'_\infty - T') + \frac{\nu}{\rho\ Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{1}{\sigma\ \rho\ Cp}\left(\frac{\partial H'_{x'}}{\partial y'}\right)^2 \tag{10}$$

Using these transformations

$$\begin{aligned} &y = \frac{y' v_0}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ Pm = \nu\sigma\mu_0, \\ &H = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}}\frac{H_{x'}}{U_0},\ u = \frac{u'}{U_0}, \\ &\Pr = \frac{\rho\nu Cp}{\kappa}, M = \left(\frac{\mu_0}{\rho}\right)^{1/2}\frac{H_0}{v_0}, \\ &G = \frac{\nu\ g\ \beta_1(T'_w - T'_\infty)}{U_0 v^2{}_0},\ S = \frac{16\ a\ \sigma\ T'^3{}_\infty v^2}{\kappa\ v^2{}_0} \\ &Ec = \frac{U^2{}_0}{Cp(T'_w - T'_\infty)} \end{aligned} \tag{11}$$

Eqs. (2-4) and Eq. (10) reduces to

$$\left(1 + \frac{1}{\beta}\right)\frac{d^2u}{dy^2} + \frac{du}{dy} - M\ \frac{dH}{dy} + G\ \theta = 0 \tag{12}$$

$$\frac{d^2H}{dy^2}+Pm\frac{dH}{dy}+M\ Pm\frac{du}{dy}=0 \tag{13}$$

$$\frac{d^2\theta}{dy^2}+\Pr\frac{d\theta}{dy}-S\ \theta+\Pr\ Ec\left(1+\frac{1}{\beta}\right)\left(\frac{du}{dy}\right)^2 + \frac{\Pr\ Ec}{Pm}\left(\frac{dH}{dy}\right)^2=0 \tag{14}$$

The matching conditions at the boundary are

$$u=0,\quad \frac{dH}{dy}=0,\quad \theta=1,\quad at\quad y=0,$$
$$u\to 1,\ H\to 0,\ \theta\to 0,\ as\ y\to\infty, \tag{15}$$

3. Results and discussion

The numerical calculations are carried out for different values of the parameters of the problem to get the physical insight. Eqs. (12-14) were solved numerically using the boundary condition Eq. (15) by the technique of shooting and the results are as follows:

Figs. 1-5, velocity distributions are plotted for viscous and non-viscous (Casson) fluids. This obeys the physical nature of Newtonian and Casson fluid flows. This figure establishes that the Casson fluid velocity decays with growth of β, M, S, $\Pr$ and Pm, for fixed values of $\beta=1$, $M=0.5$, $G=2$, $Pm=0.06$, $\Pr=0.71$, $S=1$ and $Ec=0.01$. On the other hand, Fig.6 displays the variations in Casson fluid velocity from which it is noticed that Casson fluid velocity increases with the growth of Grashoof number G. The temperature distribution θ for numerous physical parameters $\Pr$ and S are exhibited in Figs. 7 and 8. Temperature decreases with rise of $\Pr$ and S. Figs. 9 and 10 depicted the impact of Eckert number Ec and Grashoof number G on temperature θ for other fixed parameters. It is found that, temperature increases with an increase in the value of Ec and G.

Fig.11 shows that the impact of M on the induced magnetic field H is increased with decreasing of M at the value of $M(=-0.5)$. In Figs.12 and 13 we observe that the impact of Grashoof number G and magnetic Prandtl number Pm, the induced magnetic field H is increased with an increase in Grashoof number G and magnetic Prandtl number Pm at $M(=-0.5)$. In Figs. 14 and 15 the induced magnetic field is increased with decreasing Prandtl number $\Pr$ and Soret number S, at $M(=-0.5)$ and the induced magnetic field increases with an increase in Prandtl number $\Pr$ and Soret number S, at $M(=0.5)$. In Fig. 16 the induced magnetic field H is increased with growing of Casson fluid parameter β, for $M(=-0.5)$ and is increased with decreasing Casson fluid parameter β, for $M(=0.5)$. And also, we observe from the above figures that the induced magnetic field is obeying the physical nature of Newtonian and Casson fluid flows.

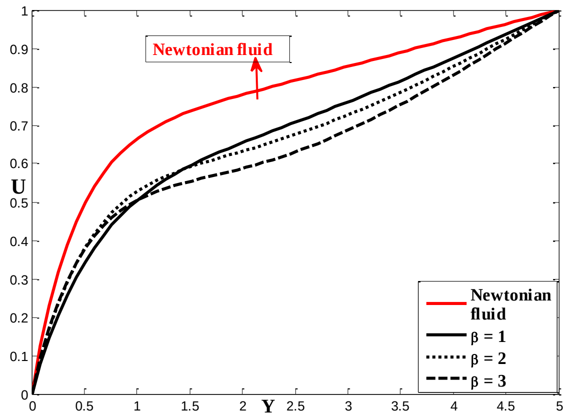

Fig. 1. Effect of **β** on velocity distribution.

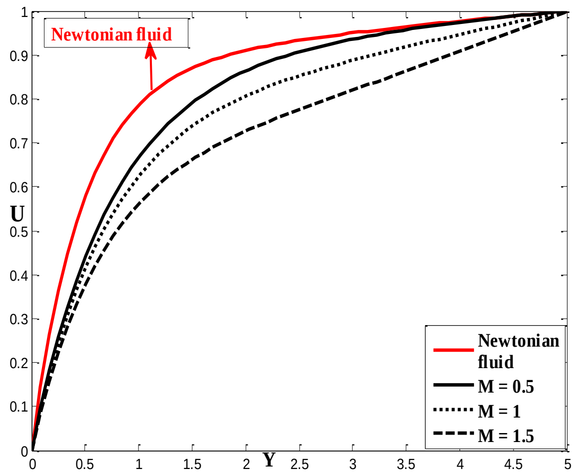

Fig. 2. Effect of Magnetic parameter M on velocity distribution.

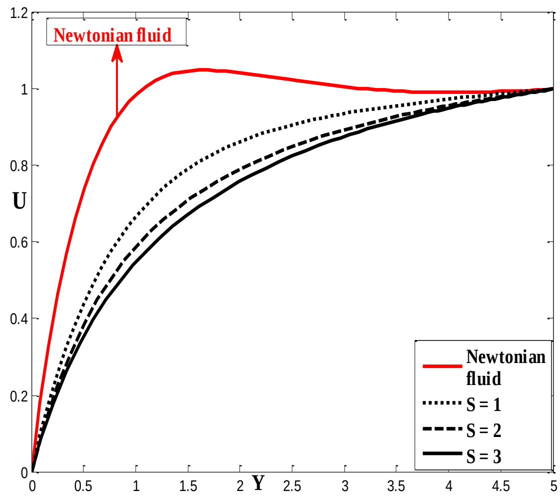

Fig. 3. The effect of Soret number S, on velocity *u*.

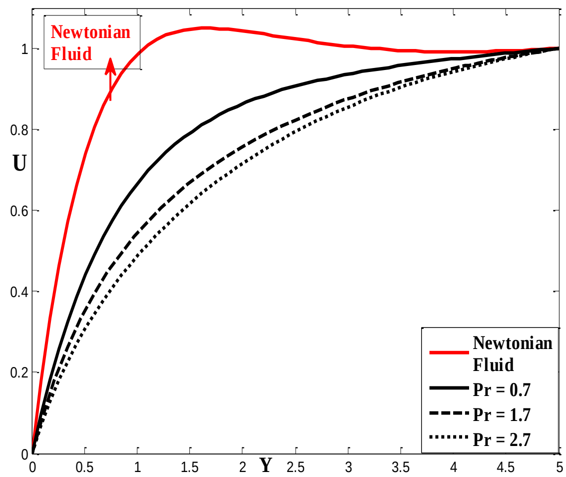

Fig. 4. The velocity distribution for different values of Prandtl number Pr.

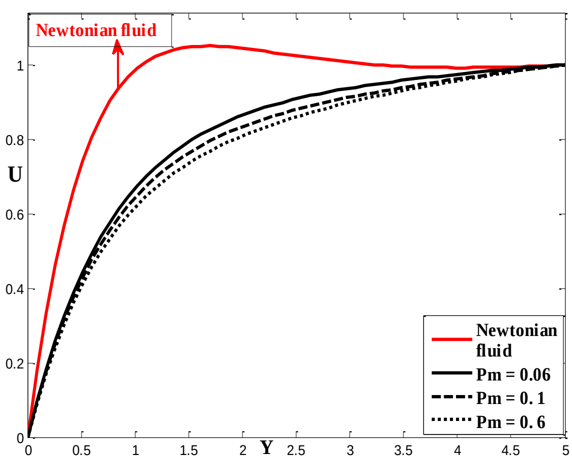

Fig. 5. Effect of Magnetic Prandtal number *Pm* on velocity.

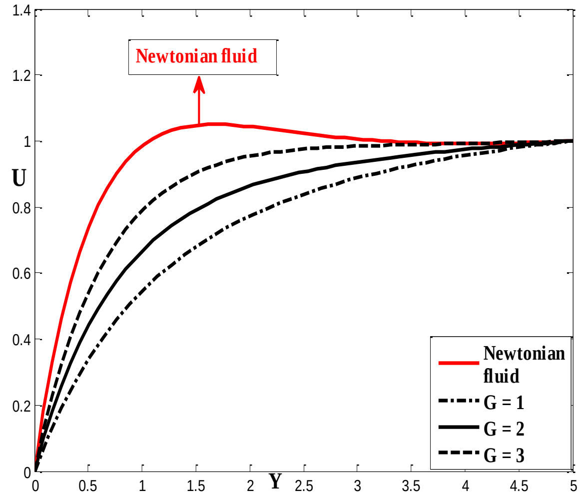

Fig. 6. The impact of Grashoof number G on velocity.

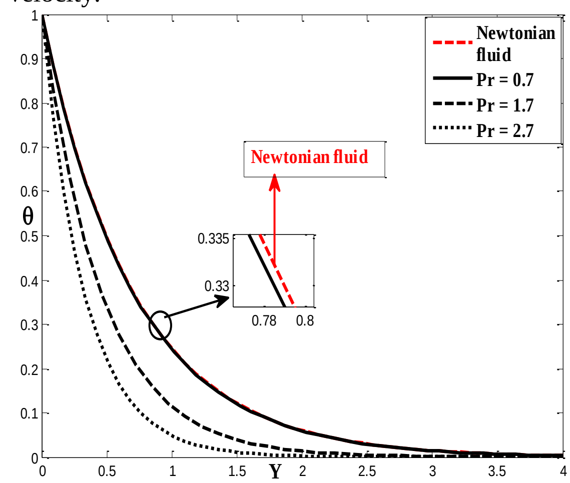

Fig. 7. The effect of Prandtl number Pr on temperature distribution.

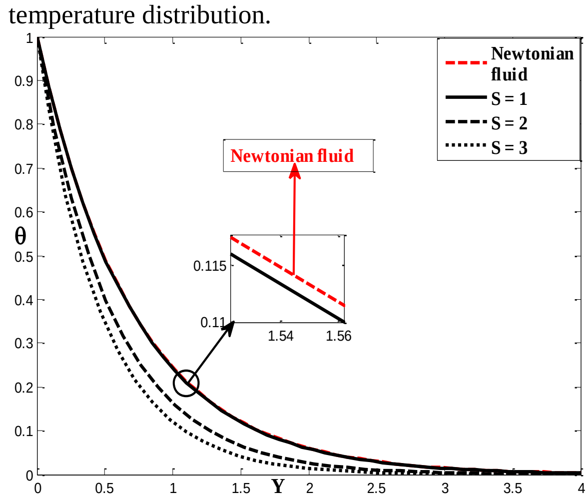

Fig. 8. Impact of Soret number *S* on temperature distribution.

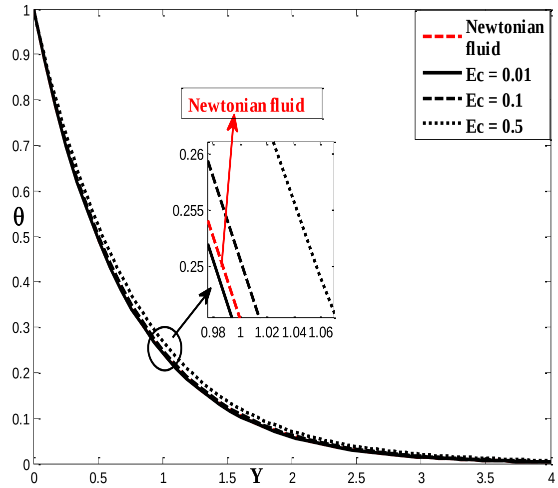

Fig. 9. Temperature distribution for various values of Ec.

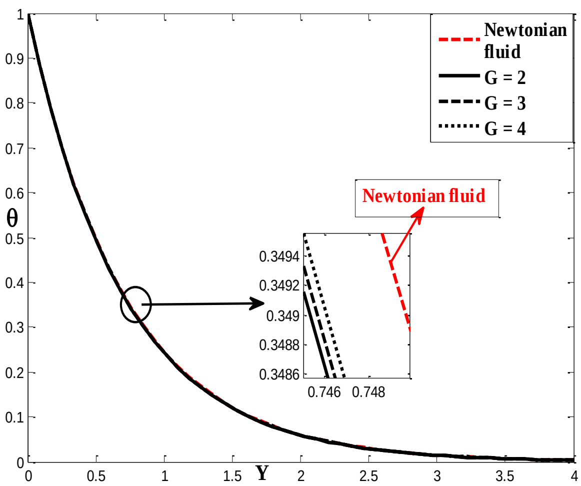

Fig. 10. Effect of Grashoof number G on temperature.

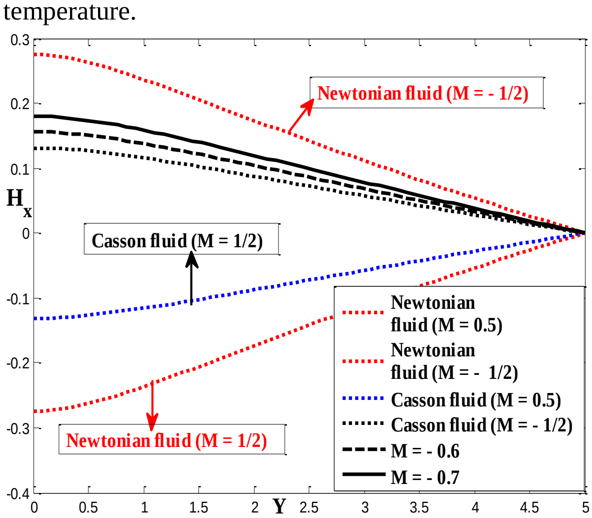

Fig. 11. The effect of M on profile H for different values of M.

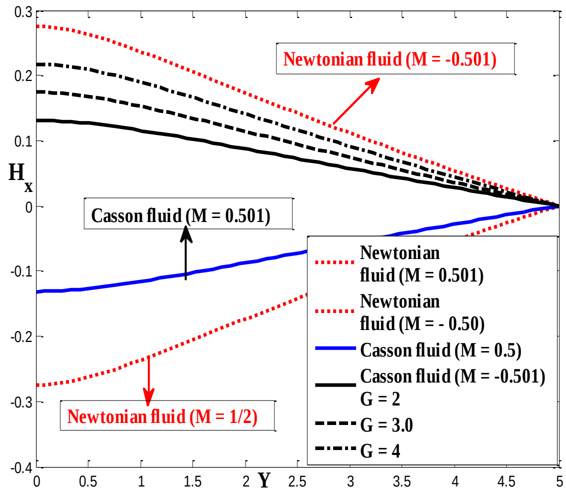

Fig. 12. Effect of Grashoof number G on induced magnetic field.

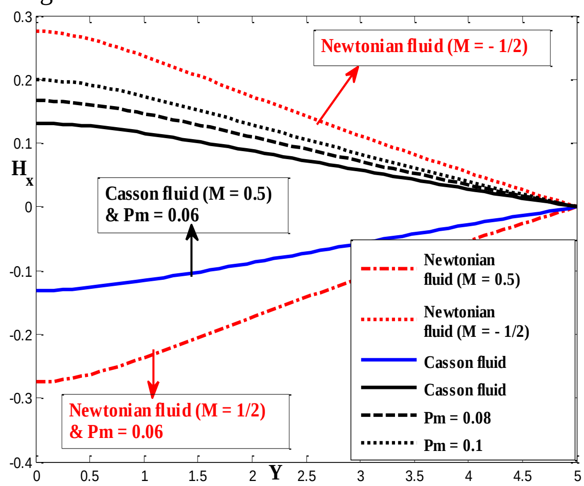

Fig. 13. The impact of various values of Pm on induced magnetic field distribution.

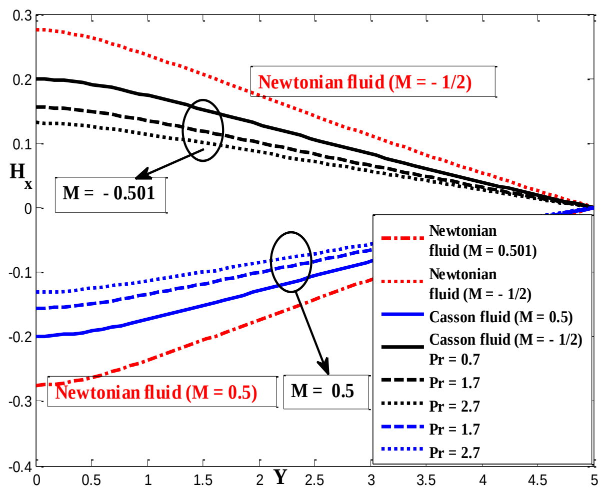

Fig. 14. The influence of numerous values of Pr on induced magnetic field.

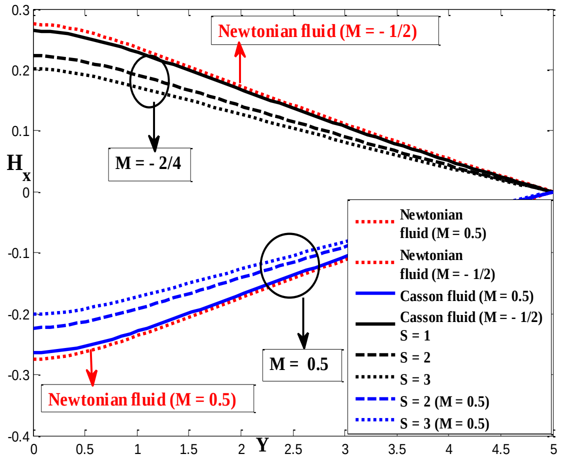

Fig. 15. The induced magnetic field profile for various values of Soret number S.

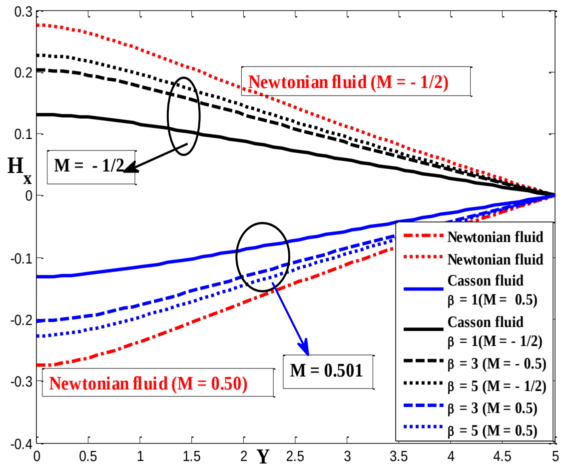

Fig. 16. The effect of Casson fluid parameter β on induced magnetic field.

4. Conclusions

A numerical study is presented to investigate the impact of induced magnetic field on Casson fluid flow past a vertical plate. The governing equations are solved numerically using the Runge- Kutta method along shooting technique. The numerical results are obtained for a wide range of values of the physical parameters.

(i) The Casson fluid velocity decreases with an increase of β, M, S, Pr and Pm.

(ii) Temperature distribution decreases with an Increase in the values of Pr and S.

(iii) The profile H is improved with decreasing of magnetic parameter M.

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How to cite this paper:

A. Parandhama, K. V. S. Raju and M. Changal Raju, "MHD Casson fluid flow through a vertical plate", *Journal of Computational and Applied Research in Mechanical Engineering*, Vol. 9, No. 2, pp. 343-350, (2019).

DOI: 10.22061/jcarme.2019.4015.1476

URL: http://jcarme.sru.ac.ir/?_action=showPDF&article=1101