



Analytical study of induced magnetic field and heat source on chemically radiative MHD convective flow from a vertical surface

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Abstract

Due to their position in various industrial applications, convective fluid flow structure is intricate and enticing to investigate. Here the flow has been made by considering multitudinous apropos parameters like induced magnetic factor, heat source and viscous dissipation effects for the mixed convective chemically radiative fluid from a vertical surface. The frame work of mathematical pattern is conferred with in the circumstances of a system of ordinary differential equations under felicitous legislation. The governed mathematical statement is handled analytically by perturbation strategy. Diagrams and numerical values of the profiles are delineated with apropos parameters. Our sketches illustrate that the induced magnetic field is perceived to be downward with intensification in magnetic parameter. Temperature profile is accelerated by rising thermal radiation and concentration distribution is decelerated by enhancing the chemical reaction and Schmidt number.

Nomenclature

B	Magnetic field ($N\ m^{-1}\ A^{-1}$)	Ec	Eckert number ($W\ m^{-2}$)
C	Concentration of the fluid ($Kg\ m^{-1}$)	Gr	Thermal Grashof number
C_w	Fluid concentration at the surface	Gm	Modified Grashof number
C_∞	Ambient fluid concentration	g	Acceleration due to gravity ($m\ s^{-2}$)
C_p	Specific heat at constant pressure ($J\ K^{-1}\ kg^{-1}$)	H	Induced magnetic factor
D	Mass diffusivity	H_0	Uniform magnetic factor
		H_x	Induced magnetic field along x -axis
		J	Current density

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k_0	Dimensional chemical reaction
k	Thermal conductivity
Kr	Chemical reaction parameter
M	Magnetic interaction parameter
Nu	Local Nusselt number
P_e	Electron pressure
Pm	Magnetic Prandtl number
Pr	Prandtl number
Q	Heat source parameter
Q^*	Heat source coefficient
R	Radiation parameter
q_r	Radiative heat flux
Sc	Schmidt number
Sh	Sherwood number
T_w	Fluid temperature at the surface
T_∞	Ambient fluid temperature
T	Temperature
u, v	Velocity components in x y directions ($m\ s^{-1}$)
U	Dimensionless free stream velocity ($m\ s^{-1}$)
v_0	Suction velocity
Greek symbols	
β	Thermal coefficient expansion
β^*	Concentration coefficient expansion
μ_0	Magnetic diffusivity
ν	Kinematic viscosity ($m^2\ s^{-1}$)
ρ	Fluid density ($Kg\ m^{-3}$)
σ	Electrical conductivity ($S\ m^{-1}$)
σ^*	Stefan-Boltzman constant
ϕ	Dimensionless concentration
k^*	Mean absorption coefficient

Subscripts

w	Condition at the wall
∞	Free stream conditions, primes denote dimensional quantities

1. Introduction

Technological utility and academic inquisitiveness have provided prodigious attention to the exposure of convective flows with heat and mass transfer from the current researchers. The research community has received spectacular absorption of convective flows with simultaneous heat and mass transfer in view of their role in abounding slots of science and technology such as chemical industry, chemical industry, cooling and heating of buildings, condensers and cooling of nuclear reactors, etc. Choudhary and Sharma [1] analyzed the flow dynamics on cooled and heated plates and found very low current density for cooled plate and very high for heated plate. Hady et al. [2] showed local skin friction coefficient ascension with variable viscosity. Numerical simulation of laminar flow over a wedge was conveyed by Gebhart and Pera [3]. Sharma and Chaudhary [4] illustrated the essence of mixed convective Newtonian fluid over cooled and heated plate through porous medium.

Being multifaceted and having an ample scope of application in industry, MHD problems with heat and mass transfer have captured great attention by the current research community. Recently the repercussions of heat transfer on electrically conducting fluid flow past a vertical either infinite or semi-infinite was studied by [5-7]. Magnetic reciprocation is a primordial episode that arises in materials when the magnetic field is set up on magnetic-fluid. The ramification of magnetic fluid in peristaltic transport among coaxial pumps was examined by Rathod and Asha [8]. Considering these numerous applications, flow through porous medium in electrical field have been explored by Raptis and Kafoussias [9], Raptis [10], Sattar and Hossain [11], Sattar [12] etc. In free convective flow from a vertical surface, Kumar and Singh [13] investigated the impact of induced magnetic factor. The detection of

peculiar features of induced magnetic field from a vertical surface was done by Chaudhary et al. [14]. Pandit and Sarma [15] and Alam et al. [16] analyzed impact of induced magnetic factor on the stream of mixed convective flow because of vertical porous plat.

High-tech advancement intensified heat generation or absorption in multifarious utilizations like in chemical reactor design, thermal power plant, dissociating fluids and manufacturing etc, entailing efficacious coolants for pertained heat dissipation. Ibrahim and Suneetha [17] and Ahmed and Sengupta [18] contemplated radiation absorbing kuvshiinshiki fluid stream in porous medium. They observed decay in temperature subjected to larger radiation absorption parameter. Sharma et al. [19, 20] discussed the radiation effect when MHD mixed convection stream is considered.

The research process in geophysical system and chemical engineering industry has given a way to concentrate on transport activity in porous materials which became a substantial field in heat transfer. Due to its prodigious applicability in science and technology such as drying technologies, oil innovations, framing and structural designing, etc, the analysts are engaged with porous media to break down the liquid flow and transport procedure through it. Stangle and Aksay [21] carried out an excellent theoretical work on blinder removal process by taking disordered porous materials. The stream of viscous flow because of exponentially accelerated isothermal sheet with chemical reaction was studied by Muthucumaraswamy et al. [22]. After such appreciable awareness, many authors [23-26] carried out research on this issue.

Propelled by the precursory research, the intrusion here is to scrutinize the repercussion of induced magnetic factor on viscous stream because of vertical surface.

2. Formulation of the problem

A 2D mixed laminar electrically incompressible viscous liquid past an electrically non-conducting moving infinite vertical porous plate (see Fig. 1).

For this problem, the boundary layer expressions (by Boussinesq's approximation) are

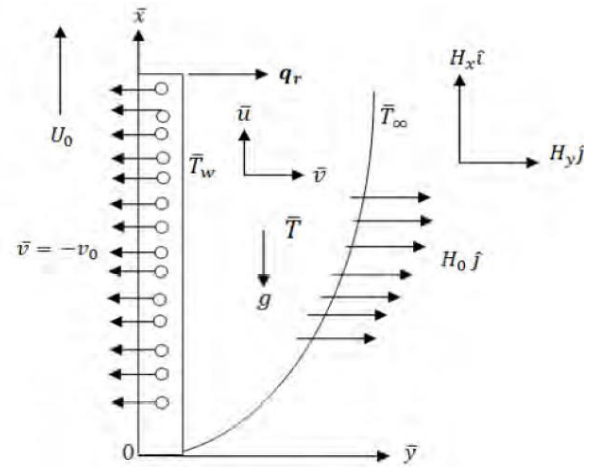


Fig. 1. Physical configuration and coordinate system.

$$-v_0 \frac{du}{dy} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + v \frac{d^2u}{dy^2} \quad (1)$$

$$+ \frac{\mu_e H_0}{\rho} \frac{dH_x}{dy} \quad (1)$$

$$-v_0 \frac{dH_x}{dy} = H_0 \frac{du}{dy} + \frac{1}{\sigma\mu_e} \frac{d^2H_x}{dy^2} \quad (2)$$

$$-v_0 \frac{dT}{dy} = \frac{\kappa}{\rho C_p} \frac{d^2T}{dy^2} - \frac{1}{\rho C_p} \frac{dq_r}{dy} + \frac{v}{C_p} \left(\frac{du}{dy} \right)^2 \quad (3)$$

$$+ \frac{1}{\sigma\rho C_p} \left(\frac{dH_x}{dy} \right)^2 - \frac{Q^*}{\rho C_p} (T_\infty - T)$$

$$-v_0 \frac{dC}{dy} = D \frac{d^2C}{dy^2} - Kr'C \quad (4)$$

The boundary restrictions are

$$\left. \begin{aligned} u = U, \frac{dT}{dy} = -\frac{Q_0}{\kappa}, \frac{dC}{dy} = -\frac{m}{D}, H_x = H_w \text{ at } y=0 \\ u = 0, T \rightarrow T_\infty, C \rightarrow C_\infty, H_x \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (5)$$

For optically thin gray gas, the local radiant is expressed by

$$\frac{\partial q_r}{\partial y} = -4a\sigma(T_\infty^4 - T^4) \quad (6)$$

By Taylor's expansion we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

Acknowledging a self-similar solution of the form

$$\left. \begin{aligned} u^* &= \frac{u}{\nu_0}, u^* = \frac{u}{\nu_0}, C^* = \frac{(C - C_\infty)\nu_0 D}{mv}, \\ \kappa^* &= \frac{\kappa \nu_0^2}{\nu^2}, \theta^* = \frac{(T - T_\infty)\kappa \nu_0}{Q\nu}, Sc = \frac{\nu}{D}, \\ Pr &= \frac{\mu C_p}{\kappa}, P_m = \sigma \nu \mu_e, Gr = \frac{g \beta Q \nu^3}{\kappa \nu_0^4}, \\ Ec &= \frac{\kappa \nu_0^3}{Q \nu C_p}, Gm = \frac{g \beta^* \nu (C_w - C_\infty)}{\nu_0^3}, \\ H &= \frac{H_x}{\nu_0} \sqrt{\frac{\mu_e}{\rho}}, R = \frac{64 a \sigma T_\infty^3}{\rho \nu C_p}, M = \frac{H_0}{\nu_0} \sqrt{\frac{\mu_e}{\rho}}, \\ Q &= \frac{Q^*}{\nu(T_\infty - T_0)}. \end{aligned} \right\} \quad (8)$$

Governing Eqs.(1-4) reduce to the form

$$\frac{d^2 u}{dy^2} + M \frac{dH}{dy} + \frac{du}{dy} = -Gr\theta - GmC \quad (9)$$

$$\frac{1}{Pm} \frac{d^2 H}{dy^2} + \frac{dH}{dy} + M \frac{du}{dy} = 0 \quad (10)$$

$$\frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + \frac{Pr R}{4} = -Pr Ec \left[\left(\frac{du}{dy} \right)^2 + \frac{1}{Pm} \left(\frac{dH}{dy} \right)^2 \right] + Q\theta \quad (11)$$

$$\frac{d^2 C}{dy^2} + Sc \frac{dC}{dy} - Kr Sc C = 0 \quad (12)$$

Boundary conditions are:

$$\left. \begin{aligned} u = U, \frac{d\theta}{dy} = -1, \frac{dC}{dy} = -1, H = h(\text{say}) \text{ at } y = 0 \\ u = 0, \theta \rightarrow 0, C \rightarrow 0, H \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (13)$$

To get the result of Eqs. (9-12) under boundary restriction (13) take

$$C = \frac{1}{m_1} e^{-m_1 y} \quad (14)$$

For getting the solutions we introduce

$$\left. \begin{aligned} u(y) &= u_1(y) + Ec u_2(y) + 0(Ec^2) + \dots \\ H(y) &= H_1(y) + Ec H_2(y) + 0(Ec^2) + \dots \\ \theta(y) &= \theta_1(y) + Ec \theta_2(y) + 0(Ec^2) + \dots \end{aligned} \right\} \quad (15)$$

With the help of Eq. (13), the Eqs. (7-9) reduces to the following form

$$u_1'' + u_1' + M H_1' = -Gr \theta_1 - Gm C \quad (16)$$

$$u_2'' + u_2' + M H_2' = -Gr \theta_2 \quad (17)$$

$$H_1'' + Pm H_1' + M Pm u_1' = 0 \quad (18)$$

$$H_2'' + Pm H_2' + M Pm u_2' = 0 \quad (19)$$

$$\theta_1'' + Pr \theta_1' + s_3 \theta_1 = 0 \quad (20)$$

$$\theta_2'' + Pr \theta_2' + s_3 \theta_2 = -Pr (u_1')^2 - \frac{Pr}{Pm} (H_1')^2 \quad (21)$$

Where $s_3 = \left(\frac{Pr R}{4} + Q \right)$

With the corresponding boundary restriction

$$\left. \begin{aligned} u_1 = U, u_2 = 0, H_1 = h, H_2 = 0, \\ \theta_1' = -1, \theta_2' = 0 \quad \text{at } y=0 \\ u_1 = 0, u_2 = 0, H_1 \rightarrow 0, H_2 \rightarrow 0, \\ \theta_1 \rightarrow 0, \theta_2 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (22)$$

$$\theta_1 = \frac{1}{m_2} e^{-m_2 y} \quad (23)$$

$$H_1 = A_3 e^{-m_3 y} - A_4 e^{-m_2 y} - A_2 e^{-m_1 y} \quad (24)$$

$$u_1 = A_7 e^{-y} + A_4 e^{-m_3 y} - A_5 e^{-m_2 y} - A_6 e^{-m_1 y} \quad (25)$$

3. Solution of the problem

$$\theta_2 = \begin{bmatrix} A_{18}e^{-m_6y} - A_8e^{-2y} - A_9e^{-2m_3y} \\ -A_{10}e^{-2m_2y} - A_{11}e^{-2m_1y} - A_{12}e^{-(m_3+1)y} \\ +A_{13}e^{-(m_2+1)y} + A_{14}e^{-(m_1+1)y} \\ +A_{15}e^{-(m_2+m_3)y} + A_{16}e^{-(m_3+m_1)y} \\ +A_{17}e^{-(m_2+m_1)y} \end{bmatrix} \quad (26)$$

$$H_2 = \begin{bmatrix} A_{30}e^{-m_7y} - A_{19}e^{-m_6y} + A_{20}e^{-2y} \\ +A_{21}e^{-2m_3y} + A_{22}e^{-2m_2y} + A_{23}e^{-2m_1y} \\ +A_{24}e^{-(m_3+1)y} - A_{25}e^{-(m_2+1)y} \\ -A_{26}e^{-(m_1+1)y} - A_{27}e^{-(m_2+m_3)y} \\ -A_{28}e^{-(m_3+m_1)y} - A_{29}e^{-(m_2+m_1)y} \end{bmatrix} \quad (27)$$

$$u_2 = \begin{bmatrix} A_{43}e^{-y} - A_{31}e^{-m_7y} - A_{32}e^{-m_6y} \\ -A_{33}e^{-2y} - A_{34}e^{-2m_3y} - A_{35}e^{-2m_2y} \\ -A_{36}e^{-2m_1y} - A_{37}e^{-(m_3+1)y} - A_{38}e^{-(m_2+1)y} \\ +A_{39}e^{-(m_1+1)y} + A_{40}e^{-(m_2+m_3)y} \\ +A_{41}e^{-(m_3+m_1)y} + A_{42}e^{-(m_2+m_1)y} \end{bmatrix} \quad (28)$$

The current density (J) is given by

$$J = -\left(\frac{dH}{dy}\right) \quad (29)$$

Stress of the shear is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_1}{\partial y}\right)_{y=0} + Ec \left(\frac{\partial u_2}{\partial y}\right)_{y=0} \quad (30)$$

The rate of heat transfer is given by

$$Q^* = -k \left(\frac{\partial T^*}{\partial y^*}\right)_{y=0} \quad (31)$$

Nusselt number (Nu), is as follows:

$$Nu = \frac{Q^* \nu}{K \nu_0 (T^* - T_{\infty}^*)} = \frac{1}{(\theta)_{y=0}} \quad (32)$$

The rate of mass transfer is given by

$$J^* = -\rho D \left(\frac{\partial C^*}{\partial y^*}\right)_{y=0} \quad (33)$$

Sherwood number (Sh), is as follows:

$$Sh = \frac{J^* \nu}{\rho \nu_0 (C^* - C_{\infty}^*)} = \frac{1}{(C)_{y=0}} \quad (34)$$

4. Results and discussions

Extensive analytical computations are done for velocities, thermal and concentration distributions together with friction factor feature, Nusselt as well as Sherwood number for distinct standards of physical constraints which illustrate the structures of flow. The problem composes of one independent variable (y), four dependent variables (u, θ, H, ϕ) with

$$M = 3.0, Gr = 5.0, Gm = 3.0, Pm = 1.0, Pr = 1.0, R = 0.5, Q = 0.1, Sc = 0.22, Ec = 0.001, h = 0.1, U = 1.0, Kr = 3.0.$$

Numerical solutions are well established in tables.

For the moment of numerous values of Gr , the velocity in the boundary layer is launched in Fig. 2. It is perceptible that a growth in Gr accompanies to a hike in u because of growth in buoyancy force. Fig. 3 explicates velocity for different values of Gm . A growth in Gm imparts a favorable acceleration in the fluid velocity. In Fig.4 we vigilance that rise in M causes the velocity to downtrend. The spire value radically declines with raise in the value of M , because, the existence of magnetic factor incites a well-known Lorentz force. The impact of Pm on the velocity profile is shown in Fig. 5. From this sketch, it is figured out that enhancing Pm implies an outstanding acceleration in u . we observe that velocity boost up with the hike of Pm . Fig. 6 enlighten us that the velocity profile decelerates with the reduction of Pr . Accelerating profiles of u with Q and R are visualized in Figs. 7-8. Decelerating nature of u with Sc is portrayed in Fig. 9.

Deviation of H of for various values of Gr and Gm has been portrayed in Figs. 10-11. They

appraise an enhancing environment. Fig. 12 shows the pattern of the induced magnetic profile for various values of M . It is seen that as M amplifies, H decelerates. Figs. 13 and 14 show the induced magnetic profile for multifarious values of Pm and Pr . It is exposed that H accelerates owing to Pr and Pm . From Fig. 15 it is illustrated that the H profiles narrated rising nature due to enhancing values of R . The impression of uplifting values of the heat generation parameter Q on the induced magnetic is displayed Figure 16. We perceived in this figure that enhancing the value of the heat generation Q . Figs. 17, 18 and 19 represent the temperature profiles versus y for numerous values of Pr , Q and R respectively. Fig. 17 points out the corollary of Pr on temperature. Temperature profiles indicate suppressing behaviour due to escalating values of Pr . Uplifting repercussions have been arrested for T with Q which is illustrated in Fig. 18. Fig. 19 explains, as presumed, that accelerating of R escorts to uplift in the fluid's temperature. The figures 20 and 21 depict the change of behavior of concentration profiles against y under the effect of Kr and Sc respectively. The impact of Sc on concentration is elucidated in Fig. 22. It is perceptible that raise in Sc contributes to downtrend of concentration of the fluid medium. Undifferentiated effect has been noted with Kr on concentration profile.

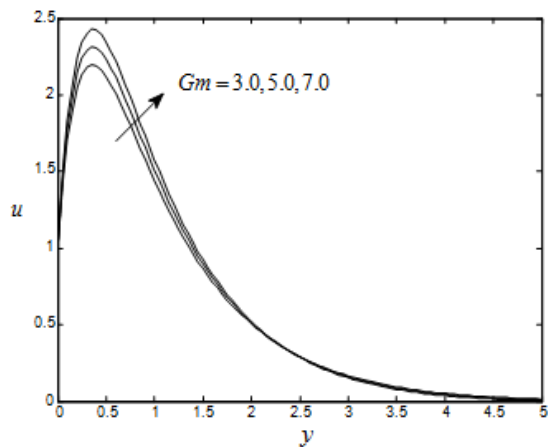


Fig. 3. Profiles of u for Gr .

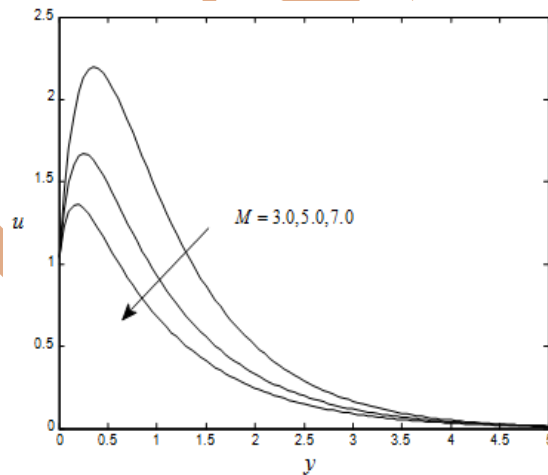


Fig. 4. Profiles of u for M .

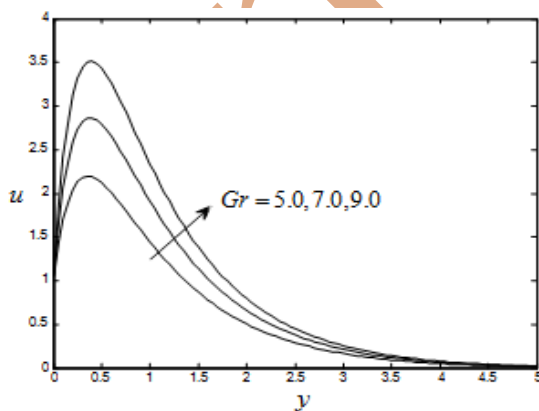


Fig. 2. Profiles of u for Gr .

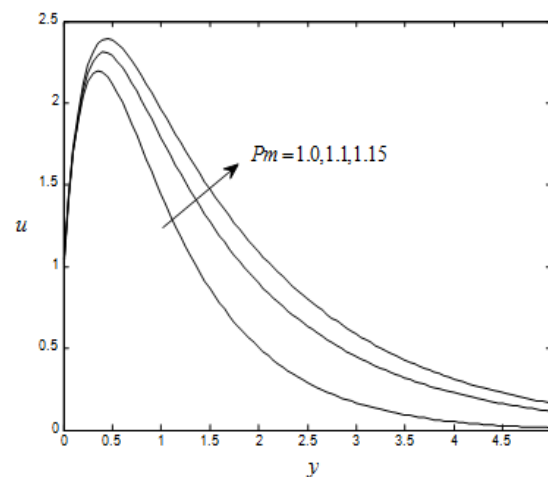


Fig. 5. Profiles of u for Pm .

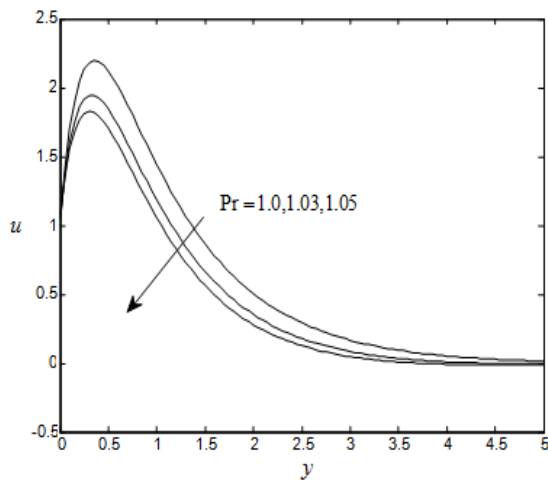


Fig. 6. Profiles of u for Pr .

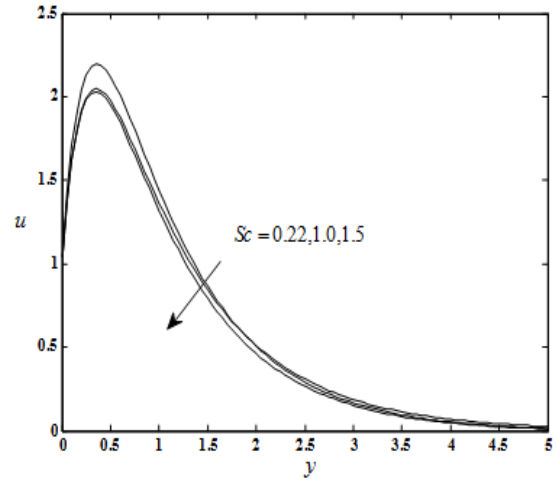


Fig. 9. Profiles of u for Sc .

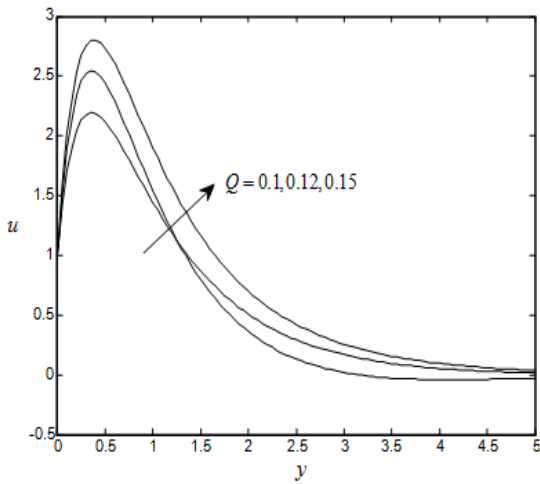


Fig. 7. Profiles of u for Q .

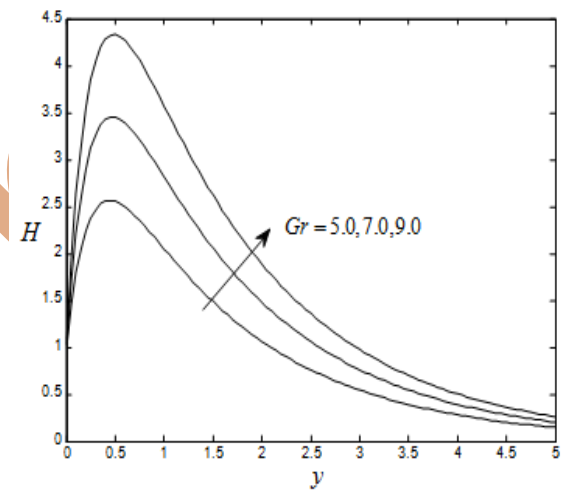


Fig. 10. Profiles of H for Gr .

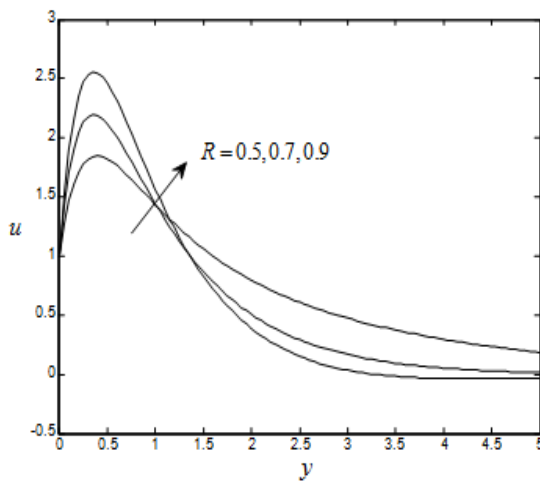


Fig. 8. Profiles of u for R .

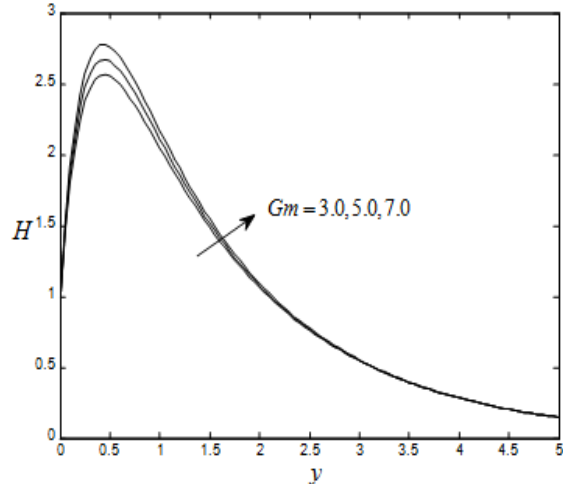


Fig. 11. Profiles of H for Gm .

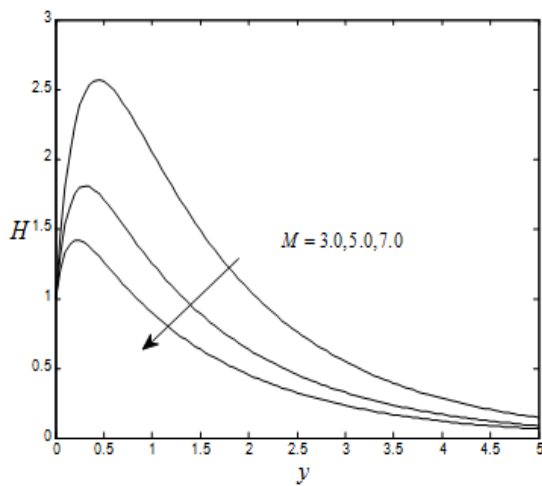


Fig. 12. Profiles of H for M .

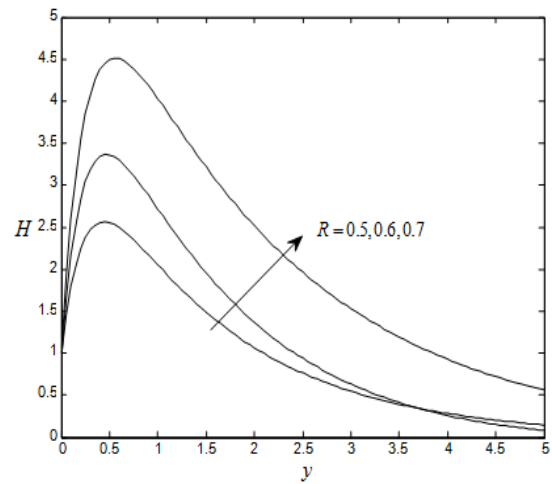


Fig. 15. Profiles of H for R .

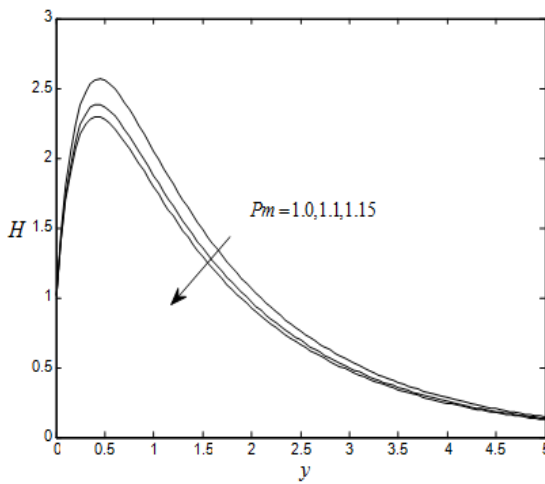


Fig. 13. Profiles of H for Pm .

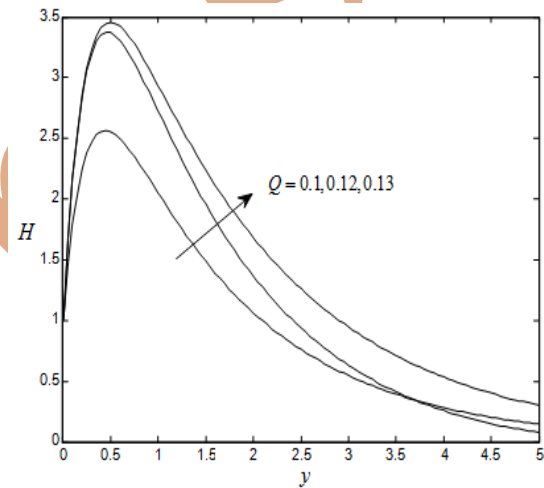


Fig. 16. Profiles of H for Q .

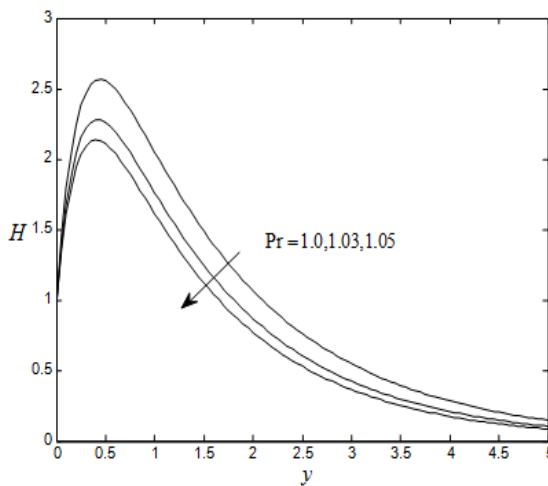


Fig. 14. Profiles of H for Pr .

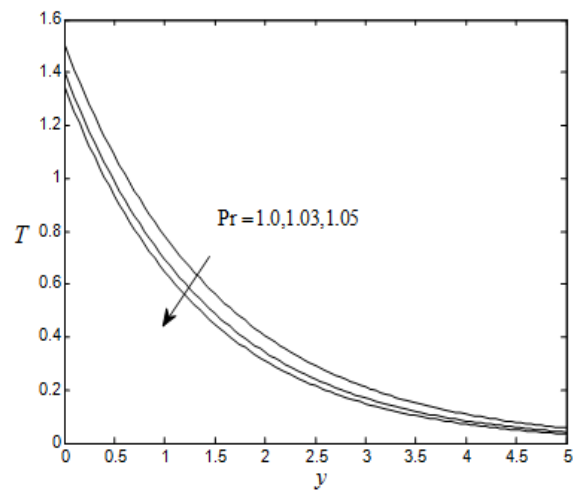


Fig. 17. Profiles of T for Pr .

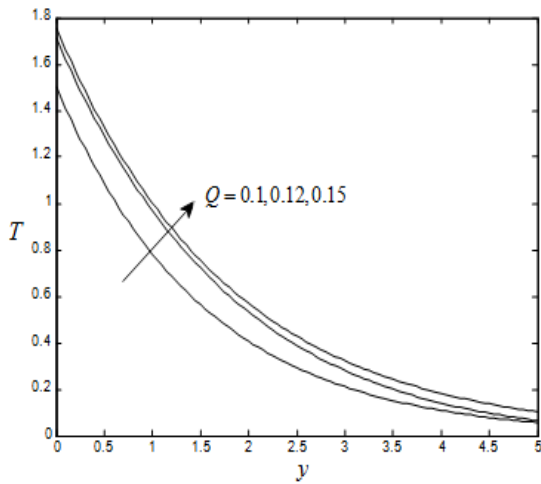


Fig.18. Profiles of T for Q .

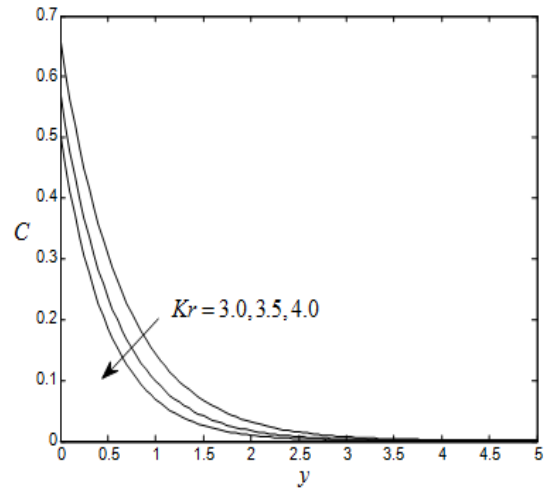


Fig. 21. Profiles of C for Kr .

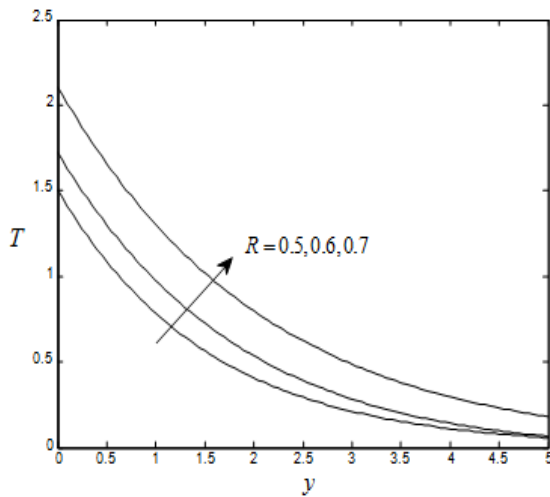


Fig.19. Profiles of T for R .

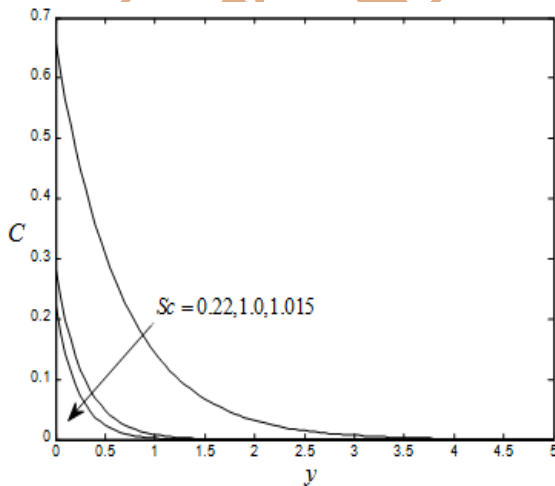


Fig. 20. Profiles of C for Sc .

The nature of physical flow, heat and mass transfer gradient of the governing parameters can be understand from Tables 1-4.

Table 1. Study of τ and J with Gr, Gm, M and Pm for $Pr = 0.71, R = 0.5, Ec = 0.005, Sc = 0.22, Kr = 3.0$.

Gr	Gm	M	Pm	τ	J
2.0	2.0	1.0	1.0	4.7956	3.7030
3.0	2.0	1.0	1.0	7.0310	5.5045
4.0	2.0	1.0	1.0	9.0596	7.2978
2.0	3.0	1.0	1.0	5.0620	3.7798
2.0	4.0	1.0	1.0	5.1241	3.8452
2.0	2.0	2.0	1.0	3.7930	2.4245
2.0	2.0	3.0	1.0	2.7956	1.2921
2.0	2.0	1.0	2.0	4.7785	1.3036
2.0	2.0	1.0	3.0	4.7532	0.6145

Table 2. Study of Nu and Sh with Gr, Gm, M and Pm for $Pr = 0.71, R = 0.5, Ec = 0.005, Sc = 0.22, Kr = 3.0$.

Gr	Gm	M	Pm	Nu	Sh
2.0	2.0	1.0	1.0	-1.0003	-1.0021
3.0	2.0	1.0	1.0	-1.0005	-1.0025
4.0	2.0	1.0	1.0	-1.0006	-1.0036
2.0	3.0	1.0	1.0	-1.0003	-1.0021
2.0	4.0	1.0	1.0	-1.0002	-1.0019
2.0	2.0	2.0	1.0	-1.0007	-1.0026
2.0	2.0	3.0	1.0	-1.0008	-1.0032
2.0	2.0	1.0	2.0	-1.0037	-1.0057
2.0	2.0	1.0	3.0	-1.0053	-1.0096

Table 3. Study of τ and J with Pr, R, Sc and Kr for $Gr = 2.0, Gm = 2.0, M = 1.0, Ec = 0.005, Pm = 1.0$.

Pr	R	Q	Sc	Kr	τ	J
0.71	0.1	0.1	0.22	3.0	4.7912	3.5089
1.0	0.1	0.1	0.22	3.0	1.5552	1.3852
2.0	0.1	0.1	0.22	3.0	0.6463	0.2863
0.71	0.2	0.1	0.22	3.0	5.3678	4.0874
0.71	0.3	0.1	0.22	3.0	6.0741	4.8142
0.71	0.1	0.3	0.6	3.0	4.4752	3.3430
0.71	0.1	0.4	0.78	3.0	4.4363	3.3372
0.71	0.1	0.1	0.6	3.0	4.6348	3.4496
0.71	0.1	0.1	0.78	3.0	4.5641	3.3845
0.71	0.1	0.1	0.22	5.0	4.5678	3.3821
0.71	0.1	0.1	0.22	6.0	4.5632	3.3856

Table 4. Study of Nu and Sh with Pr, R, Sc and Kr for $Gr = 2.0, Gm = 2.0, M = 1.0, Ec = 0.005, Pm = 1.0$.

Pr	R	Q	Sc	Kr	Nu	Sh
0.71	0.1	0.1	0.22	3.0	-2.0003	-3.020
1.0	0.1	0.1	0.22	3.0	-1.9997	-3.9947
2.0	0.1	0.1	0.22	3.0	-1.9988	-3.9958
0.71	0.2	0.1	0.22	3.0	0.0003	-1.0045
0.71	0.3	0.1	0.22	3.0	0.0003	-1.0063
0.71	0.1	0.3	0.6	3.0	0.0006	-1.0216
0.71	0.1	0.4	0.78	3.0	0.0005	-1.0455
0.71	0.1	0.1	0.6	3.0	-2.9993	-3.9743
0.71	0.1	0.1	0.78	3.0	-2.0006	-3.0636
0.71	0.1	0.1	0.22	5.0	0.9993	-1.9753
0.71	0.1	0.1	0.22	6.0	-2.0006	-3.0426

5. Conclusions

Key findings are enlisted below:

- An improvement in Gm, Gr causes to improve u, H and T .
- u and H are the suppressing functions of M and Pr .
- u, H and T accelerate with a raise in Q .
- u, H and T are the enhancing function of R .

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APPENDIX

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4ScKr}}{2}, m_2 = \frac{Pr + \sqrt{Pr^2 - 4s_3}}{2},$$

$$m_3 = \frac{n_1 + \sqrt{n_1^2 - 4n_2}}{2}, m_4 = 0, m_5 = -1,$$

$$m_6 = \frac{Pr + \sqrt{Pr^2 - 4s_3}}{2}, m_7 = \frac{n_1 + \sqrt{n_1^2 - 4n_2}}{2},$$

$$A_1 = \frac{Pr MGr}{m_1^2 m_2^2 - n_1 m_2 + n_2} \cdot 1,$$

$$A_2 = \frac{Pr MGr}{m_1^2 m_1^2 - n_1 m_1 + n_2} \cdot 1, A_3 = h + A_1 + A_2,$$

$$A_4 = \frac{Mm_3 A_3}{m_3^2 - m_3}, A_5 = \frac{Mm_2 A_1 + \frac{Gr}{m_2}}{m_2^2 - m_2},$$

$$A_6 = \frac{Mm_1 A_2 + \frac{Gm}{m_1}}{m_1^2 - m_1}, A_7 = U + A_6 + A_5 - A_4,$$

$$n_3 = Pr m_3^2 \left(A_4^2 + \frac{A_3^2}{Pm} \right), n_4 = Pr m_2^2 \left(A_5^2 + \frac{A_1^2}{Pm} \right),$$

$$n_5 = Pr m_1^2 \left(A_6^2 + \frac{A_2^2}{Pm} \right),$$

$$n_6 = 2 Pr m_3 m_2 \left(A_4 A_5 + \frac{A_3 A_1}{Pm} \right),$$

$$n_7 = 2 Pr m_3 m_1 \left(A_4 A_6 + \frac{A_3 A_2}{Pm} \right),$$

$$n_8 = 2 Pr m_1 m_2 \left(A_6 A_5 + \frac{A_2 A_1}{Pm} \right), A_8 = \frac{Pr A_7^2}{4 - 2Pr + \frac{RPr}{4}}$$

$$A_9 = \frac{n_3}{4m_3^2 - 2m_3 Pr + \frac{RPr}{4}},$$

$$A_{10} = \frac{n_4}{4m_2^2 - 2m_2 Pr + \frac{RPr}{4}},$$

$$A_{11} = \frac{n_5}{4m_1^2 - 2m_1 Pr + \frac{RPr}{4}},$$

$$A_{12} = \frac{2Pr A_7 A_4 m_3}{(m_3 + 1)^2 - Pr(m_3 + 1) + \frac{RPr}{4}},$$

$$A_{13} = \frac{2Pr A_7 A_5 m_2}{(m_2 + 1)^2 - Pr(m_2 + 1) + \frac{RPr}{4}},$$

$$A_{14} = \frac{2Pr A_7 A_6 m_1}{(m_1 + 1)^2 - Pr(m_1 + 1) + \frac{RPr}{4}},$$

$$A_{15} = \frac{n_6}{(m_3 + m_2)^2 - Pr(m_3 + m_2) + \frac{RPr}{4}},$$

$$A_{16} = \frac{n_7}{(m_3 + m_1)^2 - Pr(m_3 + m_1) + \frac{RPr}{4}},$$

$$A_{17} = \frac{n_8}{(m_1 + m_2)^2 - Pr(m_1 + m_2) + \frac{RPr}{4}},$$

$$A_{18} = \frac{\begin{bmatrix} 2A_8 + 2A_9 + 2A_{10} + 2A_{11}m_1 \\ + A_{12}(m_3 + 1) - A_{13}(m_2 + 1) \\ - A_{14}(m_1 + 1) - A_{15}(m_3 + m_2) \\ - A_{16}(m_3 + m_1) - A_{17}(m_1 + m_2) \end{bmatrix}}{m_6},$$

$$n_9 = \frac{PmMGrA_{18}}{m_6}, \quad n_{10} = \frac{PmMGrA_8}{2},$$

$$n_{11} = \frac{PmMGrA_9}{2m_3}, \quad n_{12} = \frac{PmMGrA_{10}}{2m_2},$$

$$n_{13} = \frac{PmMGrA_{11}}{2m_1}, \quad n_{14} = \frac{PmMGrA_{12}}{m_3 + 1},$$

$$n_{15} = \frac{PmMGrA_{13}}{m_2 + 1}, \quad n_{16} = \frac{PmMGrA_{14}}{m_1 + 1},$$

$$n_{17} = \frac{PmMGrA_{15}}{m_3 + m_2}, \quad n_{18} = \frac{PmMGrA_{16}}{m_3 + m_1},$$

$$n_{19} = \frac{PmMGrA_{17}}{m_1 + m_2}, \quad A_{19} = \frac{n_8}{m_6^2 - n_1 m_6 + n_2},$$

$$A_{20} = \frac{n_9}{4 - 2n_1 + n_2}, \quad A_{21} = \frac{n_{10}}{4m_3^2 - 2n_1 m_3 + n_2},$$

$$A_{22} = \frac{n_{11}}{4m_2^2 - 2n_1 m_2 + n_2},$$

$$A_{23} = \frac{n_{12}}{4m_1^2 - 2n_1 m_1 + n_2},$$

$$A_{24} = \frac{n_{13}}{(m_3 + 1)^2 - n_1(m_3 + 1) + n_2},$$

$$A_{25} = \frac{n_{14}}{(m_2 + 1)^2 - n_1(m_2 + 1) + n_2},$$

$$A_{26} = \frac{n_{15}}{(m_1 + 1)^2 - n_1(m_1 + 1) + n_2},$$

$$A_{27} = \frac{n_{16}}{(m_3 + m_2)^2 - n_1(m_3 + m_2) + n_2},$$

$$A_{28} = \frac{n_{17}}{(m_3 + m_1)^2 - n_1(m_3 + m_1) + n_2},$$

$$A_{29} = \frac{n_{18}}{(m_1 + m_2)^2 - n_1(m_1 + m_2) + n_2},$$

$$A_{30} = \begin{bmatrix} A_{19} - A_{20} - A_{21} - A_{22} - A_{23} - A_{24} \\ + A_{25} + A_{26} + A_{27} + A_{28} + A_{29} \end{bmatrix},$$

$$A_{31} = \frac{MA_{30}}{m_7^2 - m_7}, \quad A_{32} = \frac{A_{19} + GrA_{18}}{m_6^2 - m_6},$$

$$A_{33} = \frac{MA_{20} - GrA_8}{2}, \quad A_{34} = \frac{MA_{21} - GrA_9}{4m_3^2 - 2m_3},$$

$$A_{35} = \frac{MA_{22} - GrA_{10}}{4m_2^2 - 2m_2}, \quad A_{35} = \frac{MA_{22} - GrA_{10}}{4m_2^2 - 2m_2},$$

$$A_{36} = \frac{MA_{23} - GrA_{11}}{4m_1^2 - 2m_1}, \quad A_{37} = \frac{MA_{24} - GrA_{12}}{(m_3 + 1)^2 - (m_3 + 1)},$$

$$A_{38} = \frac{MA_{25} - GrA_{13}}{(m_2 + 1)^2 - (m_2 + 1)},$$

$$A_{39} = \frac{MA_{26} - GrA_{14}}{(m_1 + 1)^2 - (m_1 + 1)},$$

$$A_{40} = \frac{MA_{27} - GrA_{15}}{(m_3 + m_2)^2 - (m_3 + m_2)},$$

$$A_{41} = \frac{MA_{28} - GrA_{16}}{(m_3 + m_1)^2 - (m_3 + m_1)},$$

$$A_{42} = \frac{MA_{29} - GrA_{17}}{(m_1 + m_2)^2 - (m_1 + m_2)},$$

$$A_{43} = \begin{bmatrix} A_{31} + A_{32} + A_{33} + A_{34} + A_{35} + A_{36} + A_{37} \\ + A_{38} - A_{39} - A_{40} - A_{41} - A_{42} \end{bmatrix},$$

References

[1] R. C. Choudhury, and B. K. Sharma, "Combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field", *Journal of applied physics*, Vol. 99, No. 3, pp. 03490110–10, (2006).

[2] M. Hady, A.Y. Bakien, and R.S.R. Gorla, "Mixed convection boundary layer flow on

- a continuous flat plate with variable viscosity”, *Heat mass transfer*, Vol.31, No. 3, pp. 169 – 172, (1996).
- [3] B. Gebhart, and L. Pera, “The nature of vertical natural convection flows resulting from the combined buoyancy effects on thermal and mass diffusion”, *International journal of heat and mass transfer*, Vol.14, No. 2, pp. 2025–2050, (1971).
- [4] B.K. Sharma, and R.C. Chaudhary, “Hydromagnetic unsteady mixed convection and mass transfer flow past a vertical porous plate immersed in a porous medium with hall effect”, *Engineering transactions*, Vol. 56, No. 1, pp. 3–23, (2008).
- [5] V. Sri Hari Babu, and G. V. Ramana Reddy, “Mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation”, *Advances in applied science research*, Vol. 2, No. 4, pp. 138–146, (2011).
- [6] S.K. Kango, and G.C. Rana, “Double-Diffusive convection in Walters’ B’ elastic-viscous fluid in the presence of rotation and magnetic field”, *Advances in applied science research*, Vol.2, No. 4, pp.166–176, (2011).
- [7] A. Kavitha, R. Hemadri Reddy, S. Sreenadh, R. Saravana, and A. N. S. Srinivas, “Peristaltic flow of a micropolar fluid in a vertical channel with long wave length approximation, *Advances in applied science research*, Vol.2, No. 1, pp. 269–279.
- [8] V.P. Rathod, and S.K. Asha, “Effects of magnetic field and an endoscope on peristaltic motion”, *Advances in applied science research*, Vol.2, No. 4, pp. 102–109, (2011).
- [9] A. Raptis, and N.G. Kafoussias, “Magnetohydrodynamics free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux”, *Canadian journal of physics*, Vol.60, No. 12, pp. 1725–1729, (1982).
- [10] A. Raptis, “Flow through a porous medium in the presence of magnetic field”, *International journal of energy research*, Vol.10, No.1, pp. 97–101, 1986.
- [11] M.A. Sattar and M.M. Hossain,” Unsteady hydromagnetic free convection flow with hall current mass transfer along an accelerated porous plate with time-dependent temperature and concentration”, *Canadian journal of physics*, Vol. 70, No.5, pp. 369–374, (1992).
- [12] M.A. Sattar, “Unsteady hydromagnetic free convection flow with Hall current mass transfer and variable suction through a porous medium near an infinite vertical porous plate with constant heat flux”, *International journal of energy research*, Vol. 17, No. 9, pp. 1 – 5, (1993).
- [13] Anand Kumar, and A. K. Singh, “Unsteady MHD free convection heat and mass transfer flow past a semi-infinte vertical wall with induced magnetic field”, *Applied mathematics and computation*, Vol. 222, No. 1, 2012, pp.462–471, (2012).
- [14] K. Chaudhary, A. Sharma, and A. K. Jha, “Laminar mixed convection flow from a vertical surface with induced magnetic and convective boundary”, *International journal of applied mechanics and engineering*, Vol. 23. No.2, pp. 307-326, (2018).
- [15] D. Sarma, and K.K. Pandit “Effects of Thermal Radiation and Chemical Reaction on Steady MHD Mixed Convective Flow over a Vertical Porous Plate with Induced Magnetic Field”, *International journal of fluid Mechanics Research*, Vol. 42, No. 4, pp. 315-333, (2015).
- [16] Md. M. Alam, M. R. Islan, and F. Rahman, “Steady heat and mass transfer by mixed convection flow from a vertical porous plate with induced magnetic field, constant heat and mass fluxes”, *Science & technology asia*, Vol. 13, No. 4, pp. 1-13, (2008).
- [17] S.M. Ibrahim, and K. Suneetha, “Influence of chemical reaction and heat source on MHD free convection boundary layer flow of radiation absorbing Kuvshiinshiki fluid in porous medium”, *Asian journal of*

- mathematics and computer research*, Vol. .3, No. 2, pp.87-103, (2015).
- [18]N. Ahmed, and S. Sengupta, "Thermo-Diffusion and Diffusion-Thermo effects on a three dimensional MHD mixed convection flow past an infinite vertical porous plate with thermal radiation", *Magneto hydrodynamics*, Vol. 47, No. 1, pp. 41-60, (2011).
- [19] R.C. Chaudhary, B.K. Sharma, and A.K. Jha, "Radiation Effect with Simultaneous Thermal and Mass Diffusion in MHD Mixed Convection Flow from a Vertical Surface with Ohmic Heating", *Romania journal of physics*, Vol. 51, No. 7-8, 2006, pp. 715-727, (2006).
- [20] B.K. Sharma, M. Agarwal, and R.C. Chaudhary, "MHD Fluctuating Free Convective Flow with Radiation Embedded in Porous Medium Having Variable Permeability and Heat Source/Sink", *Journal of technical physics*, Vol. 47, No. 1, pp. 47-58, (2006).
- [21]G. Stangle, and I. Aksay, "Simultaneous momentum, heat and mass transfer with chemical reaction in a disordered porous medium: application to binder removal from a ceramic green body", *Chemical engineering science*, Vol. 45. No. 7, 1990, pp. 1719-1731, (1990).
- [22]R. Muthucumarswamy, "First order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion", *Annals faculty of engineering, hunedoara, journal of engineering, tome*, Vol. 7, No.1, pp. 47-50, (2009).
- [23]V. Rajesh, and S.V.K. Varma, "Chemical reaction and radiation effects on MHD flow past an infinite vertical plate with variable temperature", *Far east journal of mathematical sciences*, Vol. 32, No. 1, pp. 87- 106, (2009).
- [24]R. Kandasamy, K. Periasamy and K.K. Prashu Sivagnana, "Effects of Chemical Reaction, Heat and Mass Transfer along Wedge with Heat Source and Concentration in the Presence of Suction or Injection", *International journal of heat and mass transfer*, Vol. 48, No. 7, pp. 1388- 1394, (2005).
- [25]F.M.N. El-Fayez, "Effects of chemical reaction on the unsteady free convection flow past an infinite vertical permeable moving plate with variable temperature", *Journal of surface engineered materials and advanced technology*, Vol. 2, No. 2, pp. 100-109, (2012).
- [26]C.S. Sravanthi, A.L. Ratnam and N.B. Reddy, "Thermo-Diffusion and Chemical reaction effects on a steady mixed convective heat and mass transfer flow with induced magnetic field", *International journal of innovative research in science and engineering*, Vol. 2, No. 9, pp. 4415-4424, (2013).