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JCARME

ISSN: 2228-7922

Journal of Computational and Applied Research in Mechanical Engineering
Vol. x, No. x, xxxx
jcarme.sru.ac.ir

Analysis of generalized compressor characteristics on surge phenomena in axial compressors

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Article info:

Received: 00/00/2000
Accepted: 00/00/2018
Online: 00/00/2018

Keywords:
Surge,
Multiple time scales (MTS),
Limit cycle oscillations (LCOs),
Moore-Greitzer model,
Bifurcations.

Abstract

The paper discusses the effect of compressor characteristic on surge phenomena in axial flow compressors. Specifically, the effect of nonlinearities on the compressor dynamics is analyzed. For this purpose, generalized multiple time scales method is used to parameterize equations in amplitude and frequency explicitly. The pure surge case of the famous Moore-Greitzer model is used as the basis of the study. The compressor characteristic used in the Moore-Greitzer model is generalized to evaluate the effect of the parameters involved. Subsequently, bifurcation theory is used to study the effect of nonlinear dynamics on surge behavior. It has been found that the system exhibits supercritical Hopf bifurcation under specific conditions in which surge manifests as limit cycle oscillations. Key parameters have been identified in the analytical solution which govern the nonlinear dynamic behavior and are responsible for the existence of limit cycle oscillations. Numerical simulations of the Moore-Greitzer model are carried out and are found in good agreement with the analytical solution

1. Introduction

Compressors and pumping systems are a critical organ of any turbine based propulsion system. The dynamics of compression systems is prone to two types of unsteady aerodynamic instabilities, stall and surge [1]. The occurrences of these instabilities generate safety critical scenarios. Moreover, the prediction of such instabilities during the preliminary design stage is an active area of research. Surge is basically a large-amplitude, one-dimensional and axisymmetric flow instability, whereby the whole engine exhibits fluctuations of mass flow rate. Surge is a chronological process, and its first episode is usually the stall (which can be either progressive or abrupt in nature) as described in [2]. These fluctuations induce undesirable vibrating stresses on compressor blades and can result in reduced off-design performance and structural damage [3].

The seminal work pertaining to the analytical prediction of surge phenomena can be attributed to Greitzer [4]. In this study, the transport equations were applied to the control volume containing compressor and a set of differential

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equations were developed that models the transient behavior of compressor instability. Subsequently, Moore and Greitzer [5, 6] presented an improved model involving a smooth cubic compressor characteristic that demonstrated separate modeling of stall and surge. The mathematical model constituted a set of three nonlinear partial differential equations. The cubic characteristic used in the original model was obtained via regression analysis of experimental results for a three-stage low speed axial compressor. These partial differential equations were then converted to ordinary differential equations via a Galerkin procedure. This subsequent set of three coupled nonlinear ordinary differential equations are recognized as Moore-Greitzer model.

The readers are encouraged to go through the reviews by Greitzer [7, 8], Longley [9], Gu et al. [10] and Paduano et al. [11]. McCaughan [12] applied nonlinear dynamic analysis tools on the Moore-Greitzer model to understand surge phenomena. This idea was further pursued with the application of control theory by Abed et al. [13] and Liaw and Abed [14], Willems et al. [15], Shehata et al. [16], Fontaine et al. [17], Sheng et al. [18] and Ziabari et al.[19]. The analysis carried out by Hos et al. [20] gave special attention to global bifurcations. Recent work by Malathi and Kushari [21] examines the effect of change in geometric parameters on axial compression systems using the Moore-Greitzer model.

It is worth mentioning that the existing Moore-Greitzer model involves a specific trend of compressor characteristic coefficient that is developed from a three-stage compression system. These characteristic curve coefficients are derived based on regression modeling. Although the qualitative trend remains the same, a minor perturbation in quantitative behavior can lead to different system dynamics. In this work, a generalized case of compressor characteristic curve in Moore-Greitzer model is considered.

The nonlinear analytical technique identified to solve the problem is known as multiple time scales (MTS) method based on the work of Ramnath [22, 23]. The MTS method belongs to the family of perturbation methods. It is an asymptotic approach to approximate the physical problems that involve perturbations about nominal states specifically in limiting cases. In the present work, method of MTS has been applied on the pure surge case of Moore-Greitzer model [5]. Subsequently, an approximate closed form analytical solution for the pure surge case is generated. Finally, bifurcation analysis is done to get a qualitative insight into the solution. Such solutions have an advantage over usual numerical solutions in that the important parameters and their effects on limit-cycle characteristics, such as amplitude and frequency, can be easily seen in explicit functional relationships. The results are validated with the numerical simulations in the end.

2. Problem formulation

The complete Moore-Greitzer model along with its derivation and assumptions worked out earlier [5, 24] is not being reproduced here for brevity. The surge phenomenon is governed by a nonlinear second order ordinary differential equation as shown in Eq (1):

$$\frac{d^2\Phi}{d\xi^2}+\frac{1}{l_C}\left(\frac{1}{4B^2k_T}-\frac{d\psi_C}{d\Phi}\right)\frac{d\Phi}{d\xi}+\frac{1}{4l_C^2B^2}\left[\left(\Phi-\overline{\Phi}\right)-\frac{\Psi-\overline{\Psi}}{k_T}\right]=0 \qquad (1)$$

where $\Phi \equiv \Phi(\xi)$ is the dimensionless flow coefficient (averaged over angle); ξ is dimensionless time variable (for wheel to rotate one radian); l_C is (dimensionless) total length of compressor and ducts; $\boldsymbol{B}$ is measure of the ratio of pressure forces to inertial forces for a given rate of mass flow change (Greitzer's Parameter); k_T is the linear throttle coefficient; ψ_C is the axisymmetric pressure-rise coefficient; $\overline{\Phi}$ is the flow coefficient averaged over both angle and time; $\Psi \equiv \Psi(\xi)$ is the total-to-static pressure-rise coefficient, and $\overline{\Psi}$ is the time-averaged value of Ψ. The throttle slopes are generally steep in nature. Moreover, the throttle mass excursion and length are significantly smaller than the compressor lengths. Therefore, a common assumption found in literature is to treat the throttle slope, $k_T \to \infty$. Further, in Eq. (2) $\overline{\Phi}$ is a constant and the term $\frac{1}{4l_C^2B^2}\overline{\Phi}$ is a non-homogeneous term. It does not affect the properties of the solution qualitatively. It merely shifts the solution on the flow coefficient

(Φ)axis by an amount equal to $\frac{\bar{\Phi}}{4l_C^2B^2}$. After shifting the mean values, $\bar{\Psi}$ *and* $\bar{\Phi}$ to origin of the axes, Eq (1) can be represented as:

$$\frac{d^2\Phi}{d\xi^2}+\frac{\Phi}{4l_C^2B^2}=\frac{1}{l_C}\frac{d\psi_C}{d\Phi}\frac{d\Phi}{d\xi} \tag{2}$$

The right-hand side of the Eq. (2) represents the system damping; where $\frac{d\psi_C}{d\Phi}$ is the slope of the compressor characteristic. The compressor characteristic is a plot between two flow properties, pressure coefficient and flow coefficient while the fluid passes through the compressor at different compressor speeds. This plot is usually obtained experimentally, and contains significant information regarding the compressor's design. The compressor characteristic, in the Moore-Greitzer, has been approximated as a cubic polynomial, as follows:

$$\psi_C\equiv\psi_C(\Phi)=\psi_{C_O}+H\left[1+\alpha\left(\frac{\Phi}{W}-1\right)+\beta\left(\frac{\Phi}{W}-1\right)^3\right] \tag{3}$$

The meanings of various parameters in Eq (3) can be understood from Fig. 1.

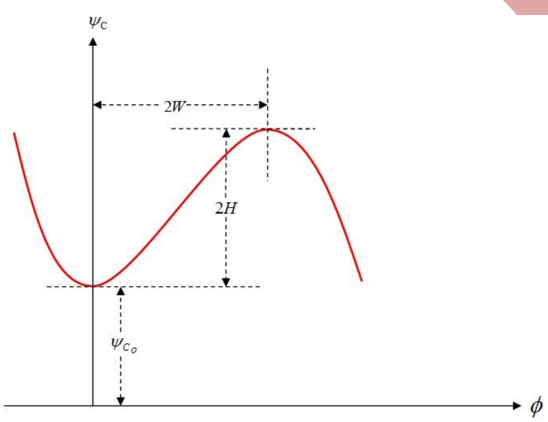

Fig 1. Axisymmetric cubic characteristics for Moore-Greitzer model

In Moore Greitzer model, the values α and β are determined from the experimental data for a three-stage compression system. Specifically, $\alpha=\frac{3}{2}$ and $\beta=-\frac{1}{2}$ are approximated through regression analysis. It should be noted that the parameters α and β vary for each compressor and are prone to uncertainty.

Therefore, in this work, these two parameters are treated in a more generalized way and a closed form expression is derived based on these generalizations. The variation of the characteristic curve subject to parametric variation of α or β is shown in Fig. 2a and Fig. 2b.

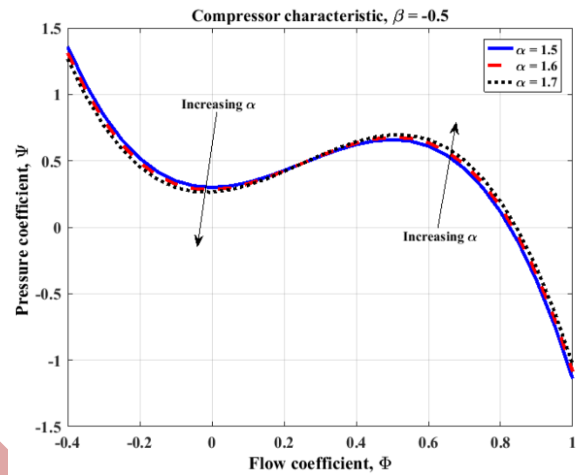

Fig 2a.Effect of varying α on compressor characteristic curve

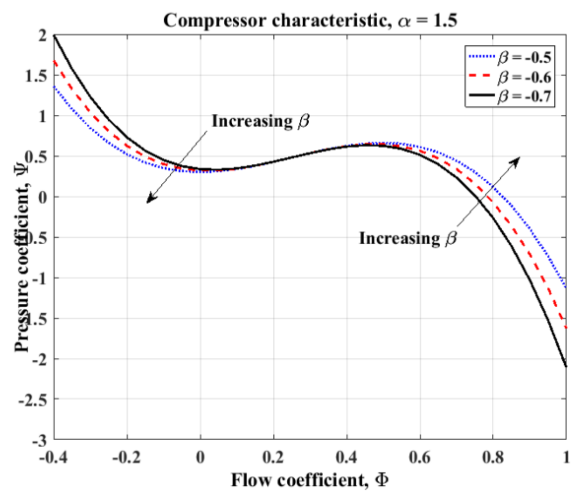

Fig 2b. Effect of varying β on compressor characteristic curve

It can be observed that under small variations, the generic shape of the characteristic curve is preserved. However, the onset, amplitude and frequency of the surge can dramatically change as will be seen ahead. Moreover, β has more

pronounced effect on the shape of compressor characteristic as compared to α.
Differentiating compressor characteristic with flow coefficient, we get:

$$\frac{d\psi_C}{d\Phi} = \left(\frac{3\beta H}{W^3}\right)\Phi^2 + \left(\frac{-6\beta H}{W^2}\right)\Phi + \frac{H}{W}(\alpha + 3\beta) \quad (4)$$

Under specific values of α and β (originally proposed), the last term in Eq. (4) goes to zero. Substituting $\frac{d\psi_C}{d\Phi}$ from Eq. (4) in Eq. (2) for the generalized case:

$$\frac{d^2\Phi}{d\xi^2} + \frac{1}{4l_C^2B^2}\Phi = \frac{1}{l_C}\left\{\left(\frac{3\beta H}{W^3}\right)\Phi^2 + \left(\frac{-6\beta H}{W^2}\right)\Phi + \frac{H}{W}(\alpha + 3\beta)\right\}\frac{d\Phi}{d\xi} \quad (5)$$

Eq. (5) can also be represented in following simplified form

$$\ddot{\Phi} + \omega^2\Phi = K_1\Phi^2\dot{\Phi} + K_2\Phi\dot{\Phi} + K_3\dot{\Phi} \quad (6)$$

where $\omega = \frac{1}{2l_CB}$, $K_1 = \frac{3\beta H}{l_CW^3}$, $K_2 = \frac{-6\beta H}{l_CW^2}$, $K_3 = \frac{H}{l_CW}(\alpha + 3\beta)$ and. It can be noted that $K_3 = 0$ if and. Eq. (6) forms the generalized governing dynamics for further surge analysis.

3. Results and Discussion

The dynamic analysis of surge is done using MTS method. The MTS method is an asymptotic approach to generate approximate analytical closed form solutions specifically for limiting cases. The readers are referred to the details of this technique in [22]. The technique has been used extensively by Go [25-28] for studying the wing rock problem and Maqsood & Go [29] for aircraft longitudinal dynamics using aerodynamic vectoring and Ahmed et al. [30] for pressure oscillations in solid rocket motors.
Here, the MTS analysis is applied to the axial compression system. The parameterization of the dynamic equation (see Eq. (6)) in terms of small perturbation parameter ε is done to carry out the MTS analysis.

$$\ddot{\Phi} + \omega^2\Phi = \varepsilon\left(K_1\Phi^2\dot{\Phi} + K_2\Phi\dot{\Phi} + K_3\dot{\Phi}\right) \quad (7)$$

where $0 < \varepsilon << 1$.The MTS method is now invoked. Two time scales are selected in this analysis and, therefore, the non-dimensional time ξ is expanded in this form:

$$\xi \to \{\tau_o, \tau_1\}; \quad \tau_o = \xi; \quad \tau_1 = \varepsilon\xi \quad (8)$$

The dependent variable is now expanded in the following manner:

$$\Phi(\xi) = \Phi_o(\tau_o, \tau_1) + \varepsilon\Phi_1(\tau_o, \tau_1) + O(\varepsilon^2) \quad (9)$$

Substituting the first and second order derivatives in the dynamics of flow coefficient, the expanded version becomes:

$$\left(\frac{\partial^2}{\partial{\tau_o}^2} + \varepsilon\frac{\partial^2}{\partial\tau_o\partial\tau_1} + \varepsilon\frac{\partial^2}{\partial\tau_1\partial\tau_o}\right)(\Phi_o + \varepsilon\Phi_1) + \omega^2(\Phi_o + \varepsilon\Phi_1)) = \varepsilon K_1\left({\Phi_o}^2 + \varepsilon^2{\Phi_1}^2 + 2\varepsilon\Phi_o\Phi_1\right)\left(\frac{\partial}{\partial\tau_o} + \varepsilon\frac{\partial}{\partial\tau_1}\right)(\Phi_o + \varepsilon\Phi_1) + \varepsilon K_2(\Phi_o + \varepsilon\Phi_1)\left(\frac{\partial}{\partial\tau_o} + \varepsilon\frac{\partial}{\partial\tau_1}\right)(\Phi_o + \varepsilon\Phi_1) + \varepsilon K_3\left(\frac{\partial}{\partial\tau_o} + \varepsilon\frac{\partial}{\partial\tau_1}\right)(\Phi_o + \varepsilon\Phi_1) \quad (10)$$

Equating order by order analysis of ε on both sides in Eq. (10) will reveal several equations. The zeroth order and first order approximations are only considered in the analysis. The zeroth order approximation can be written as:

$$\varepsilon^0: \quad \frac{\partial^2\Phi_o}{\partial{\tau_o}^2} + \omega^2\Phi_o = 0 \quad (11)$$

Correspondingly, the solution of Eq. (11) can be written as

$$\Phi_o(\tau_o, \tau_1) = A(\tau_1)\sin\eta$$
$$\eta \equiv \eta(\tau_o, \tau_1) = \omega\tau_o + P(\tau_1) \quad (12)$$

where A is the amplitude, η is the phase angle and P is the phase correction of the solution. It can be seen that the amplitude and phase of the solution vary with the slow time scale τ_1. Once these variations are known, the zeroth order approximation to the flow coefficient dynamics can be considered complete. Now the first order approximation from Eq. (10) is expressed as:

ε^1:

$$\frac{\partial^2 \Phi_1}{\partial {\tau_o}^2} + \omega^2 \Phi_1 = K_1 {\Phi_o}^2 \frac{\partial \Phi_o}{\partial \tau_o} + K_2 \Phi_o \frac{\partial \Phi_o}{\partial \tau_o} + K_3 \frac{\partial \Phi_o}{\partial \tau_o} - 2\frac{\partial^2 \Phi_o}{\partial \tau_o \partial \tau_1} \quad (13)$$

The variation of the amplitude and phase with the slower time scale can be found by substituting the solution (Eq. (12)) to the $O(\varepsilon)$ group of Eq. (10) that is represented as:

$$\frac{\partial^2 \Phi_1}{\partial {\tau_o}^2} + \omega^2 \Phi_1 = K_1(A^2 \sin^2 \eta)(\omega A \cos \eta) + K_2(A \sin \eta)(\omega A \cos \eta) + K_3(\omega A \cos \eta) - 2\omega \frac{dA}{d\tau_1} \cos \eta + 2\omega A \frac{dP}{d\tau_1} \sin \eta \quad (14)$$

Using trigonometric identities, Eq. (14) can be alternatively written as:

$$\frac{\partial^2 \Phi_1}{\partial {\tau_o}^2} + \omega^2 \Phi_1 = \left(\frac{K_1 \omega A^3}{4} + K_3 \omega A - 2\omega \frac{dA}{d\tau_1}\right) \cos \eta + 2\omega A \frac{dP}{d\tau_1} \sin \eta - \frac{K_1 \omega A^3}{4} \cos 3\eta + \frac{K_2 \omega A^2}{2} \sin 2\eta \quad (15)$$

If the coefficients of first harmonic terms on the right-hand side of Eq. (15) are non-zero, secular terms will appear and collapse the uniformity of the solution. The secular terms, $\tau_o \cos \eta$ and $\tau_o \sin \eta$, will be unbounded as time approaches infinity. Therefore, the terms on the right hand side involving $\cos\eta$ and $\sin\eta$ must be put equal to zero in order to maintain the uniformity of the solution. This obtains:

$$\frac{dA}{d\tau_1} = \zeta A^3 + \sigma A \Rightarrow \frac{dP}{d\tau_1} = 0 \quad (16)$$

where $\zeta = \frac{K_1}{8}$ and $\sigma = \frac{K_3}{2}$. It can be noted that the phase angle variable is not a function of τ_1; implying there is a constant magnitude of phase correction in the phase equation. The equilibria for the amplitude equation exist at $A = 0$ and $A = \sqrt{\frac{-\sigma}{\zeta}}$. These equilibria predict the stability properties of the solution. The equilibria consist of the σ axis and the parabola $\sigma = -\zeta A^2$. These equilibria are plotted in Fig. 3 for the values of $\zeta < 0$.

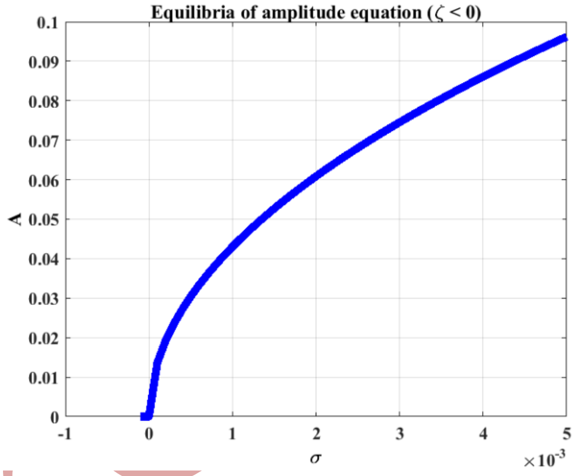

Fig 3.Equilibria of amplitude equation for $\zeta<0$

It should be noted that $\zeta > 0$ is not possible because $\zeta = \frac{(3\beta H)}{(8 l_C W^3)}$ and β cannot have positive value.

Fig. 3 implies that finite amplitude limit cycle oscillations (LCOs) appear and disappear in the system as σ is varied across $= 0$. This phenomenon is termed as Hopf bifurcation. In fact, for $\zeta < 0$ it is supercritical Hopf bifurcation, since the new branch of equilibria exists only for the values of σ which are larger than that at the onset of bifurcation. Furthermore, the limit cycle is stable only when $\zeta < 0$.. Thus, the amplitude of sustained oscillations is given by:

$$A = \sqrt{\frac{-\sigma}{\zeta}} \quad (17)$$

By separating the variables and using partial fraction expansion, Eq. (17) can be solved to get the complete (transient) amplitude of surge oscillations as:

$$A(\tau_1) = \sqrt{\frac{\sigma C e^{2\sigma\tau_1}}{C^2 - \zeta C e^{2\sigma\tau_1}}} \quad (18)$$

where C is a constant that will be determined by the initial conditions. The solution of phase correction equation can be written as:

$$P(\tau_1) = C \tag{19}$$

From the closed form analytical solution, it can be deduced that ζ and σ are the key parameters that affect the surge dynamics in terms of the onset and the appearance of limit cycles. Once σ is expressed in terms of more intuitive quantities, it becomes:

$$\sigma = \frac{H(\alpha+3\beta)}{l_C W} \tag{20}$$

whereas, ζ is expressed as:

$$\zeta = \frac{3\beta H}{8 l_C W^3} \tag{21}$$

The scope of this study is on the behavioral change of compressor characteristic, represented by α and β. It should be noted that all other parameters in Eq. (20) and (21) are positive therefore the signs of ζ and σ are explicitly dependent on compressor characteristic curve quantities. In order to preserve the qualitative shape of the compressor characteristic, the value of β should always be negative as elaborated in Fig. 1 and Fig. 2 as well. The occurrence of limit cycle (surge) is subject to positive sign of σ and that occurs only when the condition $(\alpha + 3\beta) > 0$, is satisfied.

4. Comparison with numerical simulations

In order to approximate the accuracy of analytical results obtained in Eq (18) and Eq (19), a set of two initial conditions are required. For flow coefficient Φ initial condition as given in [22] are $\Phi(0) = 0.5$ and $\dot{\Phi}(0) = 0$. The values of other parameters also taken from [22] are $H = 0.18$, $W = 0.25$, $l_C = 8$, $B = 1$ and for the numerical simulations, fourth-order Runge-Kutta method is used to integrate the governing equation of surge (Eq. (6)). Subroutine *ode45* of MATLAB® is used for this purpose. A simulation of surge phenomena is shown in Fig. 4.

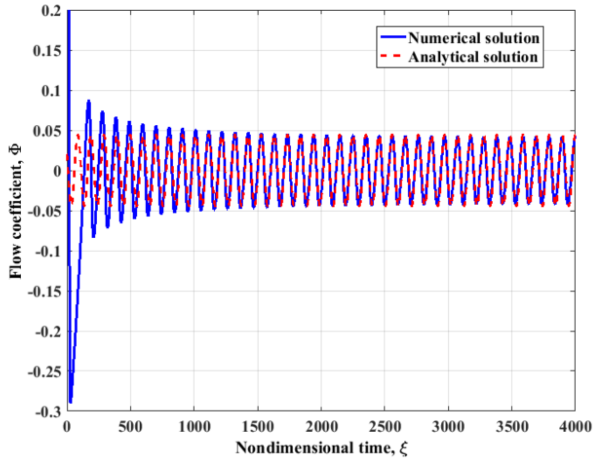

Fig 4a. Comparison of numerical and analytical solutions.

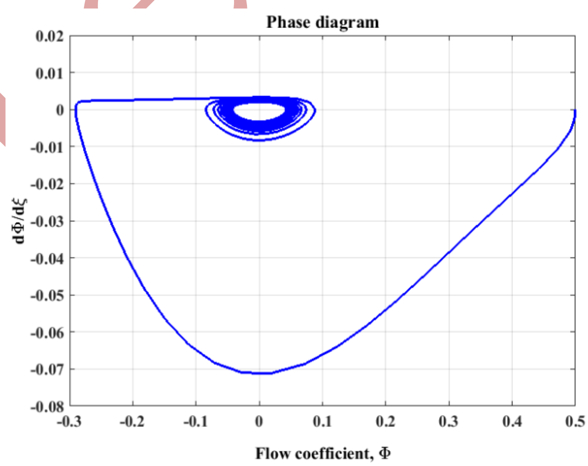

Fig.4b. Phase portrait under given initial conditions

It is evident that the analytical solution is in well agreement with the numerical solution. It should be noted that there is some discrepancy in the transient behavior but an excellent agreement for the steady state dynamics demonstrates the adequacy of the analytical model derived in this paper.

5. Conclusions

The effect of compressor characteristic on surge phenomena in axial flow compressors is analyzed in detail. The generalized effect of compressor characteristic under cubic nonlinearity on the behavior of limit-cycle oscillations is considered. Using the multiple time scales method, approximate solutions are obtained, and from these solutions, stability criterion for each case as well as the necessary conditions for sustained limit-cycle-oscillations are derived. It is observed that the behavior of surge oscillations is significantly dependent on the choice of compressor characteristic. The amplitude equation gives birth to supercritical Hopf bifurcation phenomena. MTS has successfully captured the qualitative and quantitative aspects of the phenomena. The analytical solution obtained via MTS is well in agreement to the numerical results. Such knowledge can be very useful in turbomachinery design to avoid surge associated problems during conceptual design phase.

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