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Derivation of turbulent energy of fiber suspensions

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Abstract

The energy equation for turbulent flow of fiber suspensions was derived in terms of second order correlation tensors. Fiber motion of turbulent energy including the correlation between pressure fluctuations and velocity fluctuations was discussed at two points of flow field, at which the correlation tensors were the functions of space coordinates, distance between two points, and time.

1. Introduction

Turbulent energy of fiber suspensions can be found in many areas of industry, such as production of composite materials, environmental engineering, chemical engineering, textile industry, paper making, and so on. In all the composite materials and paper, the overall orientation of the reinforcing part (fibers) plays a crucial role in deciding on the mechanical properties of the final product. Thus, the fiber suspensions property has a significant effect on the quality of products. In papermaking processes, mechanical properties of manufactured paper are deeply influenced by anisotropic fiber orientation induced by the carrier flow [1]. The quality of the fabric, in terms of fiber density uniformity, is governed by the fibre-flow interaction which determines the fiber movement in the pipeline and in the laydown area. Fibers in the flow play a role in suppressing turbulence and reducing drag; such effects become obvious with the increase of fiber mass concentration. Stability and drag reduction are revealed in transient channel flow of fiber suspension [2,3]. At great concentrations, there is an interaction between the fibers through collisions and through the effects on the flow of the fluid in the neighborhood of the particles. Bracco et al. [4] described a scenario in which turbulence mediated a process by aggregating particles into anti-cyclonic regions. The simulation result of this study suggested that the anti-cyclonic vortices were formed as long-lived coherent structures and the process became more powerful, because such vortices effectively trapped the particles.
Turbulence is maintained by turbulent energy production, where dissipation and the buoyancy flux act as sinks for the turbulent energy. The kinetic energy dissipation was determined from batchelor curve fitting [5]. Energetic of the current and balanced the turbulent energy

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equation was examined to justify using dissipation by turbulence per unit of mass as an estimation of the local production [6]. Numerical models for turbulent fluid-particle flows were reviewed by Crowe et al. [7]. The review was structured according to the turbulence models used for the continuous phase: turbulence energy dissipation models, large-eddy simulations, direct numerical simulations, and discrete vortex models. Oakey [8] examined the dissipation rate of turbulent energy from simultaneous temperature and velocity shear microstructure measurements. Spectra of turbulence were examined for both temperature gradient and velocity shear. Saito and Lemos [9] derived a macroscopic two-energy equation model for turbulent flow in a highly porous medium and applied it to a porous channel bounded by parallel plates. Macroscopic continuity, momentum, and energy equations were presented and local non-thermal equilibrium was considered by means of independent equations for the solid matrix and the working fluid.

The motion between a fluid particle and suspended fibers based on the basic fluid dynamics in order to behavior of turbulence with the correlations between pressure fluctuations and velocity fluctuations. The fiber orientation is an important physical quantity, which does not just refer to the rheology of fiber suspensions. The orientation distribution of fibers has been examined in a pipe flow [10,11]. Motion of particles was explored in the turbulent pipe flow of fiber suspensions [12]. Equation for turbulent flow of fiber suspensions and its solution and application were discussed to the pipe flow by Jian-Zhong et al. [13]. Observations of fiber suspensions were also studied in turbulent motion by Anderson [14]. An expression was derived for turbulent motion with the correlation between pressure fluctuations and velocity fluctuations at two points of the flow field [15], at which the correlation tensors were the functions of space coordinates, distance between two points, and time. An independent variable was introduced in order to differentiate between the effects of distance and location [16]. In the limiting case of zero viscosity and infinite electrical conductivity, there exist two distinct modes of turbulence, which have been distinguished as the velocity mode and the magnetic mode, respectively. Equation of fiber motion for turbulent flow was derived by Ahmed and Sarker in terms of second order correlation tensors, in which the correlation tensors were the functions of space coordinates, distance between two points, and the time [17]. The turbulent fiber motion has been also derived in a rotating system [18-20] and in the presence of dust particles [21-23]. Considering all these works, the main aim of the present study was to derive energy equation for turbulent flow of fiber suspensions in terms of second order correlation tensor, in which the correlation tensors were the functions of space coordinates, distance between two points, and the time.

2. Mathematical model of the problem

Assume that the fluid is incompressible. The energy equations of motion and continuity for turbulent flow of a viscous incompressible fluid are given by [6]:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j}-2\varepsilon_{ijl}\Omega_j u_l \quad (1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \quad (2)$$

Fiber suspensions into the flow equation of motion is given by [13]:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j\partial x_j}-2\varepsilon_{ijl}\Omega_j u_l$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \quad (3)$$

where u_i are the fluid velocity components, p is the unknown pressure field, υ is the kinematical viscosity of the suspending fluid, ρ is the density of the fluid particle, and ε_{ijl} is the three-dimensional permutation symbol, where

ε is the dissipation by turbulence per unit of mass, and Ω_j is the rotation vector [13];

$\mu_f = \dfrac{\pi n L^3 \mu}{3[\log(1/c) + \log\log(1/c) + A(c)]}$ is the apparent viscosity of fiber semi-dilute suspensions, in which n is the number of fibers per unit volume, L and d are the length and diameter of the fibers, respectively, μ is the dynamical viscosity of the suspending fluid, $c = (\pi/4)nLd^2$ is the volume fraction of the fibers, and A(c) is a constant. $\varepsilon_{lm} = \dfrac{1}{2}\left(\dfrac{\partial u_l}{\partial x_m} + \dfrac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate, I_{ij} is the turbulent intensity of suspensions, a_{lm} and a_{ijlm} are the second- and fourth-orientation tensors of the fiber, respectively, and t is the time.

Assume A and B as two points in the flow field and let a and b be two given directions at points A and B, respectively, where U_a and U_b are the velocity components along these directions. Assume that the mean velocity $\overline{U}_i$ is constant throughout the considered region and independent from time; thus:

$$\left(U_i = \overline{U}_i + u_i\right)_A,\ \left(U_j = \overline{U}_j + u_j\right)_B.$$

where the value of each term can be obtained by the equations of motion for U_j at point B and for U_i at point A.

The energy equation for u_i at point A is obtained from Eq. (3):

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{ikl}\Omega_k u_l + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] \quad (4)$$

For an incompressible fluid $\left(u_i \dfrac{\partial u_k}{\partial x_k}\right)_A = 0$ so that Eq. (4) can be written as:

$$\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A + \left(u_i\frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A - \left(2\varepsilon_{ikl}\Omega_k u_l\right)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A \quad (5)$$

Multiplying Eq. (5) by $\left(u_j\right)_B$ results in:

$$\left(u_j\right)_B\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A\left(u_j\right)_B + (u_i)_A\left(\frac{\partial}{\partial x_k}\right)_A (u_k)_A\left(u_j\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A\left(u_j\right)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A\left(u_j\right)_B - \left(2\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A\left(u_j\right)_B \quad (6)$$

where $\left(u_j\right)_B$ can be treated as a constant in a differential process at point A.

Similarly, the energy equation for u_j at point B is obtained as:

$$\frac{\partial u_j}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} - 2\varepsilon_{jkl}\Omega_k u_l + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right] \quad (7)$$

For an incompressible fluid $\left(u_j \dfrac{\partial u_k}{\partial x_k}\right)_B = 0$ so that Eq. (7) can be written as:

$$\frac{\partial}{\partial t}(u_j)_B+\left[\overline{U}_k+(u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B(u_j)_B+\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B$$
$$=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B(u_j)_B-\left(2\varepsilon_{jkl}\Omega_k u_l\right)_B$$
$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B \quad (8)$$

Multiplying Eq. (8) by $(u_i)_A$ results in:

$$(u_i)_A\frac{\partial}{\partial t}(u_j)_B+\left[\overline{U}_k+(u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B(u_j)_B(u_i)_A$$
$$+(u_j)_B\left(\frac{\partial}{\partial x_k}\right)_B(u_k)_B(u_i)_A=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B(u_i)_A$$
$$+\upsilon\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B(u_j)_B(u_i)_A-\left(2\varepsilon_{jkl}\Omega_k u_l\right)_B(u_i)_A$$
$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B(u_i)_A \quad (9)$$

where $(u_i)_A$ can be treated as a constant in a differential process at point B.

Adding Eqs. (6) and (9) gives the result:

$$\frac{\partial}{\partial t}(u_i)_A(u_j)_B+\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_k)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_k)_B(u_j)_B\right]$$
$$+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_j)_B\right]$$
$$=-\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B+\left(\frac{\partial}{\partial x_j}\right)_B p_B(u_i)_A\right]$$
$$+\upsilon\left[\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A+\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\right](u_i)_A(u_j)_B$$
$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(a_{iklm}\varepsilon_{lm}-\frac{1}{3}I_{ik}a_{lm}\varepsilon_{lm}\right)_A(u_j)_B\right.$$
$$\left.+\left(\frac{\partial}{\partial x_k}\right)_B\left(a_{jklm}\varepsilon_{lm}-\frac{1}{3}I_{jk}a_{lm}\varepsilon_{lm}\right)_B(u_i)_A\right]$$
$$-2\left[\left(\varepsilon_{ikl}\Omega_k u_l\right)_A(u_j)_B+\left(\varepsilon_{jkl}\Omega_k u_l\right)_B(u_i)_A\right] \quad (10)$$

To expose the relation of turbulent energy at point B to the one at point A, no difference will be obtained if one point is taken as the origin of A or B of the coordinate system. Consider point A as the origin. In order to differentiate between the effects of distance and location, the new independent variables are introduced:

$$\zeta_k=(x_k)_B-(x_k)_A$$

Then: $\left(\frac{\partial}{\partial x_k}\right)_B=\frac{\partial}{\partial\zeta_k}$

$$\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A=\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B=\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}$$

Using the above relations in Eq. (10) and taking ensemble average on both sides, Eq. (10) becomes:

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B}-\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B}+\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B}$$
$$=-\frac{1}{\rho}\left[-\frac{\partial}{\partial\zeta_i}\overline{p_A(u_j)_B}+\frac{\partial}{\partial\zeta_j}\overline{p_B(u_i)_A}\right]+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}\overline{(u_i)_A(u_j)_B}$$
$$-\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[\overline{\left(a_{iklm}\varepsilon_{lm}\right)_A(u_j)_B}-\frac{1}{3}\overline{\left(I_{ik}a_{lm}\varepsilon_{lm}\right)_A(u_j)_B}\right]$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[\overline{\left(a_{jklm}\varepsilon_{lm}\right)_B(u_i)_A}-\frac{1}{3}\overline{\left(I_{jk}a_{lm}\varepsilon_{lm}\right)_B(u_i)_A}\right]$$
$$-2\left[\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A(u_j)_B}+\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B(u_i)_A}\right] \quad (11)$$

Eq. (11) represents the mean motion for turbulent energy of fiber suspensions.

It is noted that the coefficient of $\overline{U}_k$ is vanished. Eq. (11) describes the turbulent energy motion of fiber suspensions, in which the motions with respect to a coordinate system move with the mean velocity $\overline{U}_k$.

Eq. (11) contains the double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$, and triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$, where all the terms are apart from one another. The correlations $\overline{p_A(u_j)_B}$

and $\overline{p_B(u_i)_A}$ form the first order tensors, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form the third order tensors. The double and triple correlations at the two points A and B in the flow field are shown in Figs. 1 and 2, respectively, where r is the distance between two points A and B.

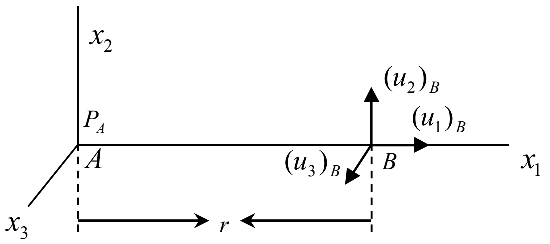

Fig. 1(a). Double correlation between pressure at A and velocity components at B.

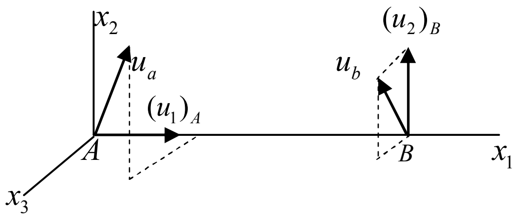

Fig. 1(b). Double velocity correlation between the velocities u_a at A and u_b at B.

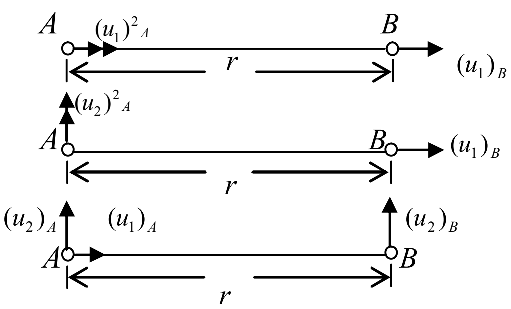

Fig. 2. Triple velocity correlation among the velocities at points A and B.

The first order correlations are designated by $(k_{p,j})_{A,B}$, second order correlations by $(Q_{i,j})_{A,B}$, and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore:

$$
\begin{aligned}
&(k_{i,p})_{A,B} = \overline{(u_i)_A p_B}, \; (k_{p,j})_{A,B} = \overline{p_A(u_j)_B}, \\
&(Q_{i,j})_{A,B} = \overline{(u_i)_A(u_j)_B}, \\
&(s_{ik,j})_{A,B} = \overline{(u_i)_A(u_k)_A(u_j)_B}, \\
&(s_{i,kj})_{A,B} = \overline{(u_i)_A(u_k)_B(u_j)_B}.
\end{aligned}
$$

where the index p indicates the pressure and is not a dummy index like i or j so that the summation convention does not apply to p.

Also, the terms $\overline{(\varepsilon_{ikl}\Omega_k u_l)_A(u_j)_B}$ and $\overline{(\varepsilon_{jkl}\Omega_k u_l)_B(u_i)_A}$ form the second order correlation tensors, designated by $M_{i,j}$ and $N_{i,j}$, respectively; the terms $\overline{(a_{jklm}\varepsilon_{lm})_B(u_i)_A}$ and $\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)_A}$ form the third order correlation tensors, designated by $D_{i,jk}$ and $H_{i,jk}$, respectively.

Thus:

$$
\begin{aligned}
&(M_{i,j})_{A,B} = \overline{(\varepsilon_{ikl}\Omega_k u_l)_A(u_j)_B}, \\
&(N_{i,j})_{A,B} = \overline{(\varepsilon_{jkl}\Omega_k u_l)_B(u_i)_A}, \\
&(D_{i,jk})_{A,B} = \overline{(u_i)_A(a_{jklm}\varepsilon_{lm})_B}, \\
&(D_{ik,j})_{A,B} = \overline{(a_{iklm}\varepsilon_{lm})_A(u_j)_B}, \\
&(H_{i,jk})_{A,B} = \overline{(u_i)_A(I_{jk}a_{lm}\varepsilon_{lm})_B}, \\
&(H_{ik,j})_{A,B} = \overline{(I_{ik}a_{lm}\varepsilon_{lm})_A(u_j)_B}.
\end{aligned}
$$

If the above relations of first, second, and third order correlations are used in Eq. (11), then:

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}S_{ik,j}+\frac{\partial}{\partial\zeta_k}S_{i,kj}=-\frac{1}{\rho}\left(-\frac{\partial}{\partial\zeta_i}K_{p,j}+\frac{\partial}{\partial\zeta_j}K_{i,p}\right)$$
$$+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-2[(M_{i,j}+N_{i,j})]$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[(D_{i,jk}-D_{ik,j})+\frac{1}{3}(H_{ik,j}-H_{i,jk})\right] \quad (12)$$

where all the correlations refer to two points A and B.

Now, for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero; ie.e:

$$(k_{p,j})_{A,B}=0,\ (k_{i,p})_{A,B}=0.$$

In the case of isotropy, the statistical features have no directional preference and the perfect disorder persists. Velocity fluctuations are independent from the axis of reference; i.e. invariant to axis rotation and reflection. From the definition of isotropy, $(Q_{i,j})_{A,B}=\overline{(u_i)_A(u_j)_B}=0$ for all $i\neq j$. In the rotating system in the flow field, 180^0 about x_1-axis, because of isotropy, results in:

$$\overline{(u_1)_A(u_2)_B}=\overline{(u_1)_A[-(u_2)]_B}=-\overline{(u_1)_A(u_2)_B}$$

which can be true only when $\overline{(u_1)_A(u_2)_B}=0$.

The isotropic turbulence in a bounded domain is a model, which is unaffected by the boundaries enclosing the fluid in the turbulence; furthermore, the statistical moments are spatially invariant and independent from orientation. Isotropic grid turbulence is a similar idealization in that the turbulence is enclosed by wind tunnel walls and the homogeneity of the turbulence in the central region is known to be unaffected by the wall boundary layers.

In an isotropic turbulence, it follows the condition of invariance under reflection in terms of point A,

$$\overline{(u_i)_A(u_k)_B(u_j)_B}=-\overline{(u_k)_A(u_j)_A(u_i)_B}$$

or, $(s_{i,kj})_{A,B}=-(s_{kj,i})_{A,B}$

In the absence of isotropic turbulence, physical properties will be different in different directions according to the direction of measurement. Anisotropic turbulence tends toward local isotropy in that the statistics of velocity differences tends toward invariance under rotation as the distance between the velocities becomes smaller. For non-isotropic (anisotropic) turbulence, constant or non-constant average velocity of pressure field will not be zero. Anisotropy is the property of being directionally dependent. It can be defined as a difference when measured along different axes in a material's physical or mechanical properties (absorbance, refractive index, conductivity, tensile strength, etc.).

Eq. (12) can be written using $(s_{i,kj})_{A,B}=-(s_{kj,i})_{A,B}$ as :

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}(S_{ik,j}+S_{kj,i})=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}$$
$$-2[(M_{i,j}+N_{i,j})]$$
$$+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial\zeta_k}(D_{jk,i}+D_{ik,j})+\frac{1}{3}\frac{\partial}{\partial\zeta_k}(H_{ik,j}+H_{jk,i})\right] \quad (13)$$

The terms $\frac{\partial}{\partial\zeta_k}(S_{ik,j}+S_{kj,i})$, $\frac{\partial}{\partial\zeta_k}(D_{jk,i}+D_{ik,j})$, $\frac{\partial}{\partial\zeta_k}(H_{ik,j}+H_{jk,i})$ and $(M_{i,j}+N_{i,j})$ form the second order tensors, designated by $S_{i,j}, D_{i,j}, H_{i,j}$, and $L_{i,j}$, respectively.

Thus we set,

$$S_{i,j}=\frac{\partial}{\partial\zeta_k}(S_{ik,j}+S_{kj,i}),$$

$$D_{i,j}=\frac{\partial}{\partial\zeta_k}(D_{jk,i}+D_{ik,j}),$$

$$H_{i,j} = \frac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right),$$
$$L_{i,j} = \left(M_{i,j} + N_{i,j}\right).$$

Therefore, Eq. (13) gives the result:

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\left[\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} - L_{i,j}\right] - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3}H_{i,j}\right) \quad (14)$$

Eq. (14) represents the energy equation for the turbulent flow of fiber suspensions.

3. Results and discussion

The resulting Eq. (14) has been developed in terms of second order correlation tensors when fiber was suspended into the turbulent flow. The equation was derived by averaging procedure, which includes the effect of fibers and the correlations between the pressure fluctuations and velocity fluctuations at two points of the flow field. Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. Velocity of fiber fluctuates around the mean velocity of flow. Fluctuation velocity of turbulence at two points A and B of the flow field leads to the weakening of the concentration of the fiber orientation distribution on small angle. This concentration leads to be weaker and orientation distribution of the fiber becomes more uniform as Reynolds numbers increase and flow fluctuation velocity becomes stringer. The fiber velocity has the same fluctuation property as fluid velocity due to its strong following ability. The fluctuation velocity of fiber on flow direction is more energetic than that on lateral direction. As Reynolds number increases, the intensity of fluctuation velocity is enhanced, flow velocity gradient becomes more irregular, and orientation distribution of fiber becomes wider.
For non-suspending fluid in the flow, the apparent viscosity of the fluid vanishes; that is $\mu_f = 0$ so that Eq. (14) takes the following form:

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\left[\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} - L_{i,j}\right] \quad (15)$$

The above equation is the energy equation for turbulent flow in terms of second order correlation tensors.
If there are no effects of dissipation ε by the turbulence per unit mass, $L_{i,j} = 0$, so that Eq. (15) takes the following form:

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} \quad (16)$$

Eq. (16) gives the turbulent motion in terms of second order correlation tensors, which is the same as the one obtained by Hinze.

4. Conclusions

Orientation and distribution of fibers are mainly affected by flow. Fiber-turbulence interaction is a complex phenomenon that is governed by many factors, including nature of flow field, turbulent length scales, concentration, and size of fibers. In presence fiber into the turbulent flow, an energy equation was developed in this study in terms of the second order correlation tensors, which corresponds to the turbulent motion in terms of second order correlation tensor as obtained by Hinze [3]. In the developed equation, all the tensors $Q_{i,j}, S_{i,j}, L_{i,j}, D_{i,j}, H_{i,j}$ are the second order correlation tensors, where $Q_{i,j}$ and $S_{i,j}$ represent the velocity correlations at two points A and B of the flow field and $D_{i,j}$ and $H_{i,j}$ stand for velocity correlations of suspending fluid, where $L_{i,j}$ signifies the correlation between angular velocity due to rotation and velocity of fluid particles at the two points. But, in the absence of fiber in the fluid and without any effect of dissipation ε by the turbulence per unit mass to the fluid velocity, the resulting Eq. (14) is reduced to Eq. (16), which represents the turbulent motion.

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