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Application of linear and nonlinear vibration absorbers for the nonlinear beam under moving load

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Abstract

Recently, a large amount of studies have been related to nonlinear systems with multi-degrees of freedom as well as continuous systems. The purpose of this paper is to optimize passive vibration absorbers in linear and nonlinear states for an Euler-Bernoulli beam with a nonlinear vibratory behavior under concentrated moving load. The goal parameter in the optimization is maximum deflection of the beam. The large deformation for beam modeling is considered, i.e. the relation between strains and deflections is nonlinear. The force magnitude and beam length are two effective factors for the beam deflection. Vibration absorber with linear damping and linear or nonlinear stiffness is also considered in this manuscript. The results show that, for normal forces and short beams, linear and nonlinear models have similar behaviors, while surveying nonlinear behavior is necessary by increasing the force and length of the beam, i.e. large deflections. Moreover, the difference between linear and nonlinear beam models for regular force magnitudes and beam lengths is negligible. For higher loads and longer beams, beam model nonlinearity can be important. Results demonstrate that,in the presented numerical values (train bridge application) for cubic nonlinear vibration absorber, there are two optimal locations for vibration absorber installation: one inclined from the middle of the beam to the direction of moving loads and the second which is more interestingly inclined from the middle of the beam to moving loads in the opposite direction. Moreover, depending on the model's numerical parameters, for short beams, linear vibration absorber is more effective, while for long beams, cubic nonlinear beam behaves better than the linear one.

1. Introduction

The dynamic response of bridges subjected to the passage of moving vehicles continues to be a subject of great interest for structural engineers. In early studies, a bridge has been modeled as a beam-like structure and a vehicle as a moving load or moving mass [1-5]. Such a model has been adopted in later studies including those by Warburton [6], Stanisic [7], Sadiku and Leipholz [8], and Akin and Mofid, [9]. Meanwhile, more delicate vehicle and bridge models that consider the effects of multi-axle loadings, multi-lane loadings, vehicle

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suspension, surface roughness have been developed and included in the analysis of the bridge response [10, 11].

Two common ways for simulating the beam model are Euler-Bernoulli and Timoshenko beam models in linear or nonlinear conditions; nonlinearity may arise from material nonlinearity or geometry nonlinearity. The relationship of material nonlinearity between stress and strain is nonlinear; also, the relationship of geometry nonlinearity between strain and displacement is nonlinear. The latter one is important for the systems with large deformations or systems that may fail due to buckling. In beams and plates, nonlinearity is from the nonlinear strain equations, in which the transverse displacement is coupled to the axial strains. As a result, mid-plane stretching of the beam or plate may occur.

Von Karman theory or large deformation theory of plates uses geometric nonlinearity in its derivations. Nonlinear moment-curvature relationship becomes significant when large deformations without stretching are considered. This theory does not consider the slope of the beam deflected middle surface. Another type of nonlinearity is related to boundary conditions, e.g. nonlinear spring or damper at plate edges or nonlinear spring in a mass-spring-damper vibration absorber. Duffing's equation is a special case of a cubic nonlinear spring in a mass-spring-damper system [12].

In this article, linear and nonlinear vibration absorbers are optimized for the nonlinear beam model. The results show that the linear vibration absorber has better performance for short and regular beam lengths, while for long beams, the vibration absorber with cubic nonlinear stiffness behaves better than the linear one.

It is demonstrated that, for the beams with ordinary lengths, it is not necessary to consider the nonlinearity for the beam equations. In this paper, nonlinear vibration absorbers behave better than the linear ones for longer beams and also the nonlinearity of the beam can be ignored for the ordinary length even if nonlinear vibration absorbers are applied. From the practical point of view, the results may become important for micro- and nano-scales, which can be a recommendation for future works.

2. Basic equations

Consider the system represented in Fig. 1: a simply supported nonlinear beam is connected to a small mass through a linear or nonlinear spring and a linear viscous damper; the beam is loaded with a traveling pointload [13, 14].

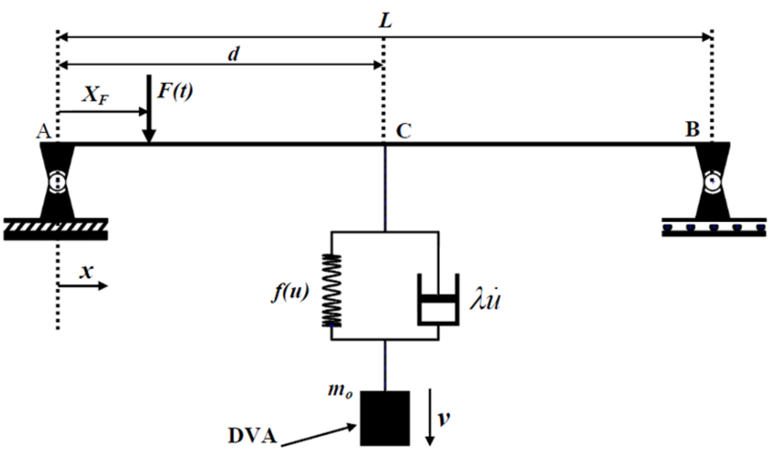

Fig. 1. The beam model.

In this article, numerical parameters of the beam without any attachment from [15] are used. L=17.4 m is the beam length, m=16300 kg/m is the mass per unit length, A=6.8 m×1m is the cross-section area, $D=EI=1.12\times10^{11}$N.m^2 is bending rigidity, E=209 GPa is Young's modulus, I is the moment of inertia of the cross-section area, ξ=0.04 is the beam damping rate, and V=72.22 m/s is the moving load speed.
Considering the Hamilton's principle for classical nonlinear Euler-Bernoulli theory in beam modeling[12], the equation of motion of the nonlinear beam without vibration absorber is given by Eq. (1):

$$m\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}-\frac{EA}{L}\left(u_L-u_0+\frac{1}{2}\int_0^L(\frac{\partial y}{\partial x})^2\mathrm{d}x\right)\frac{\partial^2 y}{\partial x^2}=F(x,t) \quad (1)$$

where u_L is the axial deformation of the beam at $x = L$ and u_0 is the axial deformation of the beam at x=0. In this manuscript, these parameters are assumed as $u_0 = u_L= 0$ [12].
The beam dynamics is governed by the PDE represented by Eq. (2) with simply supported boundary conditions (3) and initial conditions (4) (Fig. 1):

$$m\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}-\frac{EA}{L}\left(\frac{1}{2}\int_0^L(\frac{\partial y}{\partial x})^2\mathrm{d}x\right)\frac{\partial^2 y}{\partial x^2}+\left[f(u)+\lambda\frac{\partial u(t)}{\partial t}\right]\delta(x-d)=F(x,t)\,,\,x\in(0,L)\,,\,t>0 \quad (2)$$

$$y(0,t)=0\,,\,y(L,t)=0\,,\,\frac{\partial^2 y}{\partial x^2}(0,t)=0\,,\,\frac{\partial^2 y}{\partial x^2}(L,t)=0 \quad (3)$$

$$y(x,0)=0\,,\,\frac{\partial y}{\partial t}(x,0)=0 \quad (4)$$

$$m_0\frac{\partial^2 v(t)}{\partial t^2}-f(u)-\lambda\frac{\partial u(t)}{\partial t}=0$$
$$v(0)=0\,,\,\frac{\partial v}{\partial t}(0)=0\,,\,t>0 \quad (5)$$

$$u(t)=y(d,t)-v(t)\,,\,f(u)=ku \quad or \quad Cu^3 \quad (6)$$

The term $[f(u)+\lambda\,\partial u/\partial t\,]\delta(x-d)$ represents the force exerted by the vibration absorber, $f(u)$ is stiffness force, see Eq. (6) for $\lambda(\partial u/\partial t)$ definition, is viscous damping force,$\delta(x\text{-}d)$ defines the location of vibrationabsorber, and$F(x,t)$ is the external force for simulating moving load. Equation (5) governs the dynamics of the vibration absorber. $y(x,t)$ is the transverse displacement field of the beam (down is positive), $v(t)$ is the absolute position of the mass m_0, $x=d$ represents the location of the damper on the beam, $\lambda$is the damping coefficient of the viscous damper, andm_0 is the mass of the vibration absorber.
The attached mass is lightweight compared to the beam mass; indeed, using weighty masses for the vibration absorber causes a more effective vibration–reduction on the beam; however, the static deflection of the beam increases as well. Therefore, the mass of the vibration absorber cannot be too large; in this work, the lumped mass of the vibration absorber is taken as 5% of the total mass of the beam [13].
The dynamics of the system (2) is analyzed after projecting the partial differential Eq. (2) into a complete and orthonormal basis; for the present problem, the eigenfunctions of the linear operator representing the simply supported beam with no attachments can be used.

$$\varphi_r(x)=(\frac{2}{mL})^{1/2}\sin(\frac{r\pi x}{L})\quad,\quad r=1,2,\ldots$$
$$\omega_r=(r\pi)^2(\frac{EI}{mL^4})^{1/2}\quad,\quad r=1,2,\ldots \quad (7)$$

ω_r is the natural frequency of the r^{th} mode. The eigenfunctions satisfy the following orthonormality conditions.

$$\int_0^L m\varphi_i(x)\varphi_j(x)\mathrm{d}x=\delta_{ij}\quad,\quad i,j=1,2,\ldots$$
$$\int_0^L \varphi_i(x)(EI\varphi_j''(x))''\mathrm{d}x=\omega_j^2\delta_{ij}\quad,\quad i,j=1,2,\ldots \quad (8)$$

where δ_{ij} is Kronecker delta.
The transverse vibration of the beam can be assumed to be expressed as:

$$y(x,t)=\sum_{r=1}^{\infty}a_r(t)\varphi_r(x) \quad (9)$$

where $a_r(t)$ are unknown functions of time (modal coordinates) and $\phi_r(x)$ are the normalized eigenfunctions. By inserting Eq. (9) in Eq. (2), projecting into the p^{th} eigenfunctions, and using the orthonormality conditions, the following is obtained:

$$\frac{\partial^2 a_p(t)}{\partial t^2}+2\xi\omega_p\frac{\partial a_p(t)}{\partial t}+\omega_p^2 a_p(t) + \frac{A}{2mIL}\omega_p^2 a_p^3(t)+\{D(t)+ \lambda[\sum_{r=1}^{\infty}\frac{\partial a_r(t)}{\partial t}\varphi_r(d)-\frac{\partial v(t)}{\partial t}]\}\varphi_p(d)=\bar{F}(t) \quad , \quad p=1,2,\ldots \tag{10}$$

$$m_0\frac{\partial^2 v(t)}{\partial t^2}-D(t)+ \lambda[\frac{\partial v(t)}{\partial t}\sum_{r=1}^{\infty}\frac{\partial a_r(t)}{\partial t}\varphi_r(d)-\frac{\partial v(t)}{\partial t}]=0 \tag{11}$$

Linear vibration absorber:

$$D(t)=C[\sum_{r=1}^{\infty}a_r(t)\varphi_r(d)-v(t)] \tag{12}$$

Cubic nonlinear vibration absorber:

$$D(t)=C[\sum_{r=1}^{\infty}a_r(t)\varphi_r(d)-v(t)]^3 \tag{13}$$

For moving load:

$$\bar{F}(t)=F_0\varphi_p(Vt)[H(\frac{L}{V}-t)] \tag{14}$$

which are obtained by considering the following forcing in Eq. (2).

$$F(x,t)=F_0\delta(x-Vt)[H(\frac{L}{V}-t)] \tag{15}$$

$\delta$is the Dirac function and $H(t)$ is the Heaviside function.

$$H(t)=\begin{cases}0 & t<0\\ 1 & t>0\end{cases} \tag{16}$$

The transient dynamics are studied by numerically integrating the dynamical system represented by Eqs. (10) and (11) after truncating series (9); the truncation is suitably chosen by checking the convergence of the expansion. Three terms are considered for the truncation.

3. Validations

The basic model and numerical calculations for linear beam model subjected to transient moving load in this text is the same as that mentioned in [13]. Deep analyses and comparisons were carried out in [13]; therefore, the present model can be considered partially validated. Additional comparisons and validations are performed to check the nonlinear beam model. In the following section,the results of nonlinear beam model are compared with those of linear beam model. There is no vibration absorber attached to the beam. y is the beam deflection, $y_{Linear}|_{max}$ is the maximum deflection of linear beam, and $V_{Critical}$ is the critical velocity of the train.

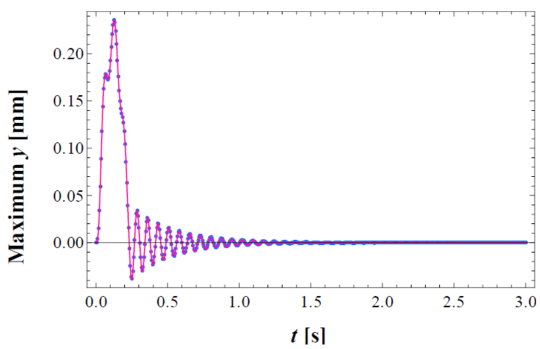

Fig. 2. Comparing deflection between linear (. . .) and nonlinear (—) beams with F_0=216kN.
$V_{Critical}=73.89$ m/s , $y_{Linear}|_{max}=0.2357$ mm , $y_{Nonlinear}|_{max}=0.2357$ mm

Figure 2 presents a comparison between linear and nonlinear beam deflections. It is obvious that beam deflection for the nonlinear beam model is almost the same as that of the linear model for regular loads. Note that, later, it will be found that, for bigger loads, the difference between these two models can be consequential. In the next sections, the effects of some factors on the beam deflection such as force and beam length are considered.

4. Evaluating critical velocity

In this section, the purpose is to determine critical velocity of moving load. Critical velocity occurs in the place of maximum beamdeflection. In Savin's bridge model[15], the minimum and maximum velocity of the train, when it passes from the bridge, is 114 and342Km/h, respectively. The following charts show that results

of linear and nonlinear beamsare the same when F_0=216kN [15]. Note that, maximum and minimum train speed is 31.67 m/s and 95m/s [15], respectively. Figure 3 demonstrates critical velocity in linear and nonlinear beam modelsas 73.89 m/s and maximum deflection of beam as 0.2473 mm in both models.

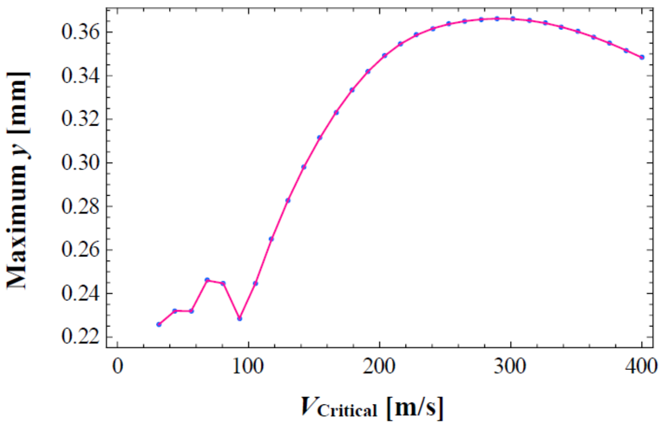

Fig. 3. Maximum deflection vs. loadspeed for linear (. . .) and nonlinear (—) beam.

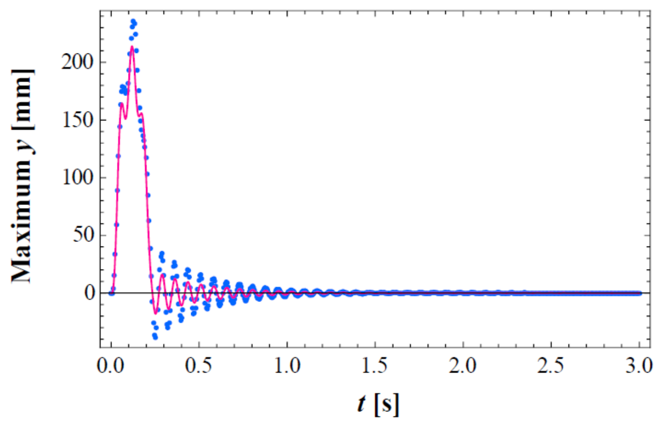

Fig. 4. Comparison linear (. . .) and nonlinear (—) beamdeflection with F=216 kN.

$V_{\text{Critical}} = 73.89$ m/s , $y_{\text{Linear}} = 235.7$ mm , $y_{\text{Nonlinear}} = 213.9$ mm

5. Effect of load magnitude

this section, the effects of force amplitude on the beam deflection are examined. By increasing the magnitude of the force, difference between linear and nonlinear beam deflections is remarkable. Validation of this subject is shown in the following charts and their results are presentedin Table 1. In this table, $F_0$is equal to 216 kN. Figure 4 demonstrates the comparison of linear and nonlinear beam deflections.

By increasing force magnitude,the difference of linear and nonlinear beam deflections increases gradually. Linear beam displacement is bigger than the nonlinear one andresults show that usage of nonlinear beam model is important, just in the case of big loads.

6. Effect of beam length

In this section, effect of length on the maximum beam deflection is surveyed. The moving load passage time is 2.94 s for the longest beam considered in this section. In order tocomparedifferentbeamlengths, execution time equal to 4s is considered. In Savin's bridge model [15], beam length is equal to 17.4 m.

The above chart shows that,with increasing beam length, deflection in the linear beam becomes higher than that of the nonlinear one and the difference of linear and nonlinear beam models increases (Fig. 5).

7. Optimizing vibration absorber

The vibration absorber is a classical device for avoiding large amplitude vibrations of a mechanical system subject to a sinusoidal varying excitation when the forcing frequency coincides or is near one of the natural frequencies of the system. Obviously, severe vibrations of a given structure or mechanical system may cause considerable disturbance, which is inconvenient from the human factors viewpoint and may be conducive to failure due to fatigue, etc. Percentage of deflection difference for the linear and nonlinear beams is shown in Fig. 6 and their results are presented in Table 2.

Table 1. Deflection difference percent in linear and nonlinear beam varying by force amplitude; F_0=216 kN.

F [kN]	F_0	$10F_0$	$100F_0$	$1000F_0$
Deflection difference percent	0 %	0 %	0.13 %	9.25 %
y_{Linear} [mm]	0.2357	2.357	23.57	235.7
$y_{\text{Nonlinear}}$ [mm]	0.2357	2.357	23.54	213.9

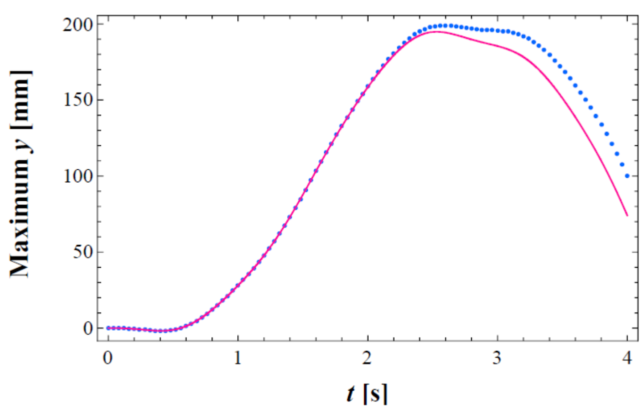

Fig. 5. Comparison linear (. . .) and nonlinear (—) beam's deflection, L=167.4m.
$V_{\text{Critical}} = 73.89$ m/s , $y_{\text{Linear}} = 207.6$ mm , $y_{\text{Nonlinear}} = 202.1$ mm.

Table 2. Deflection difference percent for linear and nonlinear beam vs. beam length.

Beam length [m]	17.4	67.4	117.4	167.4
Deflection difference percent	0 %	0.1 %	0.81 %	2.65 %
y_{Linear} [mm]	0.2357	20.11	92.23	207.6
$y_{\text{Nonlinear}}$ [mm]	0.2357	20.09	91.48	202.1

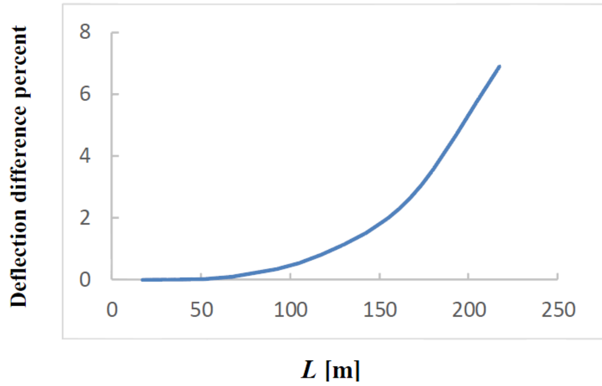

Fig. 6. Deflection difference percent of beam length between the linear and nonlinear beams.

Optimum linear and cubic nonlinear vibration absorber was obtained for a linear beam model in [13]. In this section, the importance of beam nonlinearity model is evaluated when nonlinear vibration absorber is attached and the results are gathered in Table 3. As can be observed, if F_0=216kN, results of linear and nonlinear beamswill bethe same. In Table 3,L equals 17.4 m.

The governing equation of nonlinear Euler-Bernoulli beam with linear vibration absorber is:

$$\frac{\partial^2 a_p(t)}{\partial t^2}+2\xi\omega_p\frac{\partial a_p(t)}{\partial t}+\omega_p^2 a_p(t)+\frac{A}{2mIL}\omega_p^2 a_p^3(t)+\{k[\sum_{r=1}^{\infty}a_r(t)\varphi_r(d)-v(t)]+\lambda[\sum_{r=1}^{\infty}\frac{\partial a_r(t)}{\partial t}\varphi_r(d)-\frac{\partial v(t)}{\partial t}]\}\,\varphi_p(d)=\bar{F}(t)\quad ,p=1,2,\ldots \tag{17}$$

Equation (18) is the same as Eq. (17) for the vibration absorber with cubic nonlinear stiffness:

$$\frac{\partial^2 a_p(t)}{\partial t^2}+2\xi\omega_p\frac{\partial a_p(t)}{\partial t}+\omega_p^2 a_p(t)+\frac{A}{2mIL}\omega_p^2 a_p^3(t)+\{C[\sum_{r=1}^{\infty}a_r(t)\varphi_r(d)-v(t)]^3+\lambda[\sum_{r=1}^{\infty}\frac{\partial a_r(t)}{\partial t}\varphi_r(d)-\frac{\partial v(t)}{\partial t}]\}\,\varphi_p(d)=\bar{F}(t)\quad ,p=1,2,\ldots \tag{18}$$

The goal function to define optimum vibration absorber is to minimize the maximum deflection of the beam. The optimization is carried out for the linear and nonlinear beam models. The optimal damping for the present transient load defined in [13] is zero. Small damping is considered here to prevent numerical errors. It is obvious that, for the present beam length and force magnitude, the results are the same for linear and nonlinear beam models. Figure 7(a) presents the variation of maximum beam deflection versus stiffness of linearvibration absorber. k_{Linear} and $k_{\text{Nonlinear}}$ represent the stiffness of optimal vibration absorber for linear beam models. Figure 7(b) presents the variation of maximum beam deflection versus stiffness of cubic nonlinear vibration absorber. C_{Linear} and $C_{\text{Nonlinear}}$show the optimal stiffness of vibration absorber for linear and nonlinear beam models.

Figure 8 presents the optimal location for linear and cubic nonlinear vibration absorber. *d* is the installation location of vibration absorber.

As shown in the chart, the optimum location for attaching vibration absorber on the beam is not exactly in the middle of the beam. For example, when stiffness is nonlinear, its optimum location is 0.38 *L*, which is 6.612 m from the left side of the beam, as shown in Figure 1 called point A. The results show that the linear vibration absorber has better performance for short and regular beam lengths, while for long beams, the vibration absorber with cubic nonlinear stiffness behaves better. For the present numerical values, two special beam lengths are recognized so that the linear and nonlinear vibration absorbers have the maximum influence.

Moreover, the presented results illustrate the beam length, at which the performance of the optimal linear vibration absorber and the optimal cubic nonlinear vibration absorber is equal. The goal function to define optimum vibration absorber is to minimize the maximum deflection of the beam. The optimization is carried out for the linear and nonlinear beam models. The optimal damping for the present transient load defined in [13] is zero. Small damping is considered here to prevent numerical errors. It is obvious that, for the present beam length and force magnitude, the results are the same for linear and nonlinear beam models. Figure 7(a) presents

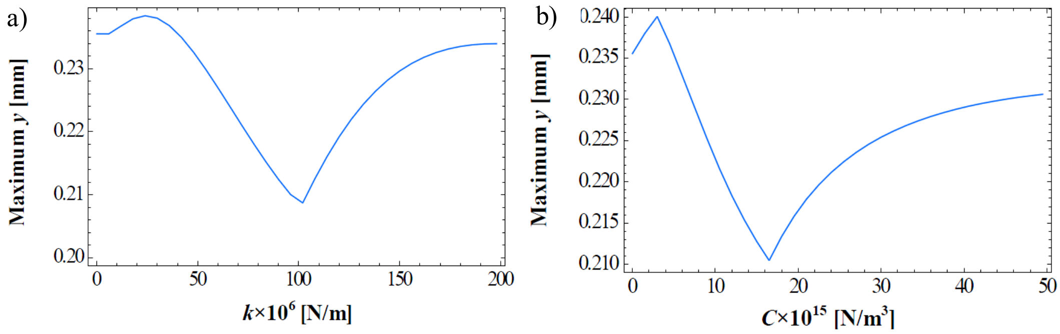

Fig. 7. Optimal stiffness of linear and nonlinear vibration absorber: maximum deflection vs. Stiffness.

a) linear vibration absorber: $d = 0.48\ L$, $k = 100 \times 10^6$ N/m , $V_{\text{Critical}} = 73.89$ m/s

b) nonlinear vibration absorber: $C_{\text{Linear}} = C_{\text{Nonlinear}} = 16.5 \times 10^{15}$ N/m^3 , $d = 0.53\ L$, $V_{\text{Critical}} = 73.89$ m/s

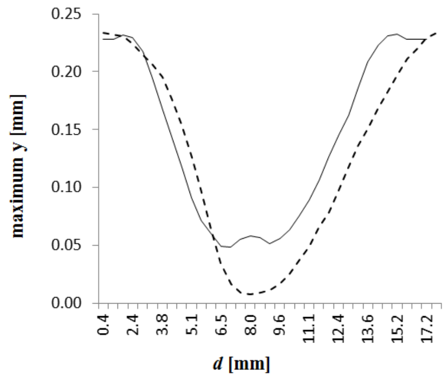

Fig. 8. Optimal location of linear and nonlinear vibration absorber: maximum deflection vs. damper location.

——— nonlinear vibration absorber , - - - - linear vibration absorber

linear vibration absorber: $d = 0.48\ L$, $k = 100 \times 10^6$ N/m , $V_{\text{Critical}} = 73.89$ m/s

nonlinear vibration absorber: $d = 0.38\,L$, $C = 16.5 \times 10^{15}$ N/m^3 , $V_{\text{Critical}} = 73.89$ m/s the variation of maximum beam deflection versus stiffness of linear vibration absorber. k_{Linear} and $k_{\text{Nonlinear}}$ represent the stiffness of optimal vibration absorber for linear beam models. Figure 7(b) presents the variation of maximum beam deflection versus stiffness of cubic nonlinear vibration absorber. C_{Linear} and $C_{\text{Nonlinear}}$show the optimal stiffness of vibration absorber for linear and nonlinear beam models. Figure 8 presents the optimal location for linear and cubic

nonlinear vibration absorber. C_{Linear} and $C_{\text{Nonlinear}}$show the optimal stiffness of vibration absorber for linear and nonlinear beam models. Figure 8 presents the optimal location for linear and cubic nonlinear vibration absorber. *d* is the installation location of vibration absorber.

As shown in the chart, the optimum location for attaching vibration absorber on the beam is not exactly in the middle of the beam. For example, when stiffness is nonlinear, its optimum location is 0.38 *L*, which is 6.612 m from the left side of the beam, as shown in Figure 1 called point A. The results show that the linear vibration absorber has better performance for short and regular beam lengths, while for long beams, the vibration absorber with cubic nonlinear stiffness behaves better. For the present numerical values, two special beam lengths are recognized so that the linear andnonlinear vibration absorbers have the maximum influence. Moreover, the presented results illustrate the beam length, at which the performance of the optimal linear vibration absorber and the optimal cubic nonlinear vibration absorber is equal.

8. Conclusions

The purpose of this paper is to optimize passive vibration absorbers in linear and nonlinear states for an Euler-Bernoulli beam with a nonlinear vibratory behavior under the concentrated moving load. The goal parameter in the optimization is maximum deflection of the beam. The large deformation is consideredfor beam modeling; i.e. the relationship between strains and deflections is nonlinear.

The presented numerical results show that critical velocity and maximum deflection of beam are identical in linear and nonlinear beam models. Moreover, by increasing beam length, deflection in the linear beam becomes higher than the nonlinear beam with vibration absorber; i.e. the difference between linear and nonlinear beam models becomes more sensible. For longer beams, the nonlinear absorber is more effective. It is demonstrated that, for the beams with ordinary lengths, it is not necessary to consider the nonlinearity for the beam equations. In this paper, nonlinear vibration absorbers behave better for longer beams and also the nonlinearity of the beam can be ignored for ordinary length, even if nonlinear vibration absorbers are applied.

In the same manner, results show that, by increasing force magnitude, linear and nonlinear beam deflection difference is increased gradually. In addition, for severe loading, nonlinear absorbers become more effective than the linear one. Similar to the conventional beam length, for the conventional forces, nonlinear terms of the beam do not considerably affect the results.

Vibration absorber with linear damping and linear or cubic nonlinear stiffness is considered. The obtained results demonstrate that the linear vibration absorber has better performance for short and regular beam lengths, while for long beams, the vibration absorber with cubic nonlinear stiffness behaves better. For the present numerical values, two special beam lengths are recognized so that the linear and nonlinear vibration absorbershave maximum influence.

Results represented that, for the presented numerical values (train bridge application) in the case of cubic nonlinear vibration absorber, there are two local optimal locations: one inclined from the middle of the beam to the direction of moving loads and the second, which is more interesting, inclined from the middle of the beam to the opposite direction of the moving loads.

Table 3. Optimization results.

F_0=215732N	Linear beam model (Damped linear beam)		Nonlinear beam model (Damped nonlinear beam)	
optimum vibration absorber	linear vibration absorber	nonlinear vibration absorber	linear vibration absorber	nonlinear vibration absorber
Optimum stiffness	100×10^6 N/m	16.5×10^{15} N/m^3	100×10^6 N/m	16.5×10^{15} N/m^3
d [m]	0.48L	0.38L	0.48L	0.38L
Δ_1 [mm]	0.2357	0.2357	0.2357	0.2357
Δ_2 [mm]	0.2083	0.2103	0.2083	0.2103
Deflection decrease percentage	11.63%	10.78%	11.63%	10.78%

d: Vibration absorber location, Δ_1: Maximum deflection without any vibration absorber, Δ_2: Maximum deflection with optimum vibration absorber.

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