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Derivation of turbulent energy in a rotating system

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Abstract

Energy equation for turbulent flow in a rotating system was derived in terms of second order correlation tensors, where the correlation tensors were functions of space coordinates, distance between two points and time. To reveal the relationship of turbulent energy between two points, one point was taken as origin of the coordinate system. Due to rotation, the Coriolis force played an important role in the rotating system of turbulent flow. The correlation between pressure fluctuations and velocity fluctuations at the two points of flow field was applied to the turbulent energy equation, in which the Coriolis force and centrifugal force acted on the fluid.

1. Introduction

Turbulent flow can be found in many areas of industry, such as production of composite materials, environmental engineering, chemical engineering, textile industry, paper making and so on. Turbulence is maintained by turbulent energy production, where dissipation and buoyancy flux act as sinks for the turbulent energy. Rotating flows are encountered in both geophysical and industrial fluid mechanics. Internal flows in rotating ducts and channels are typically found in rotating machinery, e.g. cyclone separators, pumps and turbines. Different time scales due to rotation, stratification and turbulence open up a wide field of possibilities for temporal evolution of rotating and stratified turbulence Influence of the Coriolis body force on fluid motion exhibits a number of generic features which may occur irrespective of the particular field of application. The Coriolis force due to rotation plays an important role in a rotating system of turbulent flow while the centrifugal force with potential is incorporated into pressure. When motion is referred to the axis which rotates steadily with bulk of the fluid, the Coriolis force and centrifugal force must be supposed to act on the fluid. The centrifugal force is equivalent in its effect to the contribution to pressure.

Turbulence driven by magneto rotational instability crucially affects evolution of solid bodies in protoplanetary disks. Vertical plane of a protoplanetary disk was considered by Johansen [1] to include the Coriolis force and radial advection of the Keplerian rotation flow. Crowe et al. [2] reviewed numerical models for

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turbulent fluid-particle flows. This review was structured according to the turbulence models used for continuous phase: turbulence energy dissipation models, large-eddy simulations, direct numerical simulations, and discrete vortex models. Long-range interactions in homogeneous turbulence as a consequence of the Biot–Savart law were discussed by Davidson [3]. These long-range correlations were very weak in decaying, isotropic turbulence; they argued that this case had to be also true for magneto hydrodynamic, rotating and stratified turbulence. Luketina and Imberger [4] presented an algorithm for determining rate of turbulent kinetic energy dissipation by fitting theoretical Batchelor spectrum to temperature gradient spectrum at high wavenumbers. The result showed that use of the algorithm for determining dissipation of the turbulent kinetic energy was considerably more time-efficient than manual methods. The rate of turbulent kinetic energy dissipation was estimated from pulse-coherent acoustic Doppler current profiler measurements using the inertial dissipation method by Lorke and Wüest [5]. The measurement in the bottom boundary layer of a lake revealed astonishing agreement between dissipation rates estimated from temperature microstructure profiles and those estimated by applying inertial dissipation method to the data from two different brands of acoustic Doppler current profilers. Two new formulations were examined from the measurement of energy-containing scales of turbulence in the ocean thermocline such as an inviscid estimate for the viscous dissipation rate of turbulent kinetic energy and a mixing length estimate for the turbulent heat flux [6]. It was found that energy-containing scale estimates of both dissipation rate and heat flux were in favorable comparison with dissipation scale estimates. Bourouiba et al. [7] investigated scale-locality of the energy transfers directly contributing to growth of the two-dimensional columnar structures observed in the intermediate Rossby number regime. It was shown that the dominant energy transfer responsible for the generation of a steep two-dimensional spectrum involved direct nonlocal energy transfers from small-frequency small-horizontal-scale three-dimensional waves to large-horizontal-scale two-dimensional columnar vortices. A macroscopic two-energy equation model was derived for turbulent flow in a highly porous medium and applied to a porous channel bounded by parallel plates Macroscopic continuity, momentum and energy equations presented local non-thermal equilibrium considered by means of independent equations for the solid matrix and the working fluid [8]. Gibson and Launder's model for Reynolds stress was assessed in terms of predicting rotation effects on stably stratified homogeneous shear flows by Abid and Habibi [9]. The model was found to be capable of predicting stabilizing and destabilizing effects of rotation. The transition from fluid at rest to turbulence was studied in a rotating tank by Kolvin et al. [10]. Energy was transported by inertial wave packets through the fluid volume. These high amplitude waves were propagated at velocities consistent with those calculated from linearized theory. The observed mechanisms could lead to significant differences between rotating and two-dimensional turbulent flows. Sarker and Azad [11] studied decay of temperature fluctuations in homogeneous turbulence before the final period for the multi-point and multi-time case in a rotating system using Deissler's approach. Morinishi [12] proposed a new algorithm for simulating homogeneous decaying turbulence in an incompressible fluid subjected to uniform system rotation. Using the new algorithm, direct numerical simulation of the homogeneous decaying turbulence was carried out to investigate effects of the system rotation on turbulence. The results showed that the system inhibited decay of the kinetic energy. A parametric space study of the decay of turbulence was presented in rotating flows by Teitelbaum and Mininni [13] in which direct numerical simulations, large eddy simulations and phenomenological were combined with each other. The decay laws were obtained in the simulations for the energy, helicity and anisotropy in each case explained by phenomenological arguments that considered separate decays for two-dimensional and three-dimensional models and role of helicity and rotation in slowing down the energy decay. Time evolution of the energy spectrum and development of anisotropies in the simulations were also discussed in the study.

Rotating and stably stratified turbulence exhibit not only significant anisotropies but also dynamics, which are qualitatively different from purely rotating or stratified turbulence. Liechtenstein et al. [14] retrieved standard results, such as large anisotropy for small scales in rotating turbulence and a large anisotropy for intermediate scales in the vortex mode of stratified turbulence. The anisotropic energy transfer in freely decaying turbulence was experimentally investigated subjected to a background rotation by Lamriben et al. [15]. Hinze [16] derived an expression for turbulent motion at two points of flow field in terms of correlation tensor of second order at two points of the flow field, in which the correlation tensors were functions of space coordinates, distance between two points and time. Ahmed and Sarker [17] derived an equation of fiber motion for turbulent flow at two points of the flow field in terms of second order correlation tensor, where the correlation tensors were functions of space coordinates, distance between two points and time. They derived another equation of fiber motion for turbulent flow in a rotating system at two points of the flow field in terms of second order correlation tensor, where the correlation tensors in which the correlation tensors were functions of space coordinates, distance between two points and time. However, there are few studies on the turbulent energy in a rotating system though it is prevalent in the industry. In view of all these works, the main aim of this study was to derive an energy equation for turbulent flow at two pints of the flow field in terms of second order correlation tensor, where the correlation tensors were functions of space coordinates, distance between two points and time.

2. Mathematical model of the problem

Assume that the fluid is incompressible. The energy equations of motion and continuity for turbulent flow of a viscous incompressible fluid are given by [18]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l \tag{1}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

In a rotating system, the energy equation for turbulent flow of a viscous incompressible fluid is given by [19, 20]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\varepsilon_{ijl}\Omega_j u_l - 2(\Omega_i u_i \eta_i)\sin\theta \tag{3}$$

where u_i are fluid velocity components, p is unknown pressure field, υ is kinematical viscosity of the fluid, ρ is density of the fluid particle, ε_{ijl} is three-dimensional permutation symbol, where ε is dissipation by turbulence per unit of mass, Ω_j is rotation vector, $p = \frac{p}{\rho} + \frac{1}{2}\left|\overline{\Omega} \times \overline{u}\right|^2$ stands for generalized pressure inclusive of potential centrifugal force, υ is kinematical viscosity of the suspending fluid, $-2(\Omega_i u_i \eta_i)\sin\theta = -2(\overline{\Omega} \times \overline{u})$ is the Coriolis force in which Ω_i is angular velocity, η is the unit vector perpendicular to $\overline{\Omega}$ and $\overline{u}$, θ is the angle between $\overline{\Omega}$ and $\overline{u}$ and t is time. Assume A and B are two points in the flow field and a and b are two given directions at points A and B, respectively, where U_a and U_b are velocity components along these directions. It can be also assumed that mean velocity $\overline{U_i}$ is constant throughout the considered region and is independent from time; thus:

$\left(U_i = \overline{U}_i + u_i\right)_A, \left(U_j = \overline{U}_j + u_j\right)_B$

Value of each term can be obtained using equations of motion for u_j at point B and for u_i at point A.

Energy equation for u_i at point A is obtained from Eq. (3),

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2\varepsilon_{ikl}\Omega_k u_l - 2\left(\Omega_i u_i \eta_i\right)\sin\theta \quad (4)$$

For an incompressible fluid $\left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = 0$, Eq. (4) can be written as

$$\frac{\partial}{\partial t}\left(u_i\right)_A + \left[\overline{U}_k + \left(u_k\right)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A \left(u_i\right)_A + \left(u_i\frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A \left(u_i\right)_A - 2\left(\varepsilon_{ikl}\Omega_k u_l\right)_A - 2\left(\Omega_i u_i \eta_i\right)_A \sin\theta \quad (5)$$

Multiplying Eq. (5) by $\left(u_j\right)_B$ results in

$$\left(u_j\right)_B \frac{\partial}{\partial t}\left(u_i\right)_A + \left[\overline{U}_k + \left(u_k\right)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A \left(u_i\right)_A \left(u_j\right)_B + \left(u_i\right)_A \left(\frac{\partial}{\partial x_k}\right)_A \left(u_k\right)_A \left(u_j\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A \left(u_j\right)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A \left(u_i\right)_A \left(u_j\right)_B - 2\left[\left(\varepsilon_{ikl}\Omega_k u_l\right)_A + \left(\Omega_i u_i \eta_i\right)_A \sin\theta\right]\left(u_j\right)_B \quad (6)$$

where $\left(u_j\right)_B$ can be treated as a constant in a differential process at point A.

Similarly, energy equation for u_j at point B is obtained as:

$$\frac{\partial u_j}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} - 2\varepsilon_{jkl}\Omega_k u_l - 2\left(\Omega_j u_j \eta_j\right)\sin\theta \quad (7)$$

For an incompressible fluid $\left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = 0$, Eq. (7) can be written as

$$\frac{\partial}{\partial t}\left(u_j\right)_B + \left[\overline{U}_k + \left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B \left(u_j\right)_B + \left(u_j\frac{\partial u_k}{\partial x_k}\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B \left(u_j\right)_B - 2\left(\varepsilon_{jkl}\Omega_k u_l\right)_B - 2\left(\Omega_j u_j \eta_j\right)_B \sin\theta \quad (8)$$

Multiplying Eq. (8) by $\left(u_i\right)_A$ leads to obtaining

$$\left(u_i\right)_A \frac{\partial}{\partial t}\left(u_j\right)_B + \left[\overline{U}_k + \left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B \left(u_j\right)_B \left(u_i\right)_A + \left(u_j\right)_B \left(\frac{\partial}{\partial x_k}\right)_B \left(u_k\right)_B \left(u_i\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B \left(u_i\right)_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B \left(u_j\right)_B \left(u_i\right)_A - 2\left[\left(\varepsilon_{jkl}\Omega_k u_l\right)_B + \left(\Omega_j u_j \eta_j\right)_B \sin\theta\right]\left(u_i\right)_A \quad (9)$$

where $\left(u_i\right)_A$ can be treated as a constant in a differential process at point B.

Adding of Eqs. (6- 9) leads to the result

$$\frac{\partial}{\partial t}(u_i)_A(u_j)_B + \Bigg[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A(u_k)_A(u_j)_B$$
$$+\left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A(u_k)_B(u_j)_B$$
$$+\bar{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A(u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A(u_j)_B\right]$$
$$= -\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B + \left(\frac{\partial}{\partial x_j}\right)_B p_B(u_i)_A\right]$$
$$+\upsilon\left[\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A + \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\right](u_i)_A(u_j)_B$$
$$-2[(\Omega_i u_i \eta_i)_A(u_j)_B + (\Omega_j u_j \eta_j)_B(u_i)_A]\sin\theta$$
$$-2[(\varepsilon_{ikl}\Omega_k u_l)_A(u_j)_B + (\varepsilon_{jkl}\Omega_k u_l)_B(u_i)_A] \tag{10}$$

Exposing the relation of turbulent energy at point B to that at point A will result in no difference if one point is taken as origin of A or B of the coordinate system.

Consider point A as the origin. In order to differentiate between effects of distance and location, the following are introduced as new independent variables,

$$\zeta_k = (x_k)_B - (x_k)_A$$

Then,

$$\left(\frac{\partial}{\partial x_k}\right)_A = -\frac{\partial}{\partial \zeta_k}, \left(\frac{\partial}{\partial x_k}\right)_B = \frac{\partial}{\partial \zeta_k},$$
$$\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B = \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}.$$

Using the above relations in Eq. (10) and by averaging procedure with respect to time, Eq. (10) becomes

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B} - \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B} + \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B}$$
$$= -\frac{1}{\rho}\left[-\frac{\partial}{\partial \zeta_i}\overline{p_A(u_j)_B} + \frac{\partial}{\partial \zeta_j}\overline{p_B(u_i)_A}\right]$$
$$+2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}\overline{(u_i)_A(u_j)_B}$$
$$-2[\overline{(\Omega_i u_i \eta_i)_A(u_j)_B} + \overline{(\Omega_j u_j \eta_j)_B(u_i)_A}]\sin\theta$$
$$-2[\overline{(\varepsilon_{ikl}\Omega_k u_l)_A(u_j)_B} + \overline{(\varepsilon_{jkl}\Omega_k u_l)_B(u_i)_A}] \tag{11}$$

Equation (11) represents mean motion for turbulent energy in a rotating system with pressure-velocity correlation.

It is noted that coefficient $\bar{U}_k$ is vanished. Eq. (11) describes turbulent energy motion, where motions move with mean velocity $\bar{U}_k$ with respect to a coordinate system. Eq. (11) contains double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$ and triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$, in which all the terms are apart from one another. The correlations $\overline{p_A(u_j)_B}$ and $\overline{p_B(u_i)_A}$ form first order tensors because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form third order tensors. The double and triple correlations at two points A and B in the flow field are shown in Figs.1 and 2, respectively, where r is the distance between two points A and B.

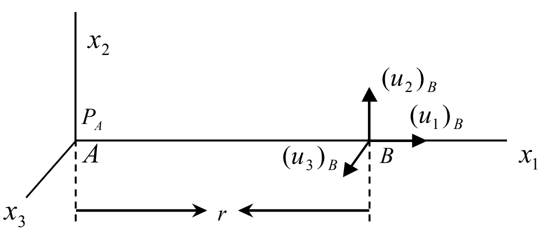

Fig. 1(a). Double correlation between pressure at A and velocity components at B.

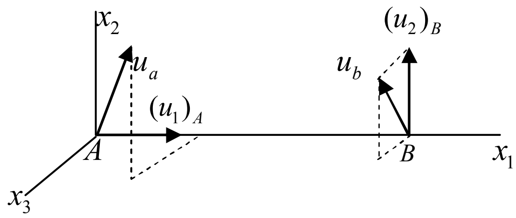

Fig. 1(b). Double velocity correlation between the velocities u_a at A and u_b at B.

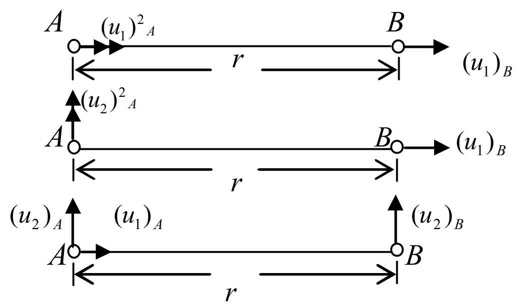

Fig. 2. Triple velocity correlation among the velocities at points A and B.

The first order correlations are designated by $\left(k_{p,j}\right)_{A,B}$, second order correlations by $\left(Q_{i,j}\right)_{A,B}$ and third order correlations by $\left(s_{ik,j}\right)_{A,B}$. Therefore:

$$\left(k_{i,p}\right)_{A,B}=\overline{\left(u_i\right)_A p_B},\ \left(k_{p,j}\right)_{A,B}=\overline{p_A\left(u_j\right)_B},$$
$$\left(Q_{i,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_j\right)_B},$$
$$\left(s_{ik,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B},$$
$$\left(s_{i,kj}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}.$$

where index p indicates pressure and is not a dummy index like i or j so that summation convention does not apply to p.

Also, terms $\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B}$ and $\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A}$ form the second order correlation tensors, designated by $D_{i,j}$ and $H_{i,j}$ respectively; $\overline{\left(\Omega_i u_i \eta_i\right)_A\left(u_j\right)_B}$ and $\overline{\left(\Omega_j u_j \eta_j\right)_B\left(u_i\right)_A}$ form the second order tensors, designated by $M_{i,j}$ and $N_{i,j}$, respectively. Thus:

$$\left(D_{i,j}\right)_{A,B}=\overline{\left(\varepsilon_{ikl}\Omega_k u_l\right)_A\left(u_j\right)_B},$$
$$\left(H_{i,j}\right)_{A,B}=\overline{\left(\varepsilon_{jkl}\Omega_k u_l\right)_B\left(u_i\right)_A},$$
$$\left(M_{i,j}\right)_{A,B}=\overline{\left(\Omega_i u_i \eta_i\right)_A\left(u_j\right)_B},$$
$$\left(N_{i,j}\right)_{A,B}=\overline{\left(\Omega_j u_j \eta_j\right)_B\left(u_i\right)_A}.$$

If the above relations of first, second and third order correlations are used in Eq. (11), the following can be obtained:

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial \zeta_k}S_{ik,j}+\frac{\partial}{\partial \zeta_k}S_{i,kj}$$
$$=-\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j}+\frac{\partial}{\partial \zeta_j}K_{i,p}\right)+2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$-2[\left(D_{i,j}+H_{i,j}\right)]-2\left(M_{i,j}+N_{i,j}\right)\sin\theta \qquad (12)$$

where all the correlations refer to two points A and B.

Now, for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is,

$$\left(k_{p,j}\right)_{A,B}=0,\ \left(k_{i,p}\right)_{A,B}=0.$$

In case of isotropy, statistical features have no directional preference and perfect disorder persists. Velocity fluctuations are independent from axis of reference; i.e. invariant to axis rotation and reflection. From the definition of isotropy, $\left(Q_{i,j}\right)_{A,B}=\overline{\left(u_i\right)_A\left(u_j\right)_B}=0$ for all $i\neq j$. In the rotating system in the flow field through 180^0 about x_1-axis must, because of isotropy, present

$$\overline{(u_1)_A(u_2)_B} = \overline{(u_1)_A[-(u_2)]}_B = -\overline{(u_1)_A(u_2)_B},$$

which can be true only when $\overline{(u_1)_A(u_2)_B} = 0$.

The isotropic turbulence in a bounded domain is a model in which turbulence is unaffected by the boundaries enclosing the fluid; furthermore, statistical moments are spatially invariant and independent from orientation. Isotropic grid turbulence is a similar idealization, in that turbulence is enclosed by wind tunnel walls and homogeneity of the turbulence in the central region is known to be unaffected by the wall boundary layers.

In an isotropic turbulence, it follows the condition of invariance under reflection with respect to point A,

$$\overline{(u_i)_A(u_k)_B(u_j)_B} = -\overline{(u_k)_A(u_j)_A(u_i)_B}$$

or, $(s_{i,kj})_{A,B} = -(s_{kj,i})_{A,B}$

Thus Eq. (12) can be written as

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right) = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} - 2[(D_{i,j} + H_{i,j})] - 2(M_{i,j} + N_{i,j})\sin\theta \tag{13}$$

Term $\frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right)$ forms second order tensor, designated by $S_{i,j}$. Also, terms $(D_{i,j} + H_{i,j})$ and $(M_{i,j} + N_{i,j})$ form the second order tensor, designated by $L_{i,j}$ and $W_{i,j}$, respectively.

Thus,

$$S_{i,j} = \frac{\partial}{\partial \zeta_k}\left(S_{ik,j} + S_{kj,i}\right),$$

$$L_{i,j} = (D_{i,j} + H_{i,j}),\ W_{i,j} = (M_{i,j} + N_{i,j}).$$

Therefore, Eq. (13) results in

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\left[\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} - L_{i,j} - W_{i,j}\sin\theta\right] \tag{14}$$

This is the energy equation for turbulent flow in a rotating system in terms of second order correlation tensors.

If there are no effects of dissipation ε by turbulence per unit mass, $L_{i,j} = 0$, so that Eq. (14) takes the form

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\left[\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} - W_{i,j}\sin\theta\right] \tag{15}$$

For the non-rotating system, $W_{i,j} = 0$ so that Eq. (15) takes the form

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} \tag{16}$$

Eq. (16) describes turbulent motion for non-rotating system in terms of second order correlation tensors. This equation represents turbulent motion in terms of second order correlation tensors, which is the same as the one obtained by Hinze.

3. Conclusions

Energy equation for turbulent flow in a rotating system was derived by averaging procedure, which consisted of the correlations between pressure fluctuations and velocity fluctuations at two points A and B of the flow field. The new mathematical model of Eq. (14) demonstrated energy equation of turbulent motion in a rotating system in terms of second order correlation tensors. In the equation, all the terms $Q_{i,j}, S_{i,j}, L_{i,j}, W_{i,j}$ were the second order correlation tensors where, $Q_{i,j}$ and $S_{i,j}$ represented velocity correlations at two points A and B of the flow field; $L_{i,j}$ was velocity correlation for turbulent energy, $W_{i,j}$ was

velocity correlation due to rotation and where the Coriolis force and centrifugal force acted on the fluid when motion rotated steadily with bulk of the fluid. But, in the absence of dissipation ε by turbulence per unit of mass, the resulting Eq. (14) was reduced to Eq. (15) which conferred turbulent motion in terms of second order correlation tensors due to rotation. In a non-rotating system, there was effect of Coriolis force in the flow field; so, Eq. (15) was reduced to Eq. (16), representing the turbulent motion in terms of second order correlation tensors.
The mathematical model in terms of second order correlation tensor which was derived in this study was modeled by imposing turbulent energy in a rotating system where a model was derived in terms of second order correlation tensor for turbulent flow only by Hinze [16].

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