SRTTU
Journal of Computational and Applied Research in Mechanical Engineering
jcarme.sru.ac.ir
JCARME
ISSN: 2228-7922

Research paper

Free vibration analysis of CFRP cylinders with torispherical heads: Experimental and numerical investigations

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Article info:

Article history:

Received:	05/04/2019
Revised:	14/01/2021
Accepted:	16/01/2021
Online:	19/01/2021

Keywords:

Torispherical heads,
Carbon fiber reinforced plastics,
Free vibration,
Modal testing.

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Abstract

Pressure vessels are used in a variety of applications in many engineering applications. The thin-walled cylinders with torispherical heads have been widely used as pressure vessels in engineering structures. The free vibration behavior of carbon fiber reinforced composite cylinders ended with torispherical heads with various boundary conditions is investigated in this paper. The shape of a torispherical head consists of a sphere of large radius and a much smaller minor radius at the knuckle. The numerical calculation with the finite element method is obtained and verified with experimental results to confirm the accuracy of the numerical solution. The acceptable accordance between experimental and numerical results leads to use of numerical model instead of expensive experimental tests. In addition, the effects of the thickness of torispherical head and cylindrical section and the lengths of cylinder and torispherical head on vibrational behavior of the structure are studied.

1. Introduction

Pressure vessels are used in a variety of applications in many engineering applications such as mining operations, oil refineries, petrochemical plants, and submarine and space ship habitats. One of the most common designs of these structures is a thin-walled cylinder with end torispherical heads. The shape of a torispherical head consists of a sphere of large radius and a much smaller minor radius at the knuckle. Because of a wide range of usage, study on mechanical characteristics and static and dynamic analyses on these structures is of vital importance.

The works on cylinders with torispherical heads mainly concentrate on stress analysis and static stability (buckling) of these structures. Gurusharan et al. [1] presented a refined finite element model and use of a special purpose subprogram to determine mixed-mode membrane and bending strength variables about an axial crack in a cylindrical shell with tori-spherical end closures. Zhu et al. [2] investigated, under outer pressure, the elastic-plastic buckling in hemispherical heads. The proposed analytical formulas for predicting non-linear buckling loads of hemispheric heads have been experimentally tested and compared to the numerical findings of this report. Wang et al. [3] proposed buckling under uniform external pressure of imperfect spherical caps with a fixed boundary. The findings suggested that the linear buckling mode-shaped imperfection provided a reasonably conservative prediction of cap buckling compared to those of the other three imperfections in the case of small-sized imperfections. The effects of perturbation forces on buckling in pressure vessel torispherical heads were studied by Senalp [4] using the finite element method. Muscat and Camilleri [5] considered various design models to avoid buckling of torispherical heads through internal pressure. Li et al. [6] combined numerical and experimental approaches for studying the buckling behavior of large-scale thin-walled ellipsoidal heads. They explained the buckling behavior via variations in compressive stresses and shapes of ellipsoidal heads. Skopinskii et al. [7] compared the stresses in torispherical and elliptical heads of pressure vessels. Blachut [8] showed that the radius of the knuckle in the torisphere is an important parameter in the design of pressure vessels. Also, Błachut [9] discussed the influences of corrosion caused by wall thinning on buckling of domed closures onto cylindrical vessels. Wang et al. [10] studied the buckling susceptibility to imperfections in the externally pressurized torispherical head. The influence of shape imperfection on the buckling of large-scale thin-walled ellipsoid heads in steel nuclear containment was investigated by Zheng et al. [11]. Zhang et al. [12] investigated the buckling of spherical caps manufactured under various wall-thickness reduction conditions. Zhang et al. [13] studied experimental and numerical collapsing characteristics of outwardly pressurized egg-shaped shells within local geometrical defects. Magnucki et al. [14] proposed a new shape of a dished head. They analytically and numerically investigated the stress state in the dished head and numerically determined the buckling problem of the head. Sharifi et al. [15] investigated the mechanical strength of internally pressurized laminated woven composite shells (hemispherical, ellipsoidal, and torispherical) using numerical and experimental methods. The experimental-numerical specification of the fracture activity of P264GH Steel notched tubes under internal pressure was investigated by Moustabchir et al. [16]. To test its reliability, this paper introduces the most reliable experimental techniques for characterizing a pipe-like structure. Senjanovic et al. [17] performed buckling analysis of toroidal shell by using the Rayleigh-Ritz process. Numerical examples compare the proposed procedure; one for a closed toroidal shell and another for a simply supported open toroidal shell. Kumar et al. [18] carried out a study on the thermal and mechanical characteristics of graphene and its hybrid polymer nanocomposites for structural usages.

There are a few studies on dynamic behavior of cylinders with torispherical heads. Static and dynamic study of pressure vessels with different stiffeners was performed by Eswara Kumar et al. [19]. In this study, considering variables such as basic structural stiffness, Von- mises stress, weight and total deformation, the best stiffener design was advised. The effect of head forms on free vibration and fatigue was studied by Abdulrazzaq Salman et al. [20] on horizontal LPG pressure vessels. However, there are several researches on the vibration of the joined cylindrical–hemispherical shells. Yusefzad and Bakhtiari nejad [21] studied the free vibration of the pre-stressed cylindrical spherical shells joined together. Kang [22, 23] suggested free vibrations, with and without a top opening, of combined hemispherical-cylindrical revolution shells. Wu et al. [24] analyzed the forced vibration of joined conical-cylindrical-spherical shells using the process of domain decomposition. They used the shell theory of Reissner-Naghdi-Berry and the constraints equations resulting via conditions of interface continuity between two adjacent shell segments. They also used this approach to investigate the vibration behavior of a spherical–cylindrical–

spherical shells [25] and joined cylindrical-spherical shell in elastic-support boundary conditions [26].

In order to investigate the free vibration of linked spherical-cylindrical shells, Lee [27] used Fourier-Chebyshev expansions. Xie et al. [28] used a semi-analytical approach to study free and forced vibration of ring-stiffened conical-cylindrical-spherical shells. Lee et al. [29] suggested the effects of a joined cylindrical-spherical shell with constant thickness for the free vibration study.

As the best knowledge of authors, no specific discussion is available concerning vibrations of cylinders with torispherical heads especially made with laminated composite materials. This research presents numerical and experimental approaches about the free vibration of the joined composite cylinder within torispherical head and examines the effect of the boundary conditions at the end of the cylindrical shell, thickness of torispherical and cylindrical shells and also the lengths of cylindrical section and torispherical head on the natural frequencies of the structure.

2. Case study

The composite cylinder with torispherical heads is shown in Fig. 1. It is made of carbon/epoxy laminates with filament winding process and the material properties are given in Table 1.

2.1. Construction method

The basic manufacturing technique for shaping load-bearing structural elements made of polymer matrix-based fibrous composites that have the shapes of revolution bodies is a winding process. The thickness of each layer is set to be 0.25 mm and the winding angle is ±75º. The epoxy resin was cured in room temperature and the test was performed at least 48 hours after the sample was made so that the resin reached its maximum strength. The volume fraction of the fibers is set to 60%.

The torispherical head consists of two joined spherical sections with 50 and 90mm radii. The quantity of material per unit fiber length is maintained constant during the filament-winding procedure, and there is no friction between the fibers and the mandrel surface so that the orientation of the fibers is determined by a geodesic path (the shortest distance on a layer between two points). Therefore, it is needed to change the fiber orientation and the laminate thickness. Table 2 presents the measured geometrical dimensions of the sample in each section of cylinder and torispherical head. The φ_r represents the angle of fiber angle with respect to axial direction.

3. Finite element analysis

The Finite Element (FE) analysis is performed by ABAQUS four-node shell element S4R software with six degrees of freedom at each node, three nodal-directional translational displacements, and three nodal-axis rotational displacements.

Table 1. Material properties of carbon/epoxy composite laminates [30].

Density, ρ (kg/m^3)	1235
Longitudinal modulus, E_1 (GPa)	131
Transverse modulus, E_2 (GPa)	8.98
Poisson's ratio, ν_{12}	0.34
Shear modulus, G_{12} (GPa)	3.74
Shear modulus, G_{13} (GPa)	3.74
Shear modulus, G_{23} (GPa)	3.4

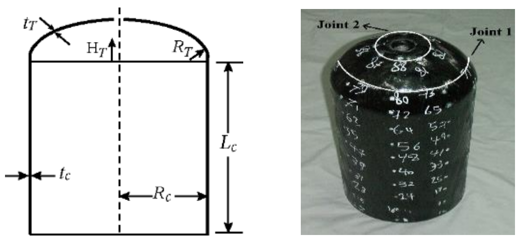

Fig. 1. Test sample and geometry.

Table 2. Dimensions of cylindrical and torispherical heads (mm).

Torispherical head				
section	H_T	R_T	t_T	φ_r
1	10	100.9	8.16	9.7
2	20	96.5	8.24	10
3	30	89	8.41	10.5
4	40	77.25	8.73	11.3
5	50	55.3	9.69	13
6	53.2	40	10.98	18.4
7	54.5	20	15.66	26
8	55.1	17.5	17	61
Cylinder				
L_C	R_C	t_C	φ_r	
380	104	9.25	90	

The S4R is a formulation of a linear element, reduced-integration, and control of the hourglass. The structure consists of laminates of carbon/epoxy and laminar layup used to model the composite shell. The problem with the eigenvalue is resolved using the LANCZOS process. On both sides, all classic boundary conditions are applied, including free (F), simply-supported (SS) and clamped (C) situations. The outcome of the FE analysis is usually influenced by the number of elements. In this way, the convergence of the results of the analysis against the number of total elements is measured and presented in Fig. 2. The final FE model of the structure is composed of 3111 elements and 3172 nodes.

4. Modal testing

4.1. Boundary conditions

The structure has F-F boundary conditions at the end of both cylinder and torispherical head. All structures with F-F boundaries in space have six rigid body modes (three translational and three rotational modes) and each of them has 0 Hz natural frequency. At rigid body modes, no bending or stretching was occurred and the motion is totally because of inertia properties of the structure. In practice, since we could not have a truly F-F condition, a simple suspension system is used which closely simulates the F-F condition [31]. The suspension system is usually a very soft spring (i.e. a light elastic band). The rigid body modes have values that are 'very low' relative to the bending modes using these supports [31]. The word 'very low' indicates that the highest rigid body mode frequency has to be less than 10 to 20 percent of that for the lowest bending mode [32].

Using this simple suspension system, we can remove the rigid body modes from the very low frequencies without any influence on the bending modes of the structure [32]. In this experiment, the rigid body modes were less than 10 Hz and far enough from bending modes. The experiment setup is shown in Fig. 3.

4.2. Sensors and excitation

An electro-dynamic shaker is used for excitation of the test specimen. The burst random signal type is selected. Eight piezoelectric accelerometers are used for response measurement at 88 points. These points consisted of 11 environmental rows and 8 columns. An analyzer with 16 input channels is employed to analyze the data from accelerometers.

To study the effects of moving mass of the shaker on modal properties of the structure, the test is repeated with roving hammer method. Results of the experiment with hammer and shaker excitation are compared (for the first mode); and it is shown that there was only 1% difference between the first natural frequencies. This leads to the fact that the effects of shaker's moving mass on the modal properties of joined shells can be neglected.

4.3. Modal properties

In order to reduce noise, the frequency response functions (FRF) were assessed after averaging over 50 times measurements; and curve fitting method is used to extract modal characteristics (frequency, damping and mode shapes) of the structure [31].

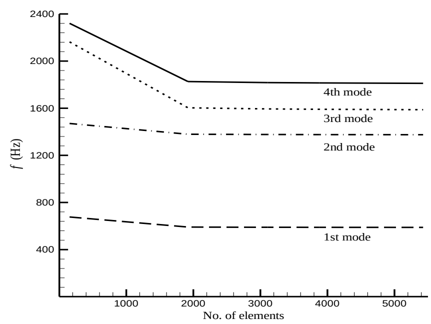

Fig. 2. Effect of number of elements on the natural frequency (Hz).

Fig. 3. Experiment set-up.

5. Results and discussion

5.1. Comparison of test and FE findings

A comparison of the FE findings with the experimental findings for the natural frequencies of the structure is illustrated in Table 3. The findings indicate strong agreement between the test and the FE calculated.

It is very important to identify the mode shape in the case of shells. Due to high modal density, it is difficult to compare the modes extracted by numerical and experimental methods using only the natural frequencies and the modes visualization is necessary. Fig. 4 shows the mode shapes of the structure with F-F boundary conditions extracted from modal testing in correspondence with FE results. The mode shapes are defined as the longitudinal and circumferential modes.

The excellent fit among numerical and experimental findings shows that for similar cases one can use the FE instead of expensive experimental method. Also, it can be shown that mode shapes are similar to cylindrical section mode shapes with C-F boundary conditions and the torispherical head acts as a clamped BC due to its higher stiffness.

5.2. Parametric study

Some examples are solved using FE in order to investigate the impact of geometric parameter and boundary condition onto modal properties.

5.2.1. Effects of cylinder length on natural frequencies

In this part, the effect of the dimensionless cylinder length (L_C/R_C) on the natural frequencies is studied using results of FEM and test. Figs. 5-7 show the effects of cylinder length on natural frequencies with F-F, SS-F and C-F boundary conditions, respectively.

As seen from these figures, the higher modes are more sensitive to length variation than the lower ones. This is due to the presence of the torispherical head, which plays the major role in the strength of the structure. In fact, due to the high strength of the torispherical section, changes in the length of the cylindrical section does not have much effect on the natural frequency of the structure (especially in lower modes). The effect of the torispherical head on the strength of the structure will be examined separately at the end of this section.

Table 3. Comparison of the numerical with the experimental data for free-free boundary condition.

	Experiment (Hz)	**FE (Hz)**	**Error (%)**
1st mode	577	589	2.1
2nd mode	1471	1376	6.5
3rd mode	1569	1593	1.5
4th mode	1850	1817	1.7

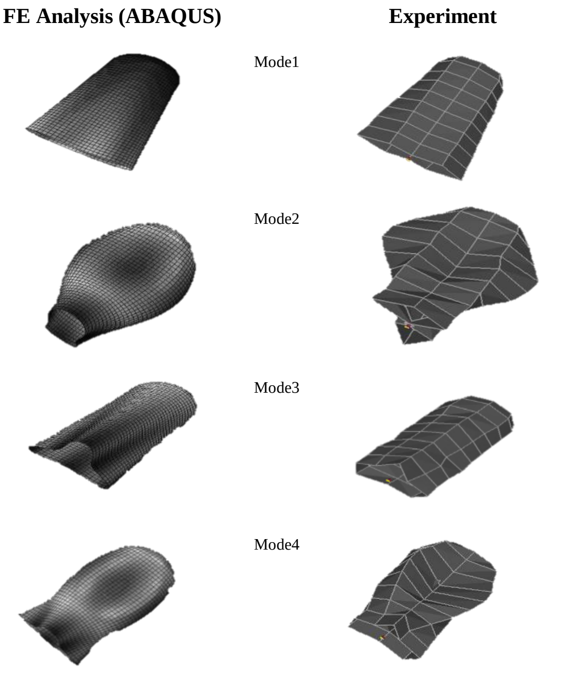

Fig. 4. Typical modal test mode shapes in correspondence with FE results (F-F boundary condition).

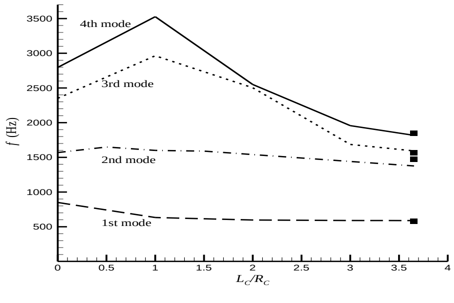

Fig. 5. Effect of cylinder length on the natural frequencies (Hz) of the structure for free-free boundary condition (solid squares are the experiment results).

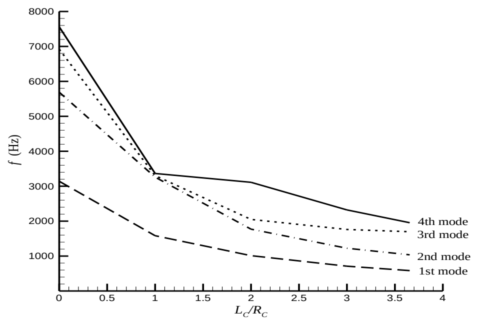

Fig. 6. Effect of cylinder length on the natural frequencies (Hz) of the structure for simply supported-free boundary condition.

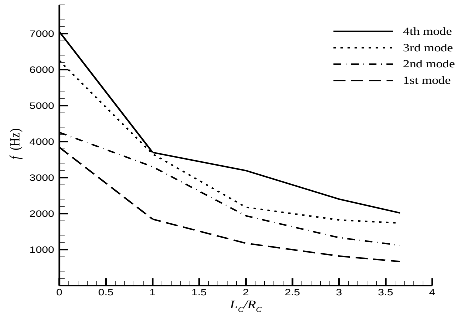

Fig. 7. Effect of cylinder length on the natural frequencies (Hz) of the structure for clamped-free boundary condition.

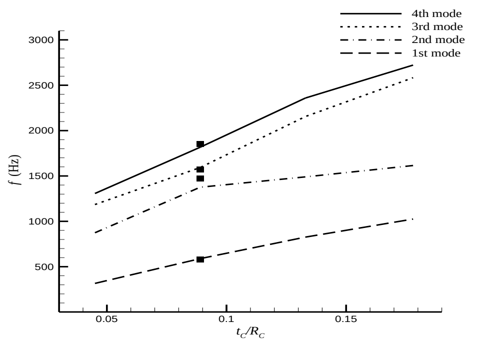

Fig. 8. Effect of the cylinder thickness on the natural frequencies (Hz) of the structure with free-free boundary condition.

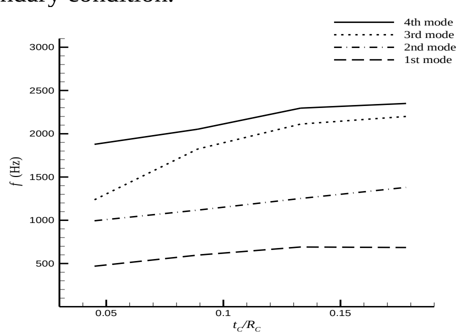

Fig. 9. Effect of the cylinder thickness on the natural frequencies (Hz) of the structure with simply supported-free boundary condition.

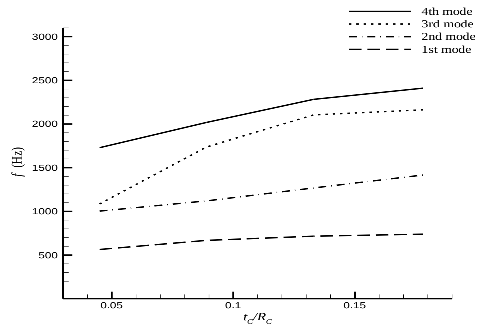

Fig. 10. Effect of the cylinder thickness on the natural frequencies (Hz) of the structure with clamped-free boundary condition.

5.2.2. Effects of cylinder thickness on the natural frequencies

The influence of cylinder thickness on the natural frequencies is evaluated using FE. The effect of this parameter for F-F, SS-F and C-F boundary conditions is shown in Figs. 8-10, respectively. It can be concluded that, for all BCs, the frequencies of the structure increase as the dimensionless thickness increases.

5.2.3. Effects of torispherical head depth on natural frequencies

The effects of torispherical head depth (H_T/R_C) on the natural frequencies are carried out using the findings of FE analysis and experiment. The effects of the torispherical head depth for F-F, SS-F and C-F boundary conditions are shown in Figs. 11-13, respectively.

It can be concluded that the frequencies of the structure are almost independent of torispherical head depth at higher depths of torisphere, because, the torispherical head of joined shells has higher stiffness relative to cylindrical section and acts as a clamped-like boundary condition. However, in lower depths of torispherical shell, the stiffness of this section get closer to cylindrical section and the mode shape of joined shells changes as seen in Fig. 14. This leads to lower frequencies of the structure in small values of torispherical head depth.

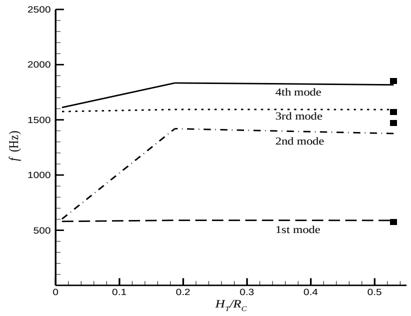

Fig. 11. Effects of torispherical head depth on the natural frequencies (Hz) of the structure with free-free boundary condition.

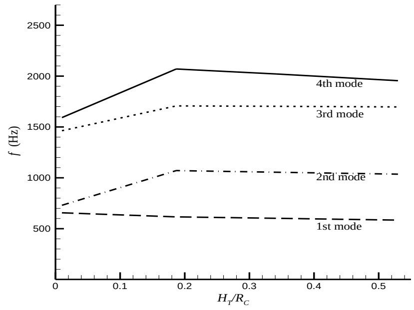

Fig. 12. Effects of torispherical head depth on the natural frequencies (Hz) of the structure with simply supported-free boundary condition.

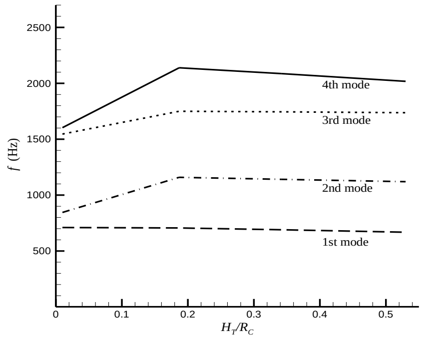

Fig. 13. Effects of torispherical head depth on the natural frequencies (Hz) of the structure with clamped-free boundary condition.

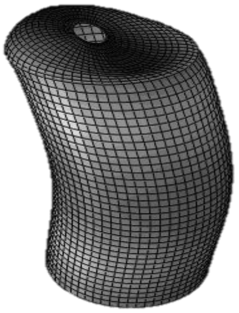

Fig. 14. Mode shape change in lower values of torispherical head depth.

6. Conclusions

In the current research, the free vibration behavior of the CFRP cylinder containing torispherical heads in various boundary conditions is studied by numerical analysis and modal testing. In modal testing, the shaker (with burst random signal) is used for excitation of test specimen and 8 accelerometers for data measurement; and the curve fitting technique is used to determine the natural frequencies and mode shapes of the joined shells. The numerical calculation with FEM is obtained and the findings validated with the experimental results to confirm the accuracy of the FE analysis. The acceptable accordance between experimental and numerical results leads to the use of numerical model instead of expensive experimental tests. In addition, the effects of the thickness and length of torispherical head and cylinder on vibrational behavior of the structure are studied. The main results of this study are as below

- The fundamental frequencies of the structure increase as the cylinder thickness increases.
- The decrease in cylinder length causes a significant increase in fundamental frequencies of the joined shells.
- The vibrational behavior of the structure is nearly independent of the thickness and length of a torispherical section. This is mainly because the stiffness of the torispherical head is much higher than the cylindrical shell and no noticeable deflection occurs in it.

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How to cite this paper:

Meisam Shakouri, Hessam Sarvahed and H. M. Navazi, "Free vibration analysis of CFRP cylinders with torispherical heads: Experimental and numerical investigations*," J. Comput. Appl. Res. Mech. Eng.*, Vol. 11, No. 1, pp. 13-21, (2021).

DOI: 10.22061/JCARME.2021.5190.1641

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