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Journal of Computational and Applied Research  
in Mechanical Engineering

jcarme.sru.ac.ir

JCARME

ISSN: 2228-7922

**Research paper**

## Reliability, availability, and maintainability modeling of multi-state systems with load-sharing structure

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**Article info:****Article history:**

Received: 05/04/2021

Accepted: 14/11/2022

Revised: 16/11/2022

Online: 19/11/2022

**Keywords:**

Reliability,  
Multi-state system,  
Load-sharing,  
Spare part inventory  
management,  
Maintenance policy.

**\*Corresponding author:**[farsi@ari.ac.ir](mailto:farsi@ari.ac.ir)**Abstract**

The reliability of manufacturing systems modeling and analysis is a complex process. Usually, their behavior is similar to multi-state systems. The configurations of such systems, possibly with load sharing and other structural dependencies, are designed to provide high reliability/availability. Consequently, this scheme can help companies to improve efficiency and reduce operating costs. Maintenance and part replacement are implemented during operation and utilization to keep their performance. Decision-making about spare ordering is difficult because of the interconnection between spare parts inventory and maintenance strategy. In this paper, the characteristic parameters of spare parts inventory management and maintenance policies are jointly considered for multi-machine systems (manufacturing systems) with different types of dependencies among them (economic, load-sharing, and multi-state configuration). Two maintenance policies are considered: condition-based and preventive maintenance. The interactions between maintenance policies and spare parts management are considered for determining a manufacturing system's cost and availability. The influence of these factors is investigated. Load sharing factor and ordering time are more important, and their influence is higher than others.

**1. Introduction**

In industrial practice, maintenance and spare parts costs often constitute a large part of the total production costs. Maintenance costs can rise to sixty percent of production costs and up to a third of these costs may be due to unnecessary or poorly executed maintenance [1]. Therefore, many studies have been performed to control and reduce maintenance costs under different maintenance policies, such as CM, PM, and CBM. An effective CBM can

reduce the number of failures and increase productivity, availability, and safety [2]. For CBM, signals and data are collected from critical units and processed to monitor the machine's state. Based on fault detection thresholds, maintenance monitors and controls degradation until the new spare part is installed to avoid catastrophic failure. Spare parts inventory and management influence maintenance effectiveness and cost. Delays in spare parts provision, logistic problems, spare

quality, and spare deterioration during storage have to be considered for a successful maintenance and production plan.

Several researchers investigated the maintenance and spare parts problems [3-5]. In real-world systems, the operation planning, maintenance, and spare parts inventory management for these systems are difficult tasks because several dependencies can exist among the system components. These dependencies can be grouped into four types [6]: structural, resource, stochastic, and economic. An optimal maintenance and spare parts inventory plan should consider these dependencies and the relations among the different components. Structural dependence concerns the relationships among the different components deriving from the system configuration: series, series-parallel, K out of N, and so on. CBM has been intensely investigated for this type of system configuration [7, 8]. Resource dependence comes from the fact that resources in an industrial plant are usually limited. Staff and labor, storage room, budget, suppliers, and logistics for a system may be restricted and, thus, need to be shared: for instance, three machines share the same specified bearing as a spare part and the number of maintenance workers is limited. This type of dependence can decrease system availability and should be considered at the planning level. Deterioration and failure processes of different units can be dependent; this dependence is called stochastic dependence. For example, in failure-induced damage, component failure can lead to damage and even failures of other components. A combined CBM and Age-Based Maintenance (ABM) policy could be implemented in this case. In load-sharing, different components share the total load so that when one component is failed the others increase their load to ensure system operation. Common cause failure modes are also critical: a failure cause can damage several units of the system and this can defeat any redundancy allocated by the designer. Economic dependence influences the relationships between maintenance processes. Sometimes, if two maintenance tasks are opportunely performed together, the total cost is decreased. In other situations, this influence is

inverse, and the total cost is increased. This dependence should be considered if it exists. Some of these dependencies have been studied by researchers in the past years. Marseguerra *et al.* [9] determined the optimal CBM for series-parallel systems. Load-sharing dependence was considered too. Zhang *et al.* [10] investigated a system where each component can be subject to two deterioration processes (normal and accelerated). They concluded that the costs of ignoring stochastic dependence increase significantly with the number of components and the degree of dependence. Keizer *et al.* [11] considered a dynamic policy structure for a system with load sharing, economic, and performance dependencies. They demonstrated that their policy outperforms a single preventive replacement threshold policy, and those preventive replacements should be accomplished at an early stage for a system with a strong degree of load-sharing.

As mentioned above, spare parts availability influences maintenance efficiency. Studies on spare parts have been started since the 1970s [12]. The earlier studies focused on simple systems, and recently complex systems with several dependencies have been considered. Alenka *et al.* [13] investigated the joint optimization problem of periodic batch replacement and periodic spare procurement. Israel *et al.* [14] considered an intelligent maintenance system, based on a hybrid model with mixed linear programming (MILP) simulation and accommodating demand information. They studied supply chain effects on spare parts management and sought a cost reduction in the supply chain to reduce spare parts shortage and, consequently, production shutdown. Wang *et al.* [15] studied the modeling and control of the spare parts for a system monitored during operation. They found the optimal preventive maintenance threshold to satisfy the spare parts support requirements. Nguyen *et al.* [16] investigated a multi-component system to find the optimal PM and inventory strategy. They jointly applied a prognostic condition index and the structural importance measure of components to propose thresholds for maintenance and spare parts ordering. Economic dependence was also considered. Liao and Rausch [17] proposed a

joint production and spare part inventory control algorithm for manufacturing equipment with a critical unit controlled by CBM. They used a two-stage algorithm wherein the stock level is first determined, and the maintenance threshold is calculated in the second stage. Wang *et al.* [18] investigated systems that consist of multiple series-parallel degrading components. They proposed a policy for spare parts ordering based on criticality importance measures. Their policy mainly consists of two steps: (1) determine which components to be replaced; (2) determine when to order spares for selected components. Their method can minimize the expected replacement cost during the once-replacement cycle. Chen *et al.* [19] proposed a new failure probability estimation function developed simultaneously based on component service time and degradation extent. They determined the optimal replacement and spare parts ordering times according to the estimated failure probability. Zhang and Zeng [20] investigated identical multi-unit systems to find the best periodic condition-based opportunistic preventive maintenance (CBOM) and safety policy for spare parts management. Jiang *et al.* [21] provided a joint optimization policy for maximum inventory level and maintenance cost. By using the Monte Carlo method, their policy found the best total cost rate for a system. They showed that the proposed policy was better than policies that do not consider the deteriorating inventory information. Dreyer *et al.* [22] discussed the challenge of determining the optimal number of spare parts for a machine using condition monitoring data. They proposed a service concept to optimize the number of spare parts by minimizing the total costs. Keizer *et al.* [23] applied Markov Decision Processes to obtain the optimal replacement decisions that minimize the long-run average cost per time unit. They investigated a load-sharing system with economic dependence and demonstrated that the load-sharing effects among components could lead to a significantly more expensive maintenance policy. Although load-sharing and multi-state configuration are frequently applied as a solution to improve the system and increase performance, these types of dependencies modeling via joint maintenance policy and

spare parts strategy are rare. This paper investigates this subject, and modeling of maintenance and spare parts strategy is carried out. The main contributions of this paper to the existing literature are in the following areas:

- Multi-state systems with load-sharing dependence are considered, and maintenance policies and spare part inventory management for this system are jointly studied.
- Significant parameters' effects on the system are studied. The existing literature has not considered supplier and storage influences on the system performance. The result shows that these parameters influence the system performance and a new equation for the total cost is proposed based on storage, spare parts, shutdown time, and replacement costs.
- Uncertainty in the maintenance times was considered as the normal distribution. In industrial practice, this subject is very important, as it affects production and maintenance planning.

## 2. The multi-state system with load-sharing dependence

In industrial practice, a system may produce multiple levels of output. This system is called a multi-state system and its reliability is determined based on an acceptable level of output performance. Multi-level outputs may be produced by using different dependencies, such as K out of N and load-sharing configurations. In this paper, load-sharing dependence is considered.

A load-sharing system, in particular, refers to a parallel system whose machines/components share the system function [24]. In this setup, when one of the machines fails, the others also carry its load. Since these other machines/components would operate under more stressful conditions, they would experience a higher rate of failure than when operating in nominally shared conditions. For instance, a system uses three machines; when one machine fails, the other two machines operate under increased stress levels. To account for this, we introduce a parameter called the load-sharing factor. This parameter defines the stress level induced onto a machine when working in a non-nominal

situation. In this study, the load-sharing factor depends on the production capacity ratio when a machine is stopped; this factor defines the increasing stress level on the remaining machines (Eq. (1)):

$$M_{i,LSF} = 1 + \frac{(SAO - RC)}{M_i C} \times M_i P \quad (1)$$

where  $M_{i,LSF}$  is the load-sharing factor,  $M_i C$  defines machine  $i$  capacity in a normal situation and SAO is the acceptable system output that defines the system's success. The RC is the capacity reduction when a machine is stopped and  $M_i P$  defines the machine  $i$  portion of work in the system under normal conditions.

Fig. 1 illustrates a simple two-machine load-sharing behavior. This shows the stress variation on a machine according to its state and the other machine's state.

### 3. System definition

Consider a system consisting of  $k$  machines. The set  $I = \{1, 2, \dots, k\}$  denotes the set of machines. These machines are in a load-sharing configuration. When a machine is stopped or shut down for maintenance, the other machines work at a higher level of stress to compensate by taking charge of the missing load, so that the system output level is held. After repair, all machines return to work at normal stress levels. Availability (A) of the system is computed as the ratio of the working time to the total time. Each machine may be preventively stopped for maintenance based on a predefined schedule (preventive maintenance, PM) or replaced by a spare part. Preventive maintenance is performed at the defined times  $T = \{t_1, t_2, \dots, t_n\}$ . It is supposed that when a machine is preventively stopped for a spare part replacement, all PM tasks that should be performed close to this time are opportunely accomplished too. Also, each machine may have two or more components monitored for CBM.

The dependence among components of a machine is assumed as a series dependence: when a component of a machine is damaged and the machine is stopped for PM, the other

components stop too, but only the damaged component is replaced with a spare part. Times to replace the damaged parts are denoted as  $\{tr_{1,1}, tr_{1,2}, \dots, tr_{m,n}\}$ , where  $m$  is the machine index and  $n$  is the spare part index.  $Tr_{mn}$  is the Reaming Useful Lifetime (RUL), estimated in terms of a statistic distribution. In this paper, a specified Weibull distribution is applied for each component. Spare parts ordering times are also denoted as  $\{to_{1,1}, to_{1,2}, \dots, to_{m,n}\}$ . If the spare parts provision strategy is incorrect, a spare part may not be delivered on time, and this may lead to the system shutdown and inefficient production costs. While in storage, a spare part can deteriorate. In this study, we also consider the storing cost in the model.

Suppliers define a spare part's quality, price, and delivery time. If the delivered part is of inferior quality, the system life can change and the next shutdown occurs sooner than expected. Also, the supplier defines the price of spare parts; this cost directly affects the total cost of maintenance and operation. In this paper, the supplier effect is also studied, and assume that at least two suppliers can provide each part of a machine. Each supplier has a specified quality, delivery time, and cost, defined as follows:

$$\{Sup_i = (cost\ i, quality\ i, Delivery\ time\ i), i = 1, \dots, l\ number\ of\ suppliers\}$$

A normal distribution ( $t_{supi} \equiv (\mu_i, \sigma_i)$ ) describes uncertainty in the delivery time

#### 3.1. The total cost maintenance

Maintenance and spare parts costs can be the main portions of the operation cost. If an effective maintenance policy is implemented at a lower cost, the operation cost is decreased too. Although several formulas have been proposed to determine system cost, in this study, new parameters and states are considered, and a new formula is proposed.

In this formula, consideration is also given to the spare parts cost, replacement cost, PM cost, shortage cost, and storage room cost are necessary. Thus, the total cost is derived as given in Eq. (2).

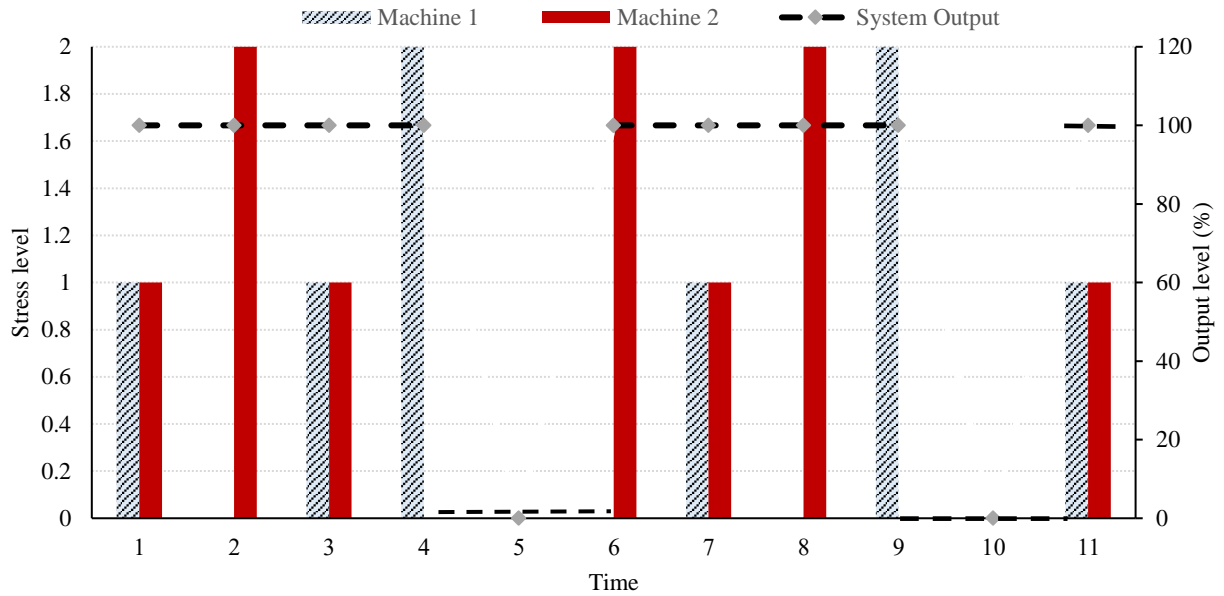


Fig. 1. Stress variation for a simple two-machine load-sharing system.

$$\begin{aligned}
 TC = & \sum_{i=1}^m \sum_{j=1}^n C_{S_{ij}} + \sum_{i=1}^m \sum_{j=1}^n C_{R_{ij}} \\
 & + \sum_{i=1}^m CPM_i \\
 & + \sum_{i=1}^m \sum_{j=1}^{s_m} ShC_{i,j} \\
 & + \sum_{l=1}^{s_n} STC_l
 \end{aligned} \tag{2}$$

where TC is the total cost, and m and n indicate the number of machines and spare parts for each machine, respectively.  $C_{S_{ij}}$  defines the spare part cost and the  $C_{R_{ij}}$  defines the replacement action cost for a spare part. Stopping or shutdown events add a system cost  $ShC$  and when a spare part k needs to be stored, the  $STC_k$  cost is accounted for.  $s_m$  is the number of system stops and  $s_n$  defines storage times. Also,  $ShC$  can be calculated by:

$$ShC_{i,j} = ST_{i,j} \times C_s \tag{3}$$

where  $ST_{i,j}$  is the duration of stoppage of the system production because the system is down and unavailable, and  $C_s$  indicates the penalty for a one-hour production stoppage. In this paper, a framework for jointly modeling CBM and spare parts strategy parameters for a multi-component system are proposed.

#### 4. Multi-state load-sharing system modeling

In this paper, multi-component systems with multi-state configuration and load-sharing dependence are considered. Also, opportunistic maintenance is considered as economic dependence. Furthermore, uncertainty is accounted for because of its effect on the maintenance and production programs. Monte Carlo simulation is applied to estimate system availability and cost. Around 10000 runs are performed to obtain accurate results. Component failures are randomly sampled from Weibull distributions and PM tasks, such as oil and lubricant change, are performed based on a specified timetable. It is assumed that when a spare part is replaced, the preventive maintenance tasks are also opportunistically carried out and the machine starts working as well as new. The system is assumed to have failed when the system output is less than a specified level.

##### 4.1. Reliability modeling

In load-sharing systems, stress on a component changes according to the system situation. The reliability of a component is computed as conditional reliability, as follows [25]:

$$R(\tau|t) = \frac{R(t + \tau)}{R(t)} \tag{4}$$

where  $R(\tau|t)$  is the component reliability of a component worked until  $t$  and should continue to work for a duration  $\tau$ ,  $R(t)$  is the component reliability at the specified time  $t$ , and  $R(t + \tau)$  indicates the component reliability at the end of the duration time ( $t + \tau$ ). If the stress level has varied for  $q$  times before this time, Eq. (5) is used to determine the component reliability:

$$R(t + \tau) = R(\tau|t) \times \left( \prod_{i=0}^q R(t_{i+1}|t_i) \right) \times R(t_i) \quad (5)$$

where the time duration for each stress level is derived as  $(t_i, t_{i+1})$  and  $R(t_i)$  is the reliability at time  $t_i$ .

A specified Weibull distribution determines the RUL of a component, and when stress is varied, the distribution parameters are updated. The shape parameter (Beta) is assumed constant, and only the scale parameter (Alfa) is changed. Based on the accelerated life testing principle [24, 25], the new Alfa can be derived as follows:

$$Alfa_{i+1} = Alfa_i \times M_{j,LSF} \quad (6)$$

where  $M_{j,LSF}$  is calculated by Eq. (1).

When the stress level is changed (increased or decreased), the RUL is updated and if the reliability is lower than the predefined value, a spare part should be ordered and when the machine is stopped, the component is replaced.

#### 4.2. Monte Carlo simulation

In this section, an algorithm is proposed based on the Monte Carlo method and system logic for evaluating system availability and cost. We sketch the overall framework of this algorithm in Fig. 2 and detail it as follows:

1. System definition: In this step, all system parameters are defined, such as the number of components, PM table time, number of suppliers, and so on.
2. Components lifetime, PM, and order times are computed. In this step, the expected times when machines are stopped for repair are computed, also ordering times for spare parts provision are determined. Ordering time is computed based on the reliability threshold for each machine as follows:

$$\begin{aligned} \text{Ordering time} &= \text{Exp}((\text{Ln}(\text{Ln}(1 / R^*))) \\ &+ \text{beta} \times \text{ln}(\text{Alfa})) / \text{beta}) \\ &- \text{workingtime} \end{aligned} \quad (7)$$

where  $R^*$  defines the reliability threshold for a component.

3. Determine the nearest repair time for active machines; each machine has three stop times for PM task, first spare part replacement and second spare part replacement.

4. If PM should be conducted, the time duration ( $\Delta$ ) for PM is sampled from the normal distribution. Also, the next stop time of other machines ( $T_n$ ) is computed.

5. If a critical component controlled by CBM should be replaced, the repair duration ( $\Delta$ ) is computed and spare part provision is checked too. Repair duration is defined as follows:

$$\Delta = \text{Repair time} + \text{Delay} \quad (8)$$

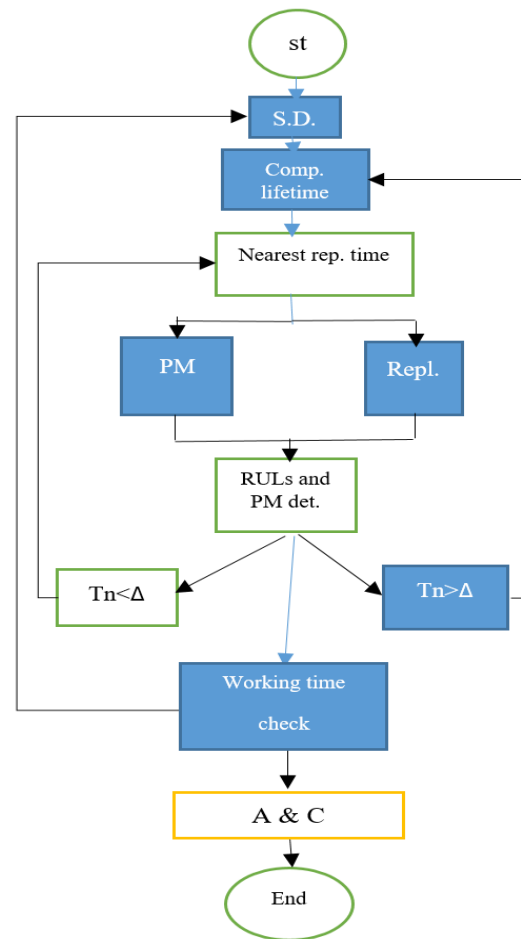


Fig. 2. An overall framework of the Monte Carlo simulation procedure.

Repair time and delay are normal random variables. The delay depends on spare parts ordering, replacement times, and the provisioning process; thus, it is defined as follows:

$$Delay = \text{Ordering time} + \text{Delivery duration} - \text{Replacement time} \quad (9)$$

Delay > 0 means that the component fails and a spare part is not yet available. Delay < 0 means that the spare part has been stored; therefore, storage cost is calculated based on storing duration and cost per hour. Delay = 0 defines that the spare part is provided on time and without delay.

6. New RULs and PM are calculated for other machines because of stress variation. The next time of stop (Tn) for these is determined again.

7. Compare Tn with Δ. If Tn is greater than the repair duration (Δ), the system continues its mission without any problem. If the repair duration (Δ) is greater than Tn, the system is stopped. Thus, the new times for spare parts provision and maintenance are determined. These new times should be compared with other machines' times for shutdown time determination. Check the working time. If working time is greater than 2000 hours (the mission time considered in this study) and the simulation iteration number is less than 10000 (the number of repeated Monte-Carlo runs considered in this study), the whole procedure is repeated. Otherwise, the simulation stops, and the system cost (C) and availability (A) are computed. Uncertainty in maintenance affects production planning. To consider the uncertainty in maintenance times, such as delivery, PM, and replacement times, these times are described by normal distributions (μ, σ) and sampled as in Eq. (8).

### 5. Results and discussion

The influence of the characteristic parameters on the system performance is studied. Assume the system includes two machines. Also, it is assumed that the other parameters are not varied to investigate the influence of one parameter on the system performance. In the rest of the paper, firstly, the simple load-sharing system is

considered, and then, the multi-state system with load-sharing dependence is studied.

#### 5.1. Load-sharing system

##### 5.1.1. Ordering time influence

The spare parts ordering time is an essential parameter of CBM and spare parts management. Often, the ordering time is strictly correlated to failure or fault detection time. Commonly, industries use expensive and advanced instruments to detect degradation, and spares are replaced before unexpected and catastrophic failures. In this paper, ten spare parts ordering times are considered to evaluate the ordering time (failure detection time) influence on system performance. These times are determined based on the failure probability. If the ordering is carried out immediately after replacement and repair, it is assumed that the failure probability is zero. If the ordering is carried out when a machine is stopped and needs a new spare part, it is assumed that it is ordered at zero reliability or failure probability = 1. The results are shown in Figs. 3 and 4.

If a spare part is ordered when reliability is high, the total cost is affected by the storage cost, whereas if it is ordered close to the breakpoint and replacement time, the total cost is controlled by the shutdown cost.

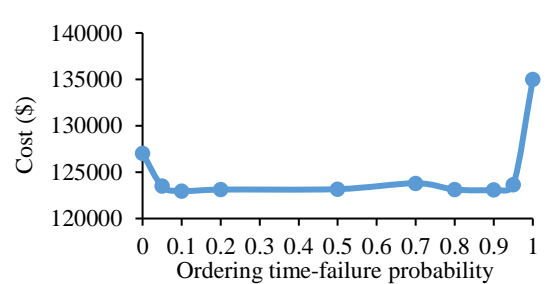


Fig. 3. Ordering time influence on the total cost of the system.

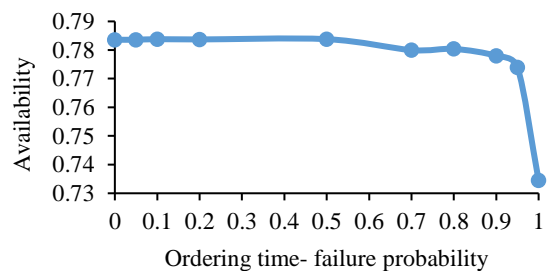


Fig. 4. Ordering time influence on the system availability.

These results show that the total cost depends on the ordering time, as expected; if a spare part is ordered at an incorrect time, the cost could be excessively increased.

However, orders should also be carried out with storing and replacement considerations, for this case, when the spare part is ordered after the shutdown, the cost is maximum [26]. Because the system waits for spare parts delivery and replacement, and the other components should work for longer times under stressful conditions so that, the failure probability is increased and system availability is decreased (Fig. 4). When storage is immediately replenished after replacement, the spare part is stored and the total cost is increased (Fig. 3). But ordering at this time cannot increase system availability more than a specified value (Fig. 4).

Fig. 4 shows that the best availability is obtained when the probability of failure is  $\leq 0.5$  and if an order is carried out in the interval (0.0, 0.5), the availability is constant and its variation can be ignored. From the technical point of view, if CBM devices cannot detect a failure in this interval, it is not very important because cost and availability are approximately constant.

### 5.1.2. Suppliers influence

Suppliers selection is the main issue in supply chain management. The influence of suppliers has been ignored in the joint analysis of spare parts inventory and maintenance management for manufacturing systems. In this paper, this significant factor is investigated. It is assumed that two suppliers can provide the spare parts; each supplier has a specified quality level, cost, and delivery time. It should be noted that the quality of both suppliers satisfies the system requirements and standards. In this section, the influence of these parameters is discussed. Fig. 5 shows the influence of the supplier on the system cost. When spare parts are provided by supplier B, the probability of selection of B is one, and when others provide these, this probability is zero.

In this case, supplier B can provide cheaper spare parts than A. Consequently, when the probability of selection for supplier B is increased, the cost is reduced. The influence of the supplier on the system cost and availability

is not constant and may vary depending on the supplier's characteristics and parameters. For instance, if supplier B provides the first spare part with better quality and the second spare part with inferior quality (each machine has two types of spare parts), this situation is captured in Fig. 6.

This variation is related to the quality and cost of spare parts, and the quality reduction may reduce system cost, but the lifetime is reduced and maintenance cost is increased. Thus, the final availability performance and cost are computed based on this interaction.

In Fig. 6, when the first spare part can compensate for the second spare part's weakness, the system availability is increased and the total cost is reduced (case 1).

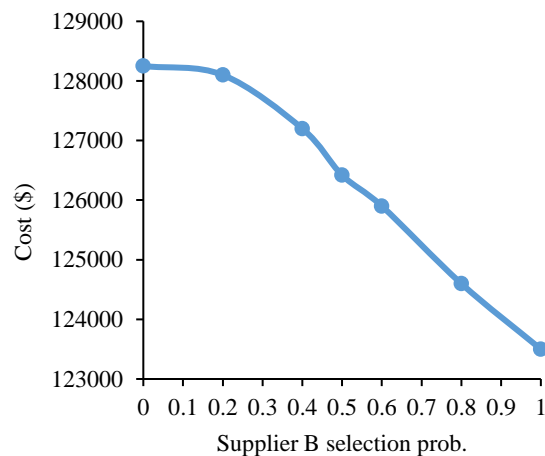


Fig. 5. Supplier selection influence on the system cost-case 2.

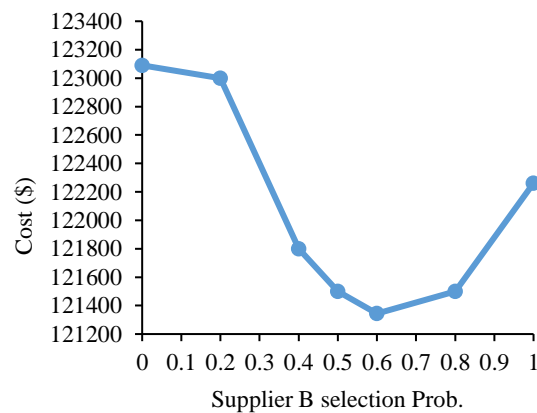


Fig. 6. Supplier selection influence on the system cost- case 1.



If the influence of the second spare part is dominant, the total cost is increased and the availability increase is stopped (case 2).

Fig. 7 shows the influence of the probability of selection of supplier B on the system availability for these cases.

In Fig. 7, it is assumed that the system includes identical machines (configuration A). Assume this configuration is changed and the production rate of a machine is two times that of the other (configuration B). When a machine is stopped, the production rate of the other needs to be increased three times. This machine uses two different spare parts, and the quality and cost of the suppliers for them are not the same. Supplier B provides the first spare part with lower quality and more cost than supplier A and delivers the second part with higher quality than it and equal cost to supplier A. Thus, the influence of these suppliers is not simple and decision-making is not easy. Fig. 8 shows the influence of these configurations on the system availability. For configuration A, when supplier B's role is increased, the availability is increased since, in this situation, quality is improved. However, for configuration B, this influence is not the same as in configuration A.

The delivery time is another significant factor that depends on supplier selection. Delay in spare part delivery increases the shutdown duration and the total cost while it reduces availability.

5.1.3. Load-sharing factor

When a machine with load-sharing dependence is stopped, the stress is increased on others [27].

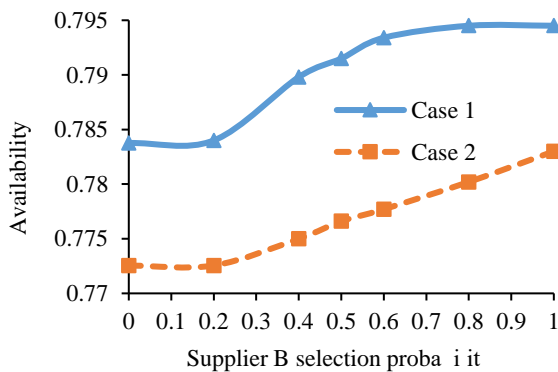


Fig. 7. Supplier selection influence on the system availability

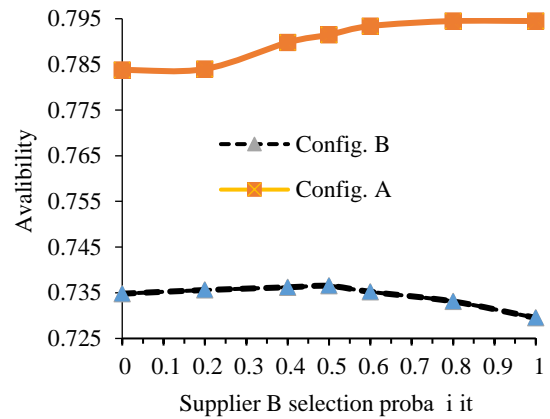


Fig. 8. System configuration and supplier influence on the availability

Assuming a system includes two machines, Fig. 9 illustrates the influence of the load-sharing factor of the first machine on the total cost of the system. For this example, when the load-sharing factor is 2, the total cost is the minimum. Because of load-sharing variations, the system stress condition is changed and the failure probability for a machine is increased. Also, this factor influences the system availability (Fig. 10).

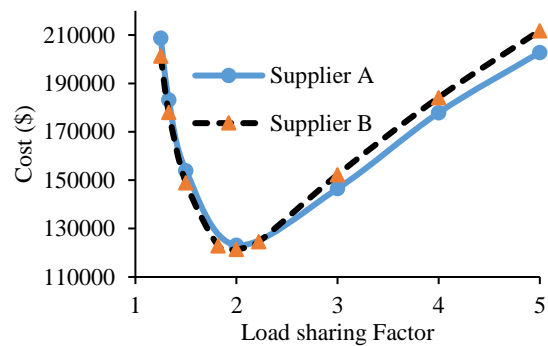


Fig. 9. Load-sharing factor influence on the system cost

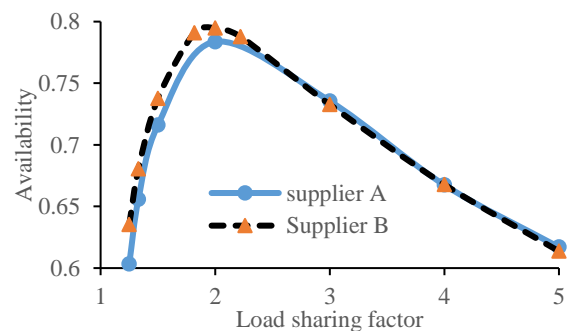


Fig. 10. Load-sharing influence on the system availability

The maximum availability is obtained when load-sharing factor is 2. Although this influence is independent of the suppliers (Figs. 9-10); the part quality defines the optimum situation. Fig. 11 shows the load-sharing factor's influence on this system's cost and availability.

5.2. Multi-state systems

Multi-state systems are designed to improve system performance and are frequently applied in different industries. If a load-sharing system can produce multiple output levels according to system configuration and load-sharing factor, we deal with a Multi-State-Load-Sharing system (MSLSS) [28-30]. Different parameters can influence this type of system performance. For instance, assume a load-sharing system with two machines, in which each machine can produce five levels of output. The system has ten states, corresponding to five levels of production output. Table 1 shows the states of the machines and the system. The system output is 100% in three states, and it is zero in one state, and in the other states, the output level depends on load-sharing and stress conditions.

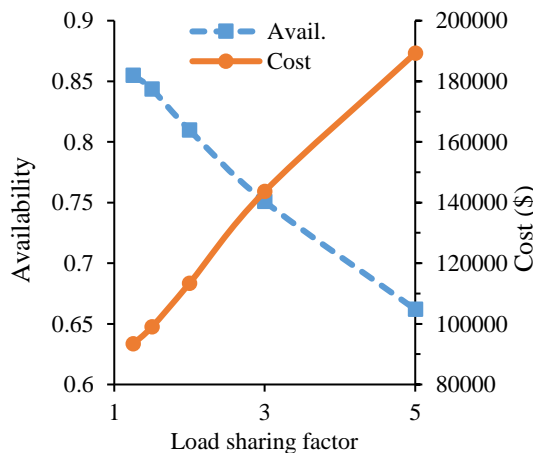


Fig. 11. Load-sharing factor influence on the system cost and availability- case 3

Fig. 12 shows the different operating states of the system. In the first state, the production rate is similar to the parallel structure, and in the fourth state, the system output is 100 %. In the parallel state, the output is at the lowest level and 50 percent is acceptable; thus, availability is at the highest level and cost is at the lowest level because of the minimum stressful condition. This result shows that when the acceptable performance level for an MSLSS is reduced, the system cost also decreases while the availability increases. In the rest of the section, the influence of significant parameters on an MSLSS is considered.

5.2.1. Ordering time effect

In the previous sections, the influence of ordering time on the load-sharing system has been investigated. If one looks as MSLSS, the ordering time effect is similar to their effect in the load-sharing system. The influence may be decreased whenever the desired level of system performance is reduced. Fig. 13 shows that for a system with 66.6% desired production output, ordering time has no impact on the availability of the system.

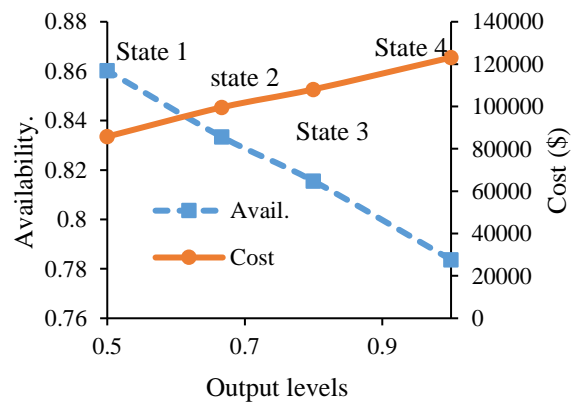


Fig. 12. The cost and availability of the MSLSS at different level outputs

Table 1. MSLSS definition.

Machine/system	Output									
Machine 1	0	0.5	0.5	0.66	0.80	1	0	0	0	0
Machine 2	0	0.5	0	0	0	0	0.5	0.66	0.80	1
System	0	1	0.5	0.66	0.80	1	0.5	0.66	0.80	1

But it becomes a crucial factor when it is close to the replacement time (e.g., probability of failure  $\geq 0.95$ ). This influence is vital for a system with 80% output if the probability of failure is higher than 0.7. Eventually, if 100% output is desired, ordering becomes significant the probability of failure is smaller than 0.5. This demonstrates that with the reduction of the stressful condition, the failure probability is also expected to decrease; therefore, spare parts can be ordered later, i.e., when the system attains 100% output.

Fig. 14 illustrates the influence of the ordering time on the system cost. It is obvious that with the reduction of the system's stressful conditions, the system cost is subsequently reduced. For all states, when the ordering time is near the failure time, the spare and repair costs are likely to increase, and consequently, the total cost starts to increase. Accordingly, the cost increase depends on the output level, and this rise is smaller for the higher output.

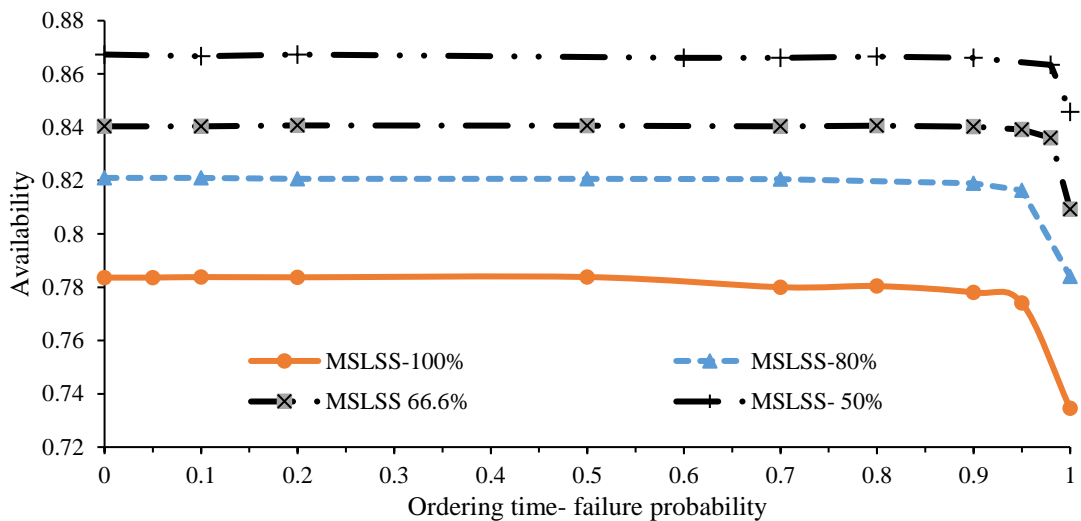


Fig. 13. The influence of ordering time on the availability for different levels of the MSLSS output

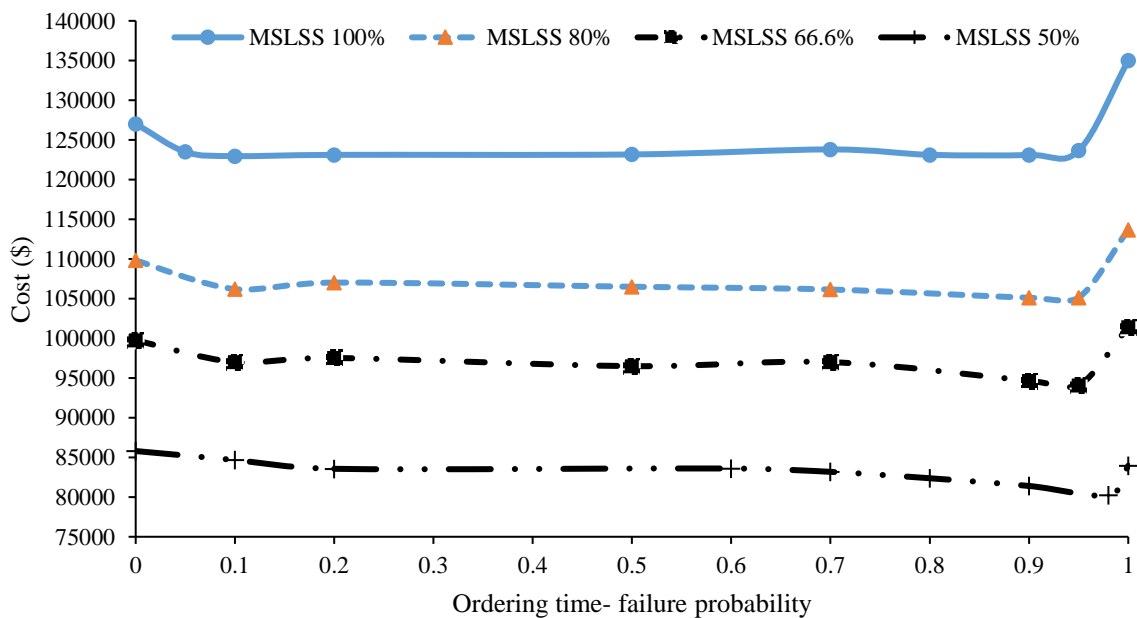


Fig. 14. Influence of ordering time on the system cost for different levels of MSLSS output.

5.2.2. Supplier selection effect

In the previous section, the influence of supplier selection on the cost and availability was investigated for the load-sharing system. In this section, two configurations of MSLSS are studied that the sum of the machine output production for these systems is equal, but the production rates of machines are different; in configuration (1), production rates for two machines are equal, and in configuration (2) the production rate of the first machine is four times of the second machine.

The spare parts of these systems are selected from two suppliers. Fig. 15 shows the system cost and availability for these cases. The influence of the supplier on the systems is similar and only when the supplier B portion is

increased, there is a decrease in the total cost and an increase in the availability.

The influence of the supplier on the cost and availability depends on the system logic. For instance, for configuration (2) and the desired output of the system are 80% of the normal condition output, the supplier effect on this system is different from it in configuration (1). In this situation, the cost and supplier selection follow a non-linear relationship (Fig. 16). Because supplier B provides two spare parts with different qualities but similar costs, i.e., the first spare part has a lower quality compared to the second spare part, but the costs are constant. The interaction between the quality and cost of the spare part overshadows the system behavior; when identical spare parts are applied, there is no such variation.

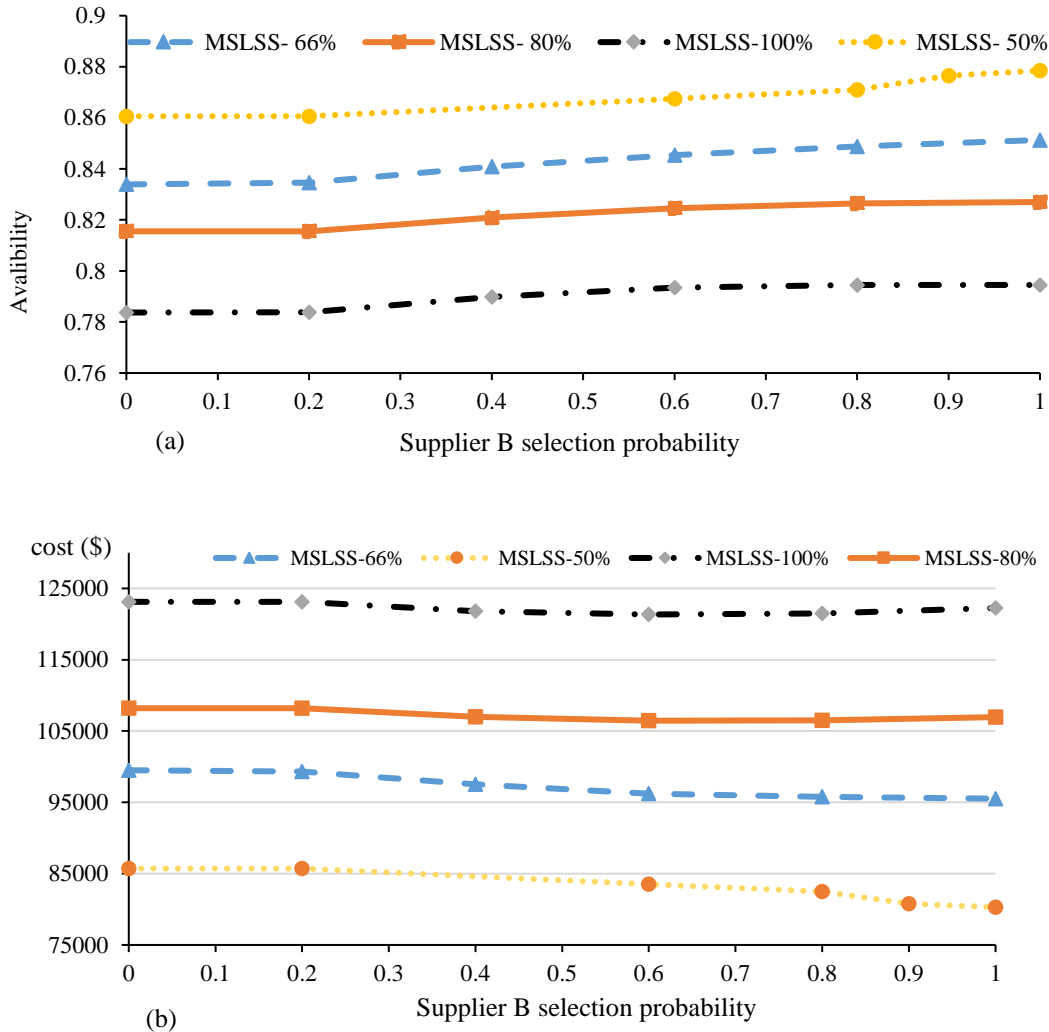
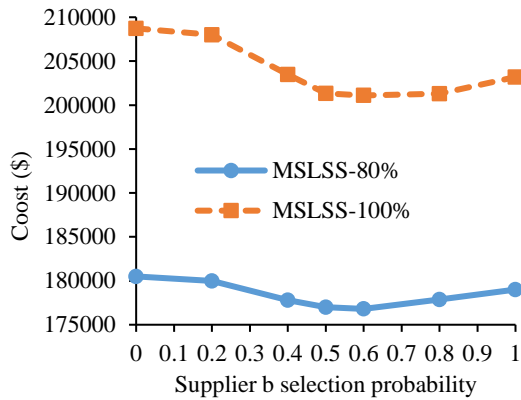


Fig. 15. The influence of the supplier selection on the system (a) MSLSS availability and (b) MSLSS cost.



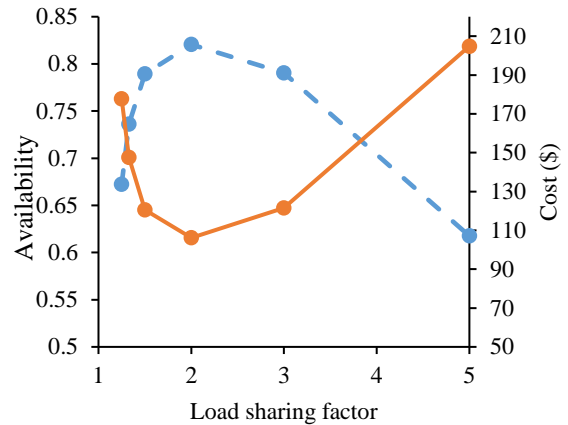
**Fig. 16.** Supplier selection influence on an MSLSS cost (configuration 2).

5.2.3. Load-sharing factor effect

In this section, the load-sharing factor is studied on an MSLSS. It is assumed that the acceptable output is 80% of the normal situation. Fig. 17 shows that the respective influence is the same as a simple load-sharing (Figs. 8 and 9) because the system structure and the relationship among load-sharing components are constant.

6. Conclusions

This paper considers spare parts inventory planning and maintenance modeling for a multi-state system with load-sharing dependence under preventive maintenance and CBM policy. Opportunity maintenance tasks are carried out when a spare part is replaced. Load-sharing, ordering time, and supplier selection factors are investigated. These factors affect the system cost and availability and should be considered in the system cost and availability analysis and optimization. The interaction among these factors should also be taken into account because it can (further) affect system performance. The results show that the influence of the ordering time of spare parts, the acceptable output level of the system, and the load-sharing factor are important. In future works, other types of dependencies among machines, such as technical and resource dependencies, and other practical issues, such as product quality under the load-sharing situation, corrective maintenance, and production planning process, will be considered.



**Fig. 17.** Influence of load sharing factor on the cost and availability of an MSLSS with 80% output.

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**How to cite this paper:**

M. A. Farsi, "Reliability, availability, and maintainability modeling of multi-state systems with load-sharing structure," *J. Comput. Appl. Res. Mech. Eng.*, Vol. 13, No. 1, pp. 39-53, (2023).

**DOI:** 10.22061/JCARME.2022.7962.2059

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