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Exploring entropy weighted TOPSIS and BHARAT approach for multi-criteria decision-making problem

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Abstract

Computational models/methods are regularly used by the engineering community to evaluate the optimal set of solutions with respect to the defined performance criteria. In the case of multi-objective optimization problems, the set of Pareto front or optimal solutions needs to be evaluated with a simple and effective search methodology to help the decision maker select the best parameters/factors. In this study, an attempt has been made to optimize the content of MWCNTs, cutting speed, and feed rate to minimize delamination factor and thrust force during drilling of MWCNTs reinforced GFRP nano-composite. Entropy-weighted TOPSIS and BHARAT approaches are implemented successfully for evaluating optimal parameters. Experimental result analysis suggests that the feed rate is the major contributing factor affecting the delamination factor and thrust force. The optimized levels for MWCNTs, cutting speed, and feed rate are 3 (1%), 3 (75 mm/min), and 1 (0.1 mm/rev), respectively, as obtained by both methods. Comparative analysis of entropy-weighted TOPSIS and the BHARAT approach has been performed in relation to multi-criteria decision-making problems. The evaluated Pearson's and Spearman's correlation coefficients for both methods are 0.98, suggesting a high correlation between both methods.

1. Introduction

Composite materials are combinations of two or more materials that are physically separable and distinct. The properties of the composite as a whole are enhanced compared to its individual constituents. Composite material generally consists of two materials with a base/bulk material known as the matrix and another reinforcing material as a fiber. Reinforcing the polymer matrix with fibers improves the base material's chemical, physical, and mechanical properties. A glass fiber-reinforced polymer composite is a type of polymer matrix composite that utilizes glass as its reinforcement. GFRP composites are widely used for various applications, including household, automobile, and aircraft components. GFRP composites are gaining popularity due to their high strength-to-weight and stiffness-to-weight ratios, lightweight, high elastic modulus, remarkable fracture toughness, and outstanding resistance to corrosion and thermal [1-3].

Over the past few decades, carbon nanotubes (CNTs) have been extensively utilized as reinforcement agents in the polymer industry, leading to multifunctional materials for a wide range of applications, including electronic devices, defence, aerospace, and automotive sectors. CNT is a rolled cylinder with several microns in length and a few nanometers in diameter of a graphite sheet. According to thickness, they are again classified as single-walled CNTs and multi-walled CNTs (MWCNTs) [4]. CNTs are the preferred reinforcing material for polymer composites due to their high aspect ratio and remarkable mechanical properties [5, 6].

In recent years, research has been focused on developing CNT-reinforced polymer composites for various applications with improved mechanical and physical properties [7-9]. After the development of CNT-reinforced polymer nano-composite, they further need machining or drilling operations for assembly in various applications. But the machining/drilling of CNT-reinforced polymer composite comes with defects like fiber tearing, cracking, delamination, and ovality. The quality of drilled holes directly affects the life of punctured joints [10]. Delamination error is a major cause of part rejection in assemblies of fiber-reinforced composite materials.

During the drilling of fiber-reinforced polymer composite, the quality of the drilled hole in terms of delamination error, surface roughness, and thrust force during the drilling operation is an outcome of different machining parameters. Selecting optimal drilling parameters for improving surface roughness, along with minimizing thrust force, is a kind of multi-criteria decision-making problem (MCDM).

Various methods and techniques are proposed and used by different researchers for solving MCDM problems in the field of engineering. The some of MCDM methods used by researchers are Teaching-Learning-Based Optimization (TLBO) [11, 12], Simulated Annealing (SA) [13, 14], Ant Colony Optimization (ACO) [15], Particle Swarm Optimization (PSO) [16], Salp Swarm Algorithm (SSA) [17], Lion Optimization Algorithm (LOA) [18], Whale Optimization Algorithm [19], Jaya Optimization algorithm [20], Grey Relational Analysis (GRA) [21, 22], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [23], Analytic Hierarchy Process (AHP) [24], BHARAT (Best Holistic Adaptable Ranking of Attributes Technique) [25, 26], etc.

During the drilling of fiber-reinforced polymer composite, various parameters come into the picture, such as proportion and nature of reinforcing material, drill speed, feed rate, drill tool, and drill diameter. Kopparthi *et al.* [27] proposed TLBO for maximizing the mechanical properties of GFRP by optimizing the number of layers and injection pressure. Rao *et al.* [28] evaluated the electro-discharge machining process for maximizing the metal removal rate (MRR), while minimizing the taper angle (ϴ), the tool wear rate (TWR), and the delamination factor (DF). The gap voltage (Vs), pulse current (Ip), pulse on time (Ton), and tool rotation speed (N) were the process parameters optimized with the aim of defining an objective function using the Jaya algorithm.

Verma *et al.* [23] applied the TOPSIS methodology to optimize time, NaOH concentration, and temperature for maximizing biomass production. Chandrasekhar and Prasad [29] applied the Entropy-VIKOR method to optimize the electro-chemical machining parameters, voltage, electrolyte concentration, and current to maximize material removal rate and minimize over-cut and delamination during micro-drilling of AA606l-TiB_2 composite. So, as presented above, various researchers proposed and applied different MCDM techniques/ methods for solving optimization problems in the field of engineering. In this study, entropy weighted TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and recently presented BHARAT (Best Holistic Adaptable Ranking of Attributes Technique) approach are implemented for optimizing the content of MWCNTs, cutting speed, and feed rate to minimize delamination factor and thrust force during drilling of MWCNTs reinforced GFRP nano-composite. Comparative analysis for both methods is presented in the area of engineering optimization.

2. Materials and methods

2.1. Sample preparation

In this study, E-glass fiber, Bisphenol A- based resin, and amine-based hardener (HY951) produced by Inovatif Material Technologies Inc., Turkey, were used as the base matrix. The MWCNTs with a purity level of 97% were used as filler material having dimensions of 5-20 nm diameter and a length of 2-10 µm. The sample preparation for MWCNTs reinforced GFRP nano-composite is presented in Fig. 1.

Fig. 2(a and b) shows the FESEM images of MWCNTs dispersed in glass-epoxy resin. FESEM images demonstrate the homogenous dispersion of MWCNTs within the polymer matrix.

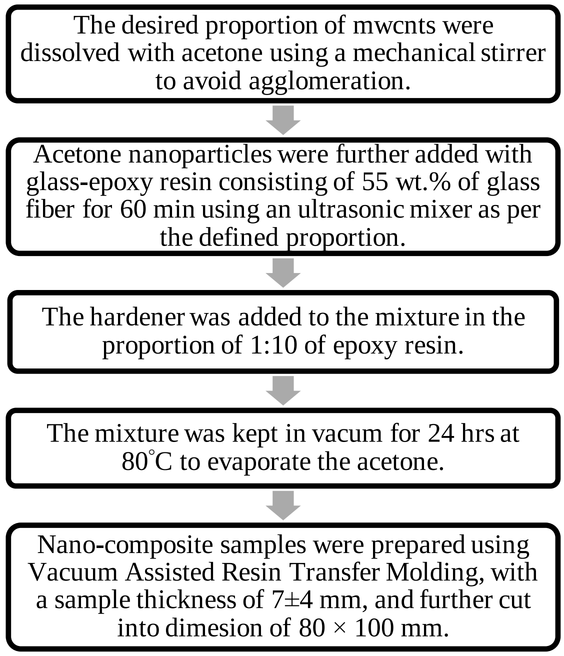

Fig. 1. Fabrication of mwcnts reinforced GFRP nano-composite.

2.2. Experimentation

The experiments were designed in accordance with DoE-Taguchi methodology. DoE-Taguchi method helps us to plan and analyse the experimental results with a minimum number of experiments [30]. Table 1 presents the controllable input factors and desired outputs.

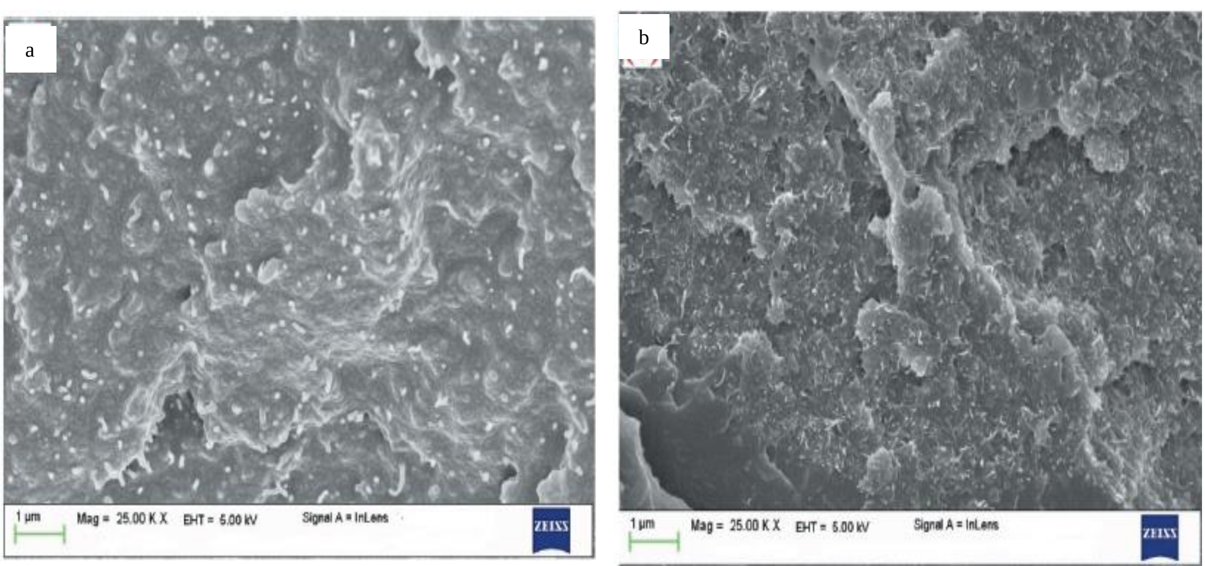

Fig. 2. (a) GFRP with 0.5 wt.% of MWCNTs and (b) GFRP with 1.0 wt.% of MWCNTs.

Table 1. Input and output parameters for experimentation.

Controllable inputs					
Controllable factors	Unit	Symbol	Levels		
			1	2	3
Mwcnts	wt.%	Cn	0	0.5	1.0
Cutting speed	m/min	Cs	25	50	75
Feed rate	mm/rev	Fr	0.10	0.15	0.20
Desired outputs					
Output parameters	Unit	Symbol			
Delamination factor	--	F_d			
Thrust force	N	F_z			

Table 2. Johnford VMC 550 CNC machine specification.

Speed	8000 – 10000 RPM
Motor power	7.5/11 kW
Travel (x/y/z)	550 mm (x)/ 400 mm (y)/ 450 mm (z)
Accuracy	±0.001 mm (x/y/z)
Control system	FANUC

The experiments were designed using an L27 orthogonal array in accordance with the DoE-Taguchi method. Each experiment was repeated three times, resulting in a total of 81 experiments, and the average results were considered for analysis. The experiments were conducted under dry conditions. The drilling operation was conducted using a Johnford VMC 550 CNC machine with an HSS drill tool, featuring an 8 mm diameter, 30° helix angle, and a 118° point angle . The specification of the VMC 550 CNC machine is given in Table 2.

Online measurement of thrust force was carried out using a Kistler 9257B dynamometer attached to the machining table, and data was recorded online with KistlerDynoWare software. The dynamometer has a clamping area of 100 × 170 mm and a force measuring range of -5 to 10kN. Fig. 3 shows the Kistler dynamometer used for thrust force measurement.

Delamination is a kind of damage occurring during the drilling operation of laminated composites, which occurs due to damage to the composite laminates. To analyse the quality of the hole, the generated delamination is quantified by the delamination factor (F_d), calculated as the ratio of maximum diameter (D_{max}) generated during drilling to nominal diameter (D). The whole delamination was quantified offline with the Euromex Holland Type PB 4161 microscope. Generated images with a microscope were transferred to a CAD environment for diameter measurement, and further delamination factor was evaluated.

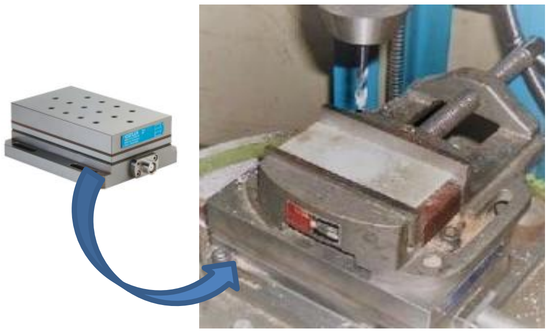

Fig. 3. Kistler 9257B dynamometer.

3. Results and discussion

3.1. Result analysis with entropy weighted TOPSIS

3.1.1. Entropy weight calculation for response variable

In decision-making problems, subjective and objective approaches are used to weigh the attributes. Nowadays, for an objective approach, various techniques are used by researchers to assign the weights. The EWM is the most preferred and effective approach for weighting attributes or responses in Multi-Criteria Decision Making (MCDM) [31-33]. EWM determines the weight objectively using the available values of the attributes, and it overcomes the shortcomings of the subjective weighing method.

Entropy weight calculating steps [34]:

Step 1: Normalization of the decision matrix

For n response variables with m values, the decision matrix is constructed as follows:

$$A = \left(a_{ij}\right)_{mn} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad (1)$$

Since response/output variables have different units, normalization is essential to enable meaningful comparison by scaling the data within the range of 0 to 1. Eq. (2) is used to normalize the desirable output, and Eq. (3) is used to normalize the undesirable output.

$$NMa_{ij} = \frac{a_{ij}}{Maxa_{ij}} \tag{2}$$

$$NMa_{ij} = \frac{Mina_{ij}}{a_{ij}} \tag{3}$$

In this work, both the delamination factor and thrust force are undesirable responses; Eq. (3) is applied for normalization.
Following normalization, the response probability is computed with Eq. (4).

$$P_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \tag{4}$$

Step 2: Computing the entropy for each index

$$E_j = -\frac{1}{lnm}\sum_{i=1}^{m} P_{ij}.lnP_{ij}, \quad j = 1,2,3 \ldots\ldots n \tag{5}$$

Step 3: Computing the degree of deviation for each response

$$D_j = |1 - E_j|, \quad j = 1,2, \ldots.n \tag{6}$$

where D_j signifies the degree of divergence of essential information provided by the j^{th} criterion.

Step 4: Computing the entropy's weight

$$w_j = \frac{D_j}{\sum_{j=1}^{m} D_j} \tag{7}$$

where w_j is the importance weight of the j^{th} criterion.
The entropy weights for each response variable are shown in Table 3.

Table 3. Entropy weights.

	Delamination factor	Thrust force (N)	Total
% Weight	97.30	2.70	100

3.1.2. Optimization using the TOPSIS method

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was proposed by Hwang and Yoon in 1995 [35]. The TOPSIS method works on the principle that the best alternative is closest to the positive ideal solution and farthest from the negative ideal solution. The TOPSIS method aims to generate an alternative that is closest to the hypothetical best solution and farthest from the hypothetical worst solution. Several researchers implemented TOPSIS in the area of optimization. The sequential steps involved in implementing the TOPSIS method are described below [36]:

Step 1: Decision matrix

In MCDM problems applying TOPSIS, the mean values of each output response obtained from each experiment are compiled into a decision matrix, as per Eq. (8):

$$D = \begin{bmatrix} x_{11}x_{12}x_{13} & \cdots & \cdots & x_{1n} \\ x_{21}x_{22}x_{23} & \cdots & \cdots & x_{2n} \\ x_{31}x_{32}x_{33} & \cdots & \cdots & x_{3n} \\ \cdots\ \cdots\ \cdots & \cdots & \cdots & \cdots \\ \cdots\ \cdots\ \cdots & \cdots & \cdots & \cdots \\ x_{m1}x_{m2}x_{m3} & \cdots & \cdots & x_{mn} \end{bmatrix} \tag{8}$$

Here, n refers to the response variables, and m represents the corresponding alternatives.

Step 2: Normalization of the decision matrix

For meaningful comparison, it is essential to convert the response variables present in different dimensions and scales into non-dimensional characteristics. The decision matrix is further normalized using Eq. (9):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1,2, \ldots.m;\ j = 1,2, \ldots.n. \tag{9}$$

Step 3: Calculation of weighted normalized matrix

At this stage, the normalized response variables are weighted with entropy-derived weights described in section 3.1(step 4). The final values of weights are 97.30% for delamination factor and 2.70% for thrust force, as presented in Table 2; the weighted normalized decision matrix is computed with Eq. (10):

$$v_{ij} = w_j * r_{ij},$$
$$i = 1,2, \ldots .m; j = 1,2, \ldots \ldots, n. \quad (10)$$

Step 4: Evaluation of positive ideal (best) and negative ideal (worst) solution

The best solution maximizes the desired response variables, whereas the worst solution minimizes them. These solutions are computed with Eqs. (11) and (12):

The best solution:

$$A^+ = \{v_1^+, v_2^+, \ldots \ldots . v_n^+\} = \{(max_i v_{ij} | j \epsilon J), (min_i v_{ij} | j \epsilon J') | 1, \ldots . m\} \quad (11)$$

The worst solution:

$$A^- = \{v_1^-, v_2^-, \ldots \ldots . v_n^-\} = \{(min_i v_{ij} | j \epsilon J'), (max_i v_{ij} | j \epsilon J) | 1, \ldots . m\} \quad (12)$$

where,

$J = \{j = 1,2, \ldots \ldots n | j\}$:

Associated with beneficial response variables.

$J' = \{j = 1,2, \ldots \ldots nj | j\}$:

Associated with non-beneficial response variables.

Step 5: Computation of the distance to ideal solution

The distance of each alternative to the positive ideal solution (S_i^+) and to the negative ideal solution (S_i^-) is calculated using Eq. (13) and (14):

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2},$$
$$i = 1,2, \ldots \ldots m \quad (13)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2},$$
$$i = 1,2, \ldots \ldots m \quad (14)$$

where S_i^+ is the distance between i^{th} alternative and the best solution, and S_i^- is the distance between i^{th} alternative and the worst solution.

Step 6: Closeness coefficient for each alternative solution

Eq. (15) is employed to compute the relative closeness of each alternative to the best solution:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad (15)$$

where, $0 < C_i \leq 1, i = 1,2, \ldots \ldots m$

Step 7: Preference order ranking

The rank preference is established by ordering the closeness coefficient (C_i) values in descending order from 1 to 0. The alternative with the closeness coefficient nearest to 1 attains the highest rank and therefore is considered the best optimal choice among multiple decision-making response variables. The calculated closeness coefficient values for each alternative/experiment with respective rankings are presented in Table 4.
An examination of the calculated closeness coefficient presented in Table 4 indicates that experiment 25 exhibits the highest value with MWCNTs: 1 wt.%, cutting speed: 75 m/min, and feed rate: 0.1 mm/rev, which is identified as the optimal set of process parameters for minimizing both the delamination factor and thrust force.

3.1.3. Anova for closeness coefficient

Analysis of variance (ANOVA) is employed to assess the effect of process parameters on multiple response variables. The ANOVA results for closeness coefficient, obtained at a 95% confidence level, are presented in Table 5.

Table 4. Values of closeness coefficient and ranking.

Test no.	Input parameters Cn	Cs	Fr	Closeness coefficient	Ranking
1	0	25	0.1	0.838	9
2	0	25	0.15	0.717	23
3	0	25	0.2	0.694	27
4	0	50	0.1	0.884	5
5	0	50	0.15	0.755	17
6	0	50	0.2	0.714	25
7	0	75	0.1	0.881	7
8	0	75	0.15	0.747	21
9	0	75	0.2	0.721	22
10	0.5	25	0.1	0.870	8
11	0.5	25	0.15	0.752	18
12	0.5	25	0.2	0.697	26
13	0.5	50	0.1	0.892	4
14	0.5	50	0.15	0.787	13
15	0.5	50	0.2	0.747	20
16	0.5	75	0.1	0.908	3
17	0.5	75	0.15	0.808	12
18	0.5	75	0.2	0.756	16
19	1	25	0.1	0.883	6
20	1	25	0.15	0.750	19
21	1	25	0.2	0.717	24
22	1	50	0.1	0.916	2
23	1	50	0.15	0.829	10
24	1	50	0.2	0.774	15
25	1	75	0.1	0.999	1
26	1	75	0.15	0.819	11
27	1	75	0.2	0.777	14

Table 5. Anova for weighted close coefficient.

Parameters	Df	Seq ss	Adj ms	Contribution (%)	F-value	P-value
Mwcnts	2	0.01458	0.00729	8.77	25.21	0.000
Cs	2	0.01512	0.00756	9.10	26.13	0.000
Fr	2	0.13077	0.06538	78.65	226.00	0.000
Error	20	0.00579	0.00029	3.48		
Total	26	0.16626		100.00		

The results for response variables are considered at 'Lower-the-Better' criteria using MINITAB software. With a P-value of 0.000, all three parameters are significant, and feed rate is a major contributing parameter with a % contribution of 78.65% for the response variable.

Table 6 presents an analysis of means with maximum values of wt. closeness coefficient for MWCNTs at level 3 (0.1 wt.%), cutting speed at level 3 (75 m/min), and feed rate at level 1 (0.1 mm/rev).

Table 6. Response table for means.

Levels	Input parameters Mwcnts	Cutting speed	Feed rate
1	0.7724	0.7686	0.8968
2	0.8019	0.8109	0.7737
3	0.8293	0.8241	0.7331
Delta	0.0569	0.0555	0.1637
Rank	2	3	1

3.2. Result analysis with BHARAT

3.2.1. Optimization with BHARAT

Best Holistic Adaptable Ranking of Attributes Technique (BHARAT) is a recent, simple, logical, effective, and powerful decision-making method proposed by Rao [25-26].

The step-wise implementation of the BHARAT approach is as follows:

Step 1: Identification of response variables

Identification of the desired response variables, O_i= [i = 1,2,3……m] and the available alternatives, Aj = [j= 1,2,3, ….. n]. The response variable may be desirable or undesirable. In this study, both response variables, delamination factor, and thrust force are undesirable.

Step 2: Weight assignment to response variables

Following the objective approach of weight assignment, the response variables are ranked according to the decision maker's choice, with options ranging from 1 to 3. So on. When two or more response variables are of equal significance, they are ranked based on their average value. After ranking, the R-method [37] is used to evaluate the objective weights.

$$w_i = \frac{1/\sum_{k=1}^{i} 1/r_k}{\sum_{i=1}^{m}[1/\sum_{k=1}^{i} 1/r_k]} \quad (16)$$

where,

w_i=weights for response variables, I (i = 1,2,..m)

r_k = rank of response variables, k (k = 1,2,3….i)

m = number of response variables.

The weights for each response variable are shown in Table 7.

Table 7. Objective weights.

	Delamination factor	Thrust force	Total
% Weight	60.00	40.00	100

Step 3: Data normalization

Since response/output variables have different units, normalization is essential to enable meaningful comparison by scaling the data within the range of 0 to 1. Normalize the response variables for different alternatives with reference to the 'Best' response variable. The word 'Best' refers to the highest value of the response variable for the desirable response variable and the lowest value of the response variable for the undesirable response variable.
Consider a decision matrix with *m* observations for *n* response variables:

$$A = (x_{ij})_{mn} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad (17)$$

The desirable response variables are normalized with Eq. (18), and the undesirable response variables are normalized with Eq. (19).

$$NMa_{ij} = \frac{a_{ij}}{Maxa_{ij}} \quad (18)$$

$$NMa_{ij} = \frac{Mina_{ij}}{a_{ij}} \quad (19)$$

In this work, all response variables, delamination factor, and thrust force are undesirable and are normalized with Eq. (19).

Step 4: Calculation of total score

The total score for the response variable is calculated using Eq. (20).

$$Total\ Score(C_i) = \sum w_i * (x_{ji})_{normalized} \quad (20)$$

Step 5: Ranking

The ranking order is established by ordering the total score (C_i) in descending order from 1 to 0.

The value that is nearest to 1 attains the first rank, and that alternative is selected as the best among multiple decision-making response variables. The evaluated values for each alternative/experiment with corresponding rankings are presented in Table 8.
An examination of the calculated total score presented in Table 8 indicates that experiment 25 exhibits the highest value with MWCNTs: 1 wt.%, cutting speed: 75 m/min, and feed rate: 0.1 mm/rev is identified as the optimal set of process parameters for minimizing both the delamination factor and thrust force.

Anova for total score

The ANOVA results for the closeness coefficient, obtained at a 95% confidence level, are presented in Table 9. The results of response variables are considered at 'Lower-the-Better' criteria using MINITAB software. With a P-value of 0.000, all three parameters are significant, and feed rate is the major contributing parameter with a % contribution of 78.34% for the response variables.

Table 8. Total score and ranking.

Test no.	Input parameters Cn	Cs	Fr	Total score	Ranking
1	0	25	0.1	0.88	9
2	0	25	0.15	0.75	23
3	0	25	0.2	0.71	27
4	0	50	0.1	0.9	7
5	0	50	0.15	0.79	19
6	0	50	0.2	0.74	25
7	0	75	0.1	0.9	8
8	0	75	0.15	0.77	21
9	0	75	0.2	0.74	24
10	0.5	25	0.1	0.91	6
11	0.5	25	0.15	0.8	16
12	0.5	25	0.2	0.73	26
13	0.5	50	0.1	0.92	5
14	0.5	50	0.15	0.82	14
15	0.5	50	0.2	0.78	20
16	0.5	75	0.1	0.93	3
17	0.5	75	0.15	0.84	12
18	0.5	75	0.2	0.79	18
19	1	25	0.1	0.92	4
20	1	25	0.15	0.8	17
21	1	25	0.2	0.76	22
22	1	50	0.1	0.94	2
23	1	50	0.15	0.87	10
24	1	50	0.2	0.82	13
25	1	75	0.1	0.99	1
26	1	75	0.15	0.86	11
27	1	75	0.2	0.81	15

Table 10 presents an analysis of means with maximum values of wt. closeness coefficient for MWCNTs at level 3 (1 wt.%), cutting speed at level 3 (75 m/min), and feed rate at level 1 (0.1 mm/rev).

Table 9. Anova for weighted total score.

Parameters	Df	Seq ss	Adj ms	Contribution (%)	F-value	P-value
Mwcnts	2	0.01949	0.00974	13.11	51.89	0.00
Cs	2	0.00896	0.00448	6.02	23.85	0.00
Fr	2	0.11647	0.05823	78.34	310.12	0.00
Error	20	0.00376	0.00019	2.53		
Total	26	0.14867		100.00		

Table 10. Response table for means.

Levels	Input parameters		
	Mwcnts	Cutting speed	Feed rate
1	0.7978	0.8067	0.9211
2	0.8356	0.8422	0.8111
3	0.8633	0.8478	0.7644
Delta	0.0656	0.0411	0.1567
Rank	2	3	1

3.3. Discussion

In the area of MCDM, various methods are proposed and implemented by researchers. In this study, entropy weighted TOPSIS and BHARAT methods are implemented for the machining/drilling process of MWCNTs reinforced GFRP nano-composite. To understand the consistency and performance, both methods are compared with the rankings provided by these methods [38]. Table 11 presents the comparative results obtained with both methods.

Table 11. Comparative results by entropy weighted TOPSIS and BHARAT.

Input parameters	Entropy weighted TOPSIS			BHARAT		
	Mwcnts	Cs	Fr	Mwcnts	Cs	Fr
Optimized levels	3	3	1	3	3	1
Contribution (%)	8.77	9.10	78.65	13.11	6.02	78.34
P- value	0.00	0.00	0.00	0.00	0.00	0.0

The results obtained with both methods are consistent with each other. The optimized input parameter levels suggested by both methods are the same as MWCNTs at level 3, Cutting Speed at level 3, and Feed Rate at level 1. Both methods also suggest that all three input parameters are significant with a P-value of 0.00. As per both methods, the feed rate plays a major role in affecting the response variable with the highest % contribution.

In the next step, to understand the consistency and correlation between entropy weight, TOPSIS and BHARAT, Pearson's correlation coefficient, and Spearman's ranking correlation coefficient were evaluated using rankings provided by both methods. A correlation coefficient is a statistical tool that helps to describe the relationship between two variables and also presents the strength of the relationship between them, but it can't be used to establish agreement between two methods [39]. Pearson's correlation coefficient is calculated using Eq. (21).

$$r_{x,y} = \frac{\sum_{i=1}^{n}((x_i-\bar{x})(y_i-\bar{y}))}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2} * \sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \tag{21}$$

where,

n = sample points,
x_i&y_i = individual sample points,
$\bar{x}$&$\bar{y}$ = sample mean

The Spearman's rank correlation coefficient is computed with Eq. (22).

$$r_k = 1 - \frac{\sum_i d_i^2}{n(n^2-1)} \tag{22}$$

where,

n = sample points,
$d_i = x_i - y_i$,

Both Pearson and Spearman coefficient can have a value between +1 to -1, where

- +1 means a perfect positive association.
- 0 means no association of values.
- -1 means a perfect negative association.

The values of the Pearson and Spearman coefficients are presented in Table 12.

Table 12. The pearson and spearman correlation coefficient.

z	Pearson's cc	Spearman's cc	Correlation
Entropy wt. TOPSIS - BHARAT	0.98	0.99	High correlation

The Pearson's and Spearman's correlation coefficient values suggest a strong correlation between Entropy wt. TOPSIS and BHARAT. The correlation coefficient quantifies the level to which the two variables or quantities are related. High-value correlation does not mean that the results of both methods are in good agreement with each other. Bland and Altman [40-41] introduced the Bland –Altman (B&A) plot to understand the agreement between the quantitative results presented by the two methods. A B&A plot is a scatter plot in the XY plane. Where the X-axis presents the average or means of two measurements ((A-B)/2) and the Y-axis presents the difference between two measurements (A-B). In other words, the B&A plot is a plot of the difference between the two measured quantities against the mean of the two measurements.

Bland & Altman recommend that if 95% of the data points lie within ±2*Std. Deviation (S) of the mean difference ($\bar{d} \pm 2S$), we can say that the quantitative measurements provided by both methods are in good agreement with each other. To evaluate the agreement between the entropy wt. in TOPSIS and BHARAT, the B&A plot is plotted with the rankings provided by these two methods. Table 13 shows the ranking data used for the B&A plot.

Fig. 4 shows the B&A plot for rankings generated by both methods. It is very clear from the B&A plot that all the data points are distributed within the range of $\bar{d} - 2S$ and$\bar{d} + 2S$. This significantly suggests that the results of both methods are in mutual agreement with each other.

4. Conclusions

Entropy-weighted TOPSIS is a well-established method in the area of MCDM, while BHARAT is a recently presented method by Rao [25, 26].

Table 13. Ranking data for and an agreement between two methods.

Expt. no.	Rank by ent. wt. TOPSIS (a)	Rank by BHARAT (b)	Mean of rankings ((a+b)/2)	Diff. (a-b)
1	9	9	9	0
2	23	23	23	0
3	27	27	27	0
4	5	7	6	-2
5	17	19	18	-2
6	25	25	25	0
7	7	8	7.5	-1
8	21	21	21	0
9	22	24	23	-2
10	8	6	7	2
11	18	16	17	2
12	26	26	26	0
13	4	5	4.5	-1
14	13	14	13.5	-1
15	20	20	20	0
16	3	3	3	0
17	12	12	12	0
18	16	18	17	-2
19	6	4	5	2
20	19	17	18	2
21	24	22	23	2
22	2	2	2	0
23	10	10	10	0
24	15	13	14	2
25	1	1	1	0
26	11	11	11	0
27	14	15	14.5	-1

Mean of Difference ($\bar{d}$): 0
Standard Deviation for difference (S): 1.30
$\bar{d} - 2S = 0 - 2 * 1.30 = -2.6$
$\bar{d} + 2S = 0 + 2 * 1.30 = 2.6$

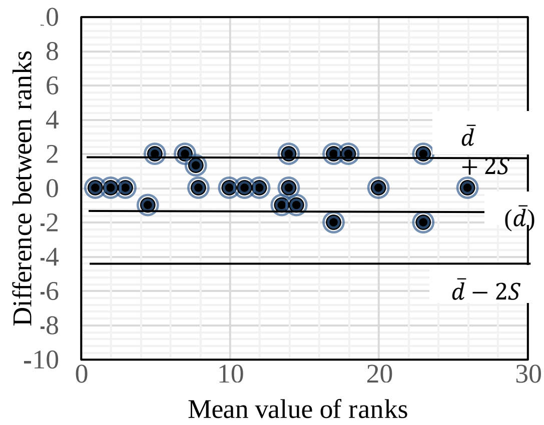

Fig. 4. Bland-altman plot.

In this study, both methods are evaluated for their applicability and feasibility of use in the area of MCDM. Based on this study, the following major conclusions are drawn:

1. Entropy-weighted TOPSIS and BHARAT approaches are successfully implemented for optimizing wt% of MWCNTs, cutting speed & feed rate during the drilling operation of MWCNTs reinforced GFRP nano-composite, intending to minimize delamination factor and thrust force.
2. The results obtained by both methods are very much in line with each other. 1 wt% of MWCNT, 75 m/min of cutting speed, and 0.1 mm/rev feed rate are optimal parameters during drilling of MWCNTs reinforced GFRP nano-composite for minimization of delamination factor and thrust force.
3. The addition of MWCNTs with GFRP nano-composite improved the quality of the drilled hole by minimizing thrust force in comparison to the GFRP composite due to improved adhesion between fiber and matrix.
4. ANOVA revealed feed rate as a major contributing parameter affecting the delamination factor and thrust force during the drilling of MWCNTs reinforced GFRP nano-composite.
5. The correlation coefficient and B&A plot presented a strong correlation and agreement between the ranking orders obtained by both methods. This supports the applicability of entropy-weighted TOPSIS and the BHARAT approach in the area of MCDM.
6. In comparison to the entropy-weighted TOPSIS method, the BHARAT approach makes it easier to assign objective weights and is stable. The BHARAT approach has presented promising results and can handle MCDM problems in every domain.

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