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Few heuristic optimization algorithms to solve the multi-period fixed charge production-distribution problem

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Abstract

This paper deals with a multi-period fixed charge production-distribution problem associated with backorder and inventories. The objective is to determine the size of the shipments from each supplier and backorder and inventories at each period, so that the total cost incurred during the entire period towards production, transportation, backorder and inventories is minimised. A 0-1 mixed integer programming problem is formulated.
Genetic algorithm based population search heuristic, Simulated annealing based neighbourhood search heuristic and Equivalent variable cost based simple heuristic are proposed to solve the formulation. The proposed methodologies are evaluated by comparing their solutions with the lower bound solutions. The comparisons reveal that Genetic algorithm and Simulated annealing algorithm generate better solutions than the Equivalent variable cost solutions and are capable of providing solutions close to the lower bound value of the problems.

1. Introduction

In a scattered production system, the production location also determines the overall production costs. It is because, the urban location will require more labour and overhead costs than rural location. Moreover, in such scattered production system with scattered customers, the production location influences the distribution schedule and thereby distribution costs. Therefore, in industrial problems where production and distribution costs are both of a similar magnitude, it is necessary to coordinate the two functions in order to limit global costs [1]. Most companies manage these two functions independently, with little or no coordination between production and distribution planning. This decoupled approach works acceptably well if there is sufficient finished goods inventory to buffer the production and distribution operations from each other. However, the cost of carrying inventory and the trend to just-in-time operations is creating pressure to reduce

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inventories in the distribution channel. As a result of this pressure, many companies are exploring closer coordination along the manufacturing/ distribution channel [2]. Many companies strive to synchronize their production, transportation and replenishment planning by adopting supply chain management practices such as vendor-managed inventory, efficient consumer response, and collaborative planning, forecasting and replenishment. The main objective of these collaborative methods is to reduce inefficiencies and to eliminate redundancies between the different partners [3].

The multi-period fixed charge production-distribution problem (MPFCPDP) is an extension of FCT (fixed charge transportation) problem, where the time based strategic decisions on size of the shipments from each supplier, inventory, and backorder can make an economical distribution. The MPFCPDP problem is difficult to solve due to the presence of fixed costs, which cause nonlinearities in the objective function and are known to be Non-deterministic Polynomial-time 'NP' hard [4]. The complexity of the problem is further increased, when supplier dependent product cost, time dependent inventory and backorder are included in the model. This limits the usage of the conventional multi-period and fixed charge solution procedures.

There are many different multi-period distribution problems (MPDP) in the literature [2, 3, 5-12] involving considerations of production and transportation, possibly together with other functions. The review on multi-period problems reveals the following: most of the models attempt to integrate inventory and transportation issues; excess in availability in any period is held as inventory at the suppliers end and is used for the subsequent periods; inventory at the demand points has not been given due consideration; admission of backorder may considerably reduce the total cost of logistics; not all the papers (except [6,8,10]) have included the fixed charge associated with transportation; though few papers have included fixed charge in their models, they do not deal exactly the multi-period fixed charge production-distribution between multiple sources (suppliers) and multiple destinations (customers) with inventory alternatives and backorder consideration to optimize the total production-distribution cost.

In the light of the above, this paper considers a MPFCPDP problem concerning the production, transportation and storage of finished goods from '*m*' suppliers (industrial producers) to '*n*' customers (demand centers like assembling centers, distribution centers etc.). The transportation cost is the main element in the proposed production-distribution model. The other considerations are storage and backorders. This paper considers a two-echelon inventory system where the suppliers' supply capacity and customers' demands are deterministic. The purpose of maintaining two-echelon inventory is to minimize the total distribution cost while integrating production, transportation, backorder and inventories. The production-distribution planning problem addressed here as MPFCPDP, is formulated as a 0-1 mixed integer programming problem. Genetic algorithm (GA) based population search heuristic, Simulated annealing algorithm (SAA) based neighbourhood search heuristic and an Equivalent variable cost (EVC) based simple heuristic are proposed to solve the formulation.

The rest of the paper is organised as follows: Section 2 addresses the problem environment and mathematical formulation of the MPFCPDP. Section 3 discusses about the proposed heuristics. Section 4 provides a numerical illustration. Section 5 discusses the computational results and performance analysis of the proposed methodologies. A summary of the present analysis and future research directions are presented in the concluding section 6.

2. Problem environment and description

There are '*m'* suppliers (industrial producers) to produce and distribute a product to '*n*' customers (demand centres like assembly centres, distribution centres etc.) in T planning periods; each supplier $i = 1,2,\ldots\ldots m$ has P_i^t units of production in each period $t = 1,2\ldots\ldots T$ and each customer $j = 1,2,\ldots\ldots n$ has D_j^t units of demand in each period $t = 1,2\ldots\ldots T$. Each supplier i can produce the product at a production cost of CU_i per unit and ship it to any customer j at a transportation cost of C_{ij} per unit for shipping from supplier i to customer j plus a fixed cost of FC_{ij} included for operating the route i to j. At any time period t, the total cumulative production of the suppliers may or may not be equal to the total demand of the customers**.** The excess or shortage of production in the period t is carried over to the subsequent period $t+1$. The excess of production in period t, addressed here as the inventory, is considered as an additional supply available for the period $t+1$. It is notified as SI_i^t at an inventory holding cost of SH_i per unit per period at i^{th} supplier's location and CI_j^t at an inventory holding cost of CH_j per unit per period at j^{th} customer's location. On the other hand, the production shortage of the period t (excess demand), addressed here as backorder, is considered as an additional demand for the period $t+1$. It is notified as BL_j^t at a penalty cost of BC_j per unit per period at j^{th} customer's location. As the proposed model considers short planning periods (days/weeks/months), the cost associated with production (CU_i), transportation (i.e. C_{ij} and FC_{ij}), inventory (i.e. SH_i and CH_j) and backorder (i.e. BC_j) are independent of period t. The beginning period's inventory and backorder (i.e., SI_i^0, CI_j^0 and BL_j^0) are known quantities. Minimization of the sum of costs of production, transportation, holding inventory and penalty for the backorder supply is considered as the objective criterion of the problem.

3. Decision variables

X_{ij} :Number of units of shipments from supplier i to customer j in period t

SI_i^t :Number of units of inventory with supplier i in period t

CI_j^t :Number of units of inventory with customer j in period t

BL_j^t :Number of units of backlog for customer j in period t

δ_{ij}^t :Binary variable that specifies the product distribution from supplier i to customer j in period t (i.e., $\delta_{ij}^t = 1$ if $X_{ij}^t > 0$ and $\delta_{ij}^t = 0$ if $X_{ij}^t = 0$)

4. Mathematical model

This model attempts to integrate production, transportation, backorder and inventory decisions monolithically from a centralized planning point of view. Let δ_{ij}^t be a binary variable to account fixed transportation cost. The mathematical model of the MPFCPDP problem is formulated as a 0-1 Mixed Integer Programming (MIP) problem as given below.

$$\sum_{t=1}^{T-1}\sum_{j=1}^{n} CH_j * CI_j^t + \sum_{t=1}^{T-1}\sum_{j=1}^{n} BC_j * BL_j^t \quad (1)$$

Subject to :

$$P_i^t + SI_i^{t-1} = \sum_{j=1}^{n} X_{ij}^t + SI_i^t$$

$$\forall_i,\ i=1\ldots m \ \ \&\ \ \forall_t,\ t=1\ldots T; \quad (2)$$

$$D_j^t + BL_j^{t-1} - CI_j^{t-1} = \sum_{i=1}^{m} X_{ij}^t + BL_j^t - CI_j^t$$

$$\forall_j,\ j=1\ldots n \ \ \&\ \ \forall_t,\ t=1\ldots T; \quad (3)$$

$$\delta_{ij}^t = 1 \text{ if } X_{ij}^t > 0;$$

$$\forall_i,\ i=1\ldots m,\ \forall_j,\ j=1\ldots n\ \&\ \forall_t,\ t=1\ldots T; \quad (4)$$

$$\delta_{ij}^t = 0 \text{ if } X_{ij}^t = 0;$$

$$\forall_i,\ i=1\ldots m,\ \forall_j,\ j=1\ldots n \ \ \&\ \ \forall_t,\ t=1\ldots T;$$

(5)

$X_{ij}^t \geq 0$ and integer;

$\forall_i$, i=1… m, $\forall_j$, j=1… n & $\forall_t$, t=1…T; (6)

$SI_i^t \geq 0$ and integer;

$\forall_i$, i=1… m, & $\forall_t$, t=1…T; (7)

$CI_j^t \geq 0$ and integer;

$\forall_j$, j=1… n & $\forall_t$, t=1…T; (8)

$BL_j^t \geq 0$ and integer;

$\forall_j$, j=1… n & $\forall_t$, t=1…T; (9)

The objective function given by Eq. (1) aims to minimize the sum of the total costs associated with production, transportation, inventory at supplier's side and customer's side and backorder. The first term of the objective function provides the total cost of production for the entire period *T* and the second term provides the total cost of transportation for the entire period *T*. The third term addresses the total cost of holding inventory at supplier's locations for the entire period *T*. The fourth and the fifth terms indicate respectively the total cost holding inventory for the entire period *T* and the total cost of backorder penalty for the entire period *T*. Constraint set given by Eq. (2) expresses the material balance at the supplier's side between any two successive time intervals. Similarly, constraint set (3) expresses the material balance at the customer's side between any two successive time intervals. In the left and right sides of the constraint set 3, either the inventory or backorder is present. Constraint sets given by Eqs. (4) and (5) return a binary value of δ_{ij}^t depending on the value of X_{ij}^t. Constraint sets given by Eqs. (6) to (9) ensure the non-negativity nature of decision variables X_{ij}^t, SI_i^t, CI_j^t, and BL_j^t.

5. Solution methodologies

In this paper, the MPFCPDP model is solved by GA and SAA based meta-heuristics and EVC based simple heuristics. They are delineated in the following sections.

5.1. GA and SAA based meta-heuristics

Over the last thirty years, there has been a growing interest in problem solving systems based on the principles of population based and neighborhood based search heuristics. In population based search heuristics, the GA has been increasingly applied to various search and optimization problems and has emerged as potential techniques to provide solutions with acceptable accuracy for NP hard problems [13-15]. In neighborhood based search heuristics, many researchers considered SAA for solving many hard optimisation problems [15-20]. The proposed GA and SAA based heuristics are structured to solve the MPFCPDP in two stages. They are as follows.

Stage I: Data input and transformation

This stage remains common in both GA and SAA based heuristics. It accepts the data of MPFCPDP under consideration as input and modulates them as a single-period fixed charge production-distribution problem (SPFCPDP) data set. The conversion provides a modulated data suitable for allocating the shipment quantities in single-period layout and subsequently deriving a feasible multi-period distribution schedule in the following GA and SAA routines, which is delineated in Stage II.

Stage II: Procedural steps of GA

Step 1: Parameters setting

The parameters of GA are:
pop_size = 10; p_cross = 0.5; p_mut = 0.1; gen_no = 1; $n\_gen = 100 + (m*T)*(n*T)$.

Step 2: Initial population generation

In this proposed GA, a chromosome c refers to a gene type representation of a distribution schedule to the SPFCPDP. The chromosome c is the permutation of cell numbers of SPFCPDP matrix, in which each cell is identified with a unique The total number of cells in the SPFCPDP, which is also equal to the length of the chromosome c, thus becomes *(m*T)*(n*T).* When the supply and demand are in same period (i.e. $t_c = t_r$), then they form T number of diagonal matrix of size $m*n$. The number of cells in the diagonal matrix thus becomes equal to $m*n*T$. Table 1 illustrates an example (*m*n*T: 3*3*2*) SPFCPDP matrix cell numbers.

A chromosome is structured as two parts. The first part is framed by the cell numbers of diagonal SPFCPDP matrix. The second part is framed by the remaining cell numbers of non-diagonal SPFCPDP matrix. Table 2 shows a randomly developed chromosome with cell number for the above example. In the same way, a randomly developed ten chromosomes form the initial population.

Table 1. SPFCPDP cell numbers matrix.

			t_c					
			1			2		
		j \ i	1	2	3	1	2	3
t_r	1	1	1	2	3	4	5	6
		2	7	8	9	10	11	12
		3	13	14	15	16	17	18
	2	1	19	20	21	22	23	24
		2	25	26	27	28	29	30
		3	31	32	33	34	35	36

Table 2. An example chromosome of SPFCPDP.

	Cell number																	
First part	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	34	23	7	3	22	9	30	13	1	28	8	15	35	2	14	24	36	29
	Cell number																	
Sec. part	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
	18	5	26	10	6	27	16	4	17	31	25	32	12	19	11	21	20	33

Step 3: Evaluation

The chromosome, on decoding, provides a feasible distribution schedule to the MPFCPDP by allocating shipment quantities to the cells of SPFCPDP based on their priority as per the cell number positions in the chromosome. Then the actual MPFCPDP distribution quantities X_{ij}^{t}, suppliers' inventory SI_i^t, customers' inventory CI_j^t, and backlog BL_j^t are derived by demodulating the allocations of SPFCPDP. The total production-distribution cost $Z(\tau m)$ corresponding to MPFCPDP distribution schedule (τm) *is* calculated from the objective Eq. (1). Each chromosome c in the initial population is evaluated in terms of Z by the same procedure.

Step 4: Updating

At the end of first generation, the followings parameters are updated.

$$pop\_best = global\_best$$
$$gen\_no = gen\_no + 1.$$

Step 5: Termination checking:

The number of generations is considered as the termination criterion. The termination criterion value of the illustration is calculated as follows. Termination criterion = n_gen =100+ *(m*T)*(n*T).* If the gen_no value is less than n_gen go to the next step, else go to step 7.

Step 6: New population generation

The generation of a new population involves three tasks: i) Selection, ii) Crossover and iii) Mutation. The selection process is repeated as many times as equal to *pop_size*. In crossover operation, each chromosome is selected with probability *p_cross*. In mutation, each gene is selected with probability p_mut . Then, that particular gene is mutated. After generating the new population the GA steps from 3 to 5 are repeated until it reaches termination.

Step 7: Output

The distribution schedule ($\tau_{(best)}$) and distribution cost ($Z_{(near\ opt)}$) in the *global_best* are the solutions to the problem and are given as output.

Stage II: Procedural steps of SAA

Step 1: Initialization of SAA parameters and counters

The parameters of SAA are:
TE = 475, ACCEPT = 0, TOTAL = 0, FREEZE = 0, α *=0.90 and*
Termination condition = (*FREEZE = 5 or TE = 20).*

Step 2: Generation of initial seed string

In this proposed SAA, a string *S* refers to a gene type representation of a distribution schedule to the SPFCPDP. The string *S* is the permutation of cell numbers of SPFCPDP matrix, in which each cell is identified with a unique The total number of cells in the SPFCPDP, which is also equal to the length of the string *S*, thus becomes *(m*T)*(n*T).* When the supply and demand are in same period (i.e. $t_c = t_r$), then they form *T* number of diagonal matrix of size *m*n*. The number of cells in the diagonal matrix thus becomes equal to *m*n*T*. Table 3 illustrates an example (*m*n*T: 3*3*2*) of SPFCPDP matrix cell numbers.

Table 3. SPFCPDP cell numbers matrix.

			t_c					
			1			2		
		j / i	1	2	3	1	2	3
t_r	1	1	1	2	3	4	5	6
		2	7	8	9	10	11	12
		3	13	14	15	16	17	18
	2	1	19	20	21	22	23	24
		2	25	26	27	28	29	30
		3	31	32	33	34	35	36

A string is structured as two parts. The first part is framed by the cell numbers of diagonal SPFCPDP matrix. The second part is framed by the remaining cell numbers of non-diagonal SPFCPDP matrix. Table 4 shows an example of a randomly developed seed string with cell number for the above example.

Table 4. An example seed string of SPFCPDP.

	Cell number																	
First part	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	34	23	7	3	22	9	30	13	1	28	8	15	35	2	14	24	36	29
	Cell number																	
Sec. part	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
	18	5	26	10	6	27	16	4	17	31	25	32	12	19	11	21	20	33

Step 3: Evaluation

The string, on decoding, provides a feasible distribution schedule to the MPFCPDP by first allocating shipment quantities to the cells of SPFCPDP based on their priority as per the positions in the string. Then the actual MPFCPDP distribution quantities X_{ij}^t, suppliers' inventory SI_i^t, customers' inventory CI_j^t, and backlog BL_j^t are derived by demodulating the allocations of SPFCPDP. The total production-distribution cost $Z(\tau m)$ corresponding to MPFCPDP distribution schedule (τm) *is* calculated from the objective Eq. (1).

Step 4: Generation of neighborhood seed string and its evaluation

A neighborhood seed string *S′* to the current seed string *S* is generated via mutation operator [21]. Mutation for this research is a unary random mutation. A random number *U* between 0 and 1 is generated corresponding to every element of the seed and if the random number *U* is less than mut *(0.5)*, then those two particular elements are interchanged (mutated). The mutation exchanges the sequence elements within the seed for maintaining the feasibility. This process is carried out separately in the first and second part of the string.

Step 5: Calculation of uphill acceptance parameter delta

The new seed string *S′* is selected by calculating the value of the delta. Delta is the cost difference between the neighborhood seed string distribution schedule and the initial seed string distribution schedule. i.e., Delta = $Z(\tau m') - Z(\tau m)$; If $Delta \leq 0$ proceed to step 6 (downhill move), else $(Delta > 0)$ go to step 7 (uphill move).

Step 6: Downhill move

Assign $\tau m = \tau m'$ $Z(\tau m) = Z(\tau m')$ and $ACCEPT = ACCEPT + 1$ If $Z(\tau m) < Z_{(near\ opt)}$ then set $\tau_{(best)} = \tau m$ and $Z_{(near\ opt)} = Z(\tau m)$, else go to step 9.

Step 7: Uphill move

Computation of $P = e^{(-Delta/TE)}$ and sample *R (Random no. generated (0, 1)).If* $R > P$ proceed to step 8 else proceed to step 9.

Step 8:

Assign $\tau m = \tau m'$, $Z(\tau m) = Z(\tau m')$ and $ACCEPT = ACCEPT + 1$.

Step 9:

Set $TOTAL = TOTAL + 1$.

Step 10: Check for termination

The algorithm is frozen. The termination of the SAA is achieved when *FREEZE* counter reaches the specified value (*FREEZE=5*) or the temperature *TE* falls to a pre-specified value (*TE*=20). Now $\tau_{(best)}$ contains the best MPFCPDP distribution schedule and $Z_{(near\ opt)}$ has the minimum $Z(\tau m)$. If ($TOTAL > (m*n*t)$) or ($ACCEPT > (m*n*t)/2$), then proceed to step 11 else go back to step 4 until it satisfies the condition in step 10.

Step 11: Output

The distribution schedule ($\tau_{(best)}$) and distribution cost ($Z_{(near\ opt)}$) in the *global_best* are the solutions to the problem and are given as output.

5.2. Equivalent variable cost heuristic

A linear distribution model can be obtained by relaxing the integral restrictions [22] of the nonlinear problem with equivalent variable transportation cost EVC_{ij}, which is defined as:

$$EVC_{ij} = C_{ij} + \left\{ FC_{ij} / Min(P_i^t, D_j^t) \right\}$$

$\forall_i$, i=1…m, $\forall_j$, j=1…n and $\forall_t$, t=1…T ; (10)

$$\text{Min } Z = \sum_{t=1}^{T}\sum_{i=1}^{m}(CU_i * \sum_{j=1}^{n} X_{ij}^t) + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}(EVC_{ij} * X_{ij}^t) + \sum_{t=1}^{T-1}\sum_{i=1}^{m} SH_i * SI_i^t + \sum_{t=1}^{T-1}\sum_{j=1}^{n} CH_j * CI_j^t + \sum_{t=1}^{T-1}\sum_{j=1}^{n} BC_j * BL_j^t \quad (11)$$

Subject to :

Contraint sets (2), (3), (6), (7), (8) and (9).

The above linear programming problem can be solved optimally using LINGO Solver. The substitution of the optimal solution in Eqs. (1) and (12) respectively provides Equivalent variable cost solution $Z_{(EVC)}$ and lower bound value $Z_{(L)}$.

5.3. Numerical illustration

An illustration for the above three methodologies is given below with an example problem. The data used for the illustration are as follows: $m = 3, n = 3, t = 3$. Tables 5a and 5c show the data related to transportation, supplier and customer respectively.

Table 5a. Transportation cost data (C_{ij} & FC_{ij}).

i \ j		1	2	3
1	FC_{ij}	900	90	100
	C_{ij}	20	35	25
2		150	1100	50
		40	5	80
3		800	70	1000
		30	70	15

Table 5b. Suppliers' data (P_i^t, CU_i, SH_i & SI_i^0).

		i			
		1	2	3	$\sum P_i^t$
P_i^t	t = 1	60	40	60	160
	2	50	30	60	140
	3	80	60	30	170
SH_i		15	12	28	-
SI_i^0		10	0	0	-
CU_i		10	12	14	-

Table 5c. Customers' data (D_j^t, CH_j, BC_j, BL_j^0 & CI_j^0).

		j			
		1	2	3	$\sum D_j^t$
D_j^t	t = 1	60	60	70	190
	2	60	20	30	110
	3	90	60	40	190
CH_j		5	10	15	-
BC_j		20	40	30	-
BL_j^0		0	20	0	-
CI_j^0		0	0	30	-

5.4. GA and SAA based meta-heuristics solutions

The distribution schedule (Tables 6 and 7) and total production-distribution cost (Z) of the above example problem are given as follows (solved using GA and SAA separately).

$Z_{(near\ opt)}$ = 27,170.00
(GAs solution)

$Z_{(near\ opt)}$ = 27,790.00
(SAAs solution)

5.5. Equivalent variable cost solution

The equivalent variable cost matrix and the optimal distribution schedule (solved using LINGO solver) of the relaxed problem are given in Tables 8 and 9 respectively. The substitution of the optimal solution in Eqs. (1) and (11) respectively provides equivalent variable cost solution $Z_{(EVC)}$ and lower bound value $Z_{(L)}$.

$Z_{(L)}$ = 27,170.00 (Lower bound value)
$Z_{(EVC)}$ = 28,510.00 (Equivalent variable cost solution)

6. Computational results and performance analysis

To evaluate the performance of the proposed heuristics, computational experiments were done on 40 test problems. Forty test problems along with their outputs are considered for this performance comparison. The comparisons reveal the followings: Equivalent variable cost method provides only approximate solutions to all the test problems but very few of them are close or equal to GA and SAA solutions. EVC heuristic can also provide the lower bound value of the problem; GA and SAA based heuristics generate better solutions than the EVC heuristic and are capable of providing solutions close or equal to lower bound values. The average percentage deviation of GA based heuristic with lower bound value is 2.12%. The average percentage deviation of SAA based heuristic with lower bound value is 2.21%. The average percentage deviation of EVC heuristic with lower bound value is 8.89%. They are depicted in Fig. 1.

Table 6. Distribution schedule using GA.

	t											
	1				2				3			
j / i	1	2	3	SI_i^1 ↓	1	2	3	SI_i^2 ↓	1	2	3	SI_i^3 ↓
1		30	40	0			40	10	90			0
2		40		0		30		0		60		0
3	60			0	60			0			30	0
BL_j^t →	0	10	0		0	0	0		0	0	0	
CI_i^t →	0	0	0		0	0	10		0	0	0	

Table 7. Distribution schedule using SAA.

	t											
	1				2				3			
j / i	1	2	3	SI_i^1 ↓	1	2	3	SI_i^2 ↓	1	2	3	SI_i^3 ↓
1		40	30	0			40	10	80		10	0
2		40		0		20		10	10	60		0
3	60			0	60			0			30	0
BL_j^t →	0	0	10		0	0	0		0	0	0	
CI_i^t →	0	0	0		0	0	0		0	0	0	

Table 8. Equivalent variable cost (EVC_{ij}^t) matrix.

	t								
	1			2			3		
j / i	1	2	3	1	2	3	1	2	3
1	35	36.28	27.5	38	39.5	28.34	30	36.5	27.5
2	43.75	32.5	81.25	45	60	81.66	42.5	23.34	81.25
3	43.34	71.17	40	43.33	73.5	48.33	56.66	72.33	48.33

Table 9. Optimal distribution schedule for the relaxed problem.

	t											
	1				2				3			
j / i	1	2	3	SI_i^1 ↓	1	2	3	SI_i^2 ↓	1	2	3	SI_i^3 ↓
1		40	30	0		20	30	0	70		10	0
2		40		0	10			20	20	60		0
3	50		10	0	60			0			30	0
BL_j^t →	10	0	0		0	0	0		0	0	0	
CI_i^t →	0	0	0		0	0	0		0	0	0	

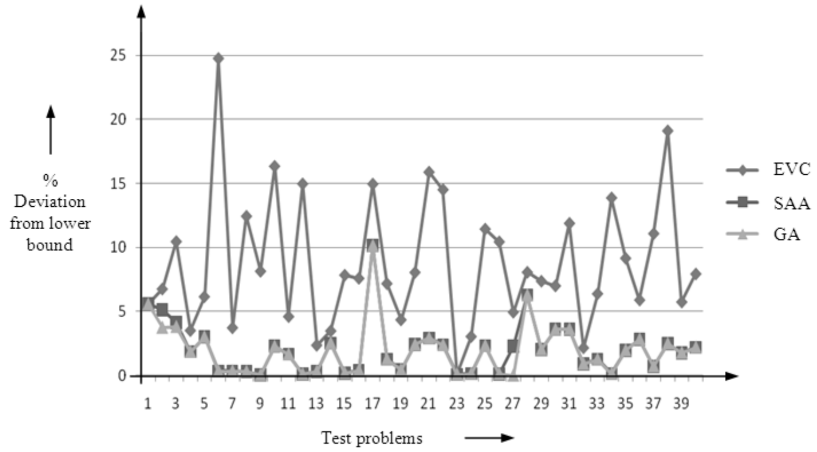

Fig. 1. Percentage deviation of proposed heuristics' solutions.

7. Conclusions

This paper proposes GA and SAA based meta-heuristics and EVC based simple heuristics to solve the MPFCPDP problem. The proposed methodologies are evaluated by comparing their solutions with lower bound values. The comparison of results reveal that the GA and SAA generate better solutions than the EVC solutions and are capable of providing solutions close or equal to the lower bound values. This paper concentrates on single-stage multi-period fixed charge model. As a future research, the single period formulation to the proposed multi-period fixed charge problem facilitates its scope for extending this to multi-stage supply chain problems.

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