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Free vibrations analysis of a sandwich rectangular plate with electrorheological fluid core

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Abstract

In this paper, a rectangular sandwich plate with a constrained layer and an electrorheological (ER) fluid core is investigated. The rectangular plate is covered an ER fluid core and a constraining layer to improve the stability of the system. The two outer layers of the sandwich structure are elastic. The viscoelastic materials express the middle layer behavior under electric field and small strain. Rheological property of an ER material, such as viscosity, plasticity, and elasticity, may be changed when applying an electric field. The ER core is found to have a significant effect on the stability of the sandwich plate. In this paper, based on the displacement field of each layer, the kinetic energy and strain energy are separately obtained for each layer. Transverse displacement of the second layer changes linearly between the transverse displacement of the first and third layers. The loss energy of the second layer consisting of the ER fluid is also calculated and, with the replacement of total kinetic energy, total strain energy, and energy dissipation in the Lagrange's equation, the structural motion equation is obtained. Natural frequencies and loss factor for the electric fields as well as the ratio of different thicknesses calculated are by Navier analytical method. As the applied electric field increases, the natural frequency of the sandwich plate increases and the modal loss factor decreases. With increasing the thickness of the ER layer, the natural frequencies of the sandwich plate are decreased. Thickness of the constrained layer also affects the stability of the sandwich plate.

Nomenclature

a	Length of the plate in x-direction
b	Length of the plate in y-direction
[c]	Sandwich structural damping matrix
d	Damping energy
E_i	Tensile modulus of the first and third layers
E_*	Applied electric field
f_n	Natural frequency
G'	Shear modulus of electrorheological layer
G''	Viscous stationary electrorheological layer
h_i	Thickness of ith layer i=1,2,3
[k]	Stiffness matrix
m	Dual expansion subtitle
[m]	Mass matrix of sandwich structure
n	Dual expansion subtitle
T	Kinetic energy of sandwich structure

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U	Strain energy of sandwich structures
u_i	Axial displacements of *i*th layer at x-direction i=1,3
v_i	Axial displacements of *i*th layer at y-direction i=1,3
w_i	Axial displacements of ith layer at z-direction i=1,3
$\overline{u}_i$	Axial displacements of each point of the layer at x-direction i=1,2,3
$\overline{v}_i$	Axial displacements of each point of the layer at y-direction i=1,2,3
$\overline{w}_i$	Axial displacements of each point of the layer at z-direction i=1,2,3
X(t)	Function of time vibrational modes
χ	Function component of time vibrational modes
ε_i	Bending strains of the*i*th layer at $i=1,2,3$
$\dot{\varepsilon}_2$	Shear strain first derivative of the middle layer versus time
ρ_i	Mass density of the*i*th layer at i=1,2,3
υ_i	Poisson's ratio of layer *i*, i=1,3
ω_*	Complex natural frequency
ξ	Loss factor

1. Introduction

Vibration behavior of various engineering systems and analysis of structures are important to the extent that, in most cases, less attention to the vibration behavior and unwanted dynamic forces will impair the efficiency of the system and make them fail, be fatigued, and have noise. In many of these applications, reduction of dynamic forces and elimination of undesirable vibrations are very important demands. Many scientists have attempted to minimize undesirable vibrant forces in structures, two examples of which are passive damping and active damping.

In a passive method, soft and viscoelastic materials are used in three-layer beams or plate structures. Sandwich structures with a high damping viscoelastic central layer like polymers, rubber, urethane, and epoxy have high damping capacity and resistance to excessive vibration and noise. Application of viscoelastic materials in the industry can be found in Rao's [1] paper. These structures typically consist of a base layer, a constrained layer, and a damping layer. They are popularly known as the sandwich plates because of their appearance.

In 1959, for the first time, three prominent researchers, Kerwin, Ross, and Engar, provided the overall analysis of sandwich structures with a viscoelastic intermediate layer. In their method in which later found fame as the RKU method, the effect of the viscoelastic material damping and dynamic behavior of the sandwich plates were studied. The experimental results were compared with their own theory and found to be in good agreement [2, 3]. Since then, many other investigators have worked on and developed the RKU theory. Ditaranto [4] developed the six-order equation of motion for the axial displacement of the closed form solution. Mead and Markus [5, 6] obtained six-order equation of motion for the lateral displacement that was previously used only for specific boundary conditions. Reissner [7] derived the governing equations for both small and finite transverse deflections of isotropic sandwich plates. He assumed that the face layers behaved like membranes and the face parallel stresses in the core were negligible. Since then, many papers have been published on various aspects of the sandwich theory. Liaw and Little [8] completed Reissner's work. Later, Azar [9] extended Liaw and Little's results to the same problems, but using orthotropic layers. Then, Yu [10, 11], Similar to Yan and Dowell [12] Khatua and Chenug [13], developed the model of the forced vibrations of the sandwich plates. Since then, energy approach was used for several issues.

In 1978, Rao [14] presented using the energy equations of motion and boundary conditions in his paper for the first time. He used the numerical method to solve his problem. Like Rao, Cottle [15] derived the equations set on Hamilton's principle. In 1988, Kanematsu [16] studied the bending and vibration of rectangular sandwich plates by the Ritz method. Numerical method such as finite element method was adopted to solve the vibration problems of the sandwich plate with a viscoelastic layer in a complex form. Determining and optimizing the location and thickness of the damping layer and layer sensitivity have been performed by Johnson [17] in 1981, O'Conner in 1987 [18], and Nakra [19] in 2001.

As mentioned, another way for removing the dynamic forces and damping vibrations is the active procedure. In this way, smart materials like piezoceramics, electrorheological fluid, magnetorheological fluid, and shape memory alloys are used as damping materials. The idea of this method is to control the structural damping characteristics by the electric field, magnetic field, or field of heat. ER fluids as a smart material have a great potential in applications for intelligent materials and structural damping. The ER fluid also has the same properties as a viscoelastic material at a small strain level. Engineering applications and advantages of the ER fluid can be found in the works by Coulter, Weiss, and Don [20-22]. Early investigations of the ER material in the structural vibration can be traced to Yalcintas and Coulter [23, 24]. In 1995, they studied the vibration problem of a sandwich beam with an ER fluid core and discussed the effects of the thickness and loss factor on the vibratory behavior of the structure. Then, Lee [25] investigated the transverse free vibration problem of a sandwich beam, in which an iterative method was explored to study the properties of the nonlinear ER fluid. By 2003, almost no research was done on the vibration of a sandwich plate with a layer of an ER fluid until two Taiwanese, Yeh and Chen Wang, studied the dynamic stability problems of the ER sandwich beam and also discussed the dynamic behavior of the sandwich plate (annular plate, orthotropic sandwich plates, orthotropic annular plate, and rotating polar orthotropic annular plate) on different thickness of the ER layer and electric filed strength [26-31]. They used two-dimensional element bounded by four nodal points and, in their investigation, each node had seven degrees of freedom by finite elements method to describe the natural frequencies and loss factors discussed in terms of the electric field and thickness ratio.

Hasheminejad and Maleki [32] used analytical solutions to obtain the forced vibration characteristics of the adaptive structure under different external transverse excitations of varying frequency (0–300 Hz) and applied electric field strength (0–3.5 kV/mm). They used classical thin plate theory to apply a set of fully coupled dynamic equations of motion. Mohammadi and Sedaghati [33] studied nonlinear vibration analysis of sandwich shell structures with a constrained electrorheological (ER) fluid for different boundary conditions. They investigated the effects of small and large displacements, core thickness ratio, and electric field intensity on the nonlinear vibration damping behavior of the sandwich shell structure.

In this paper, a rectangular sandwich plate with a constrained layer, an electrorheological (ER) fluid core, and four simply-supported end conditions is investigated using Navier analytical method. Resolution methods and assumptions used for the vertical displacement of the layers could distinguish this study from other similar works.

2. Problem formulation

Consider the geometry of the three-layered sandwich plate with an ER fluid core with length a and width b, as shown in Fig. 1. The thickness of elastic constrained layer is h_1, thickness of middle layer is h_2, and thickness of elastic base plate is h_3. The assumptions used in this derivation must be mentioned:

1. No slipping is assumed between the elastic and ER layers.
2. The transverse displacements are the same for every point on a cross-section. w_1 and w_3 are the transverse displacements of the first and third layers, respectively.
3. w_2 is the transverse displacement of the second layer. w_2 changes linearly between w_1 and w_3.
4. There exists no normal stress in the ER layer.
5. Keep shear stress assuming in the elastic layers.

Assumptions 3 and 5 are investigated for the first time in this paper. Therefore, we can use these equations for thick plates.

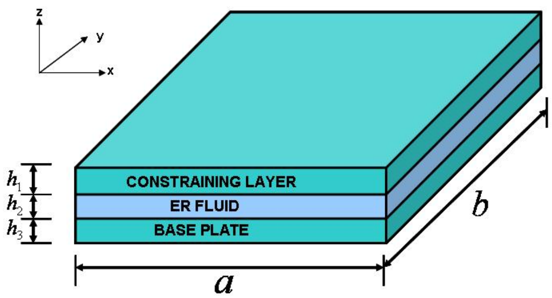


Fig. 1. Sandwich plate with an electrorheological fluid core and a constraining layer.

As shown in Fig. 2, the displacement relation of the elastic layers can be expressed as:

$$\begin{aligned}
\bar{u}_i(x,y,z,t) &= u_i(x,y,t) - z_i \frac{\partial w_i(x,y,t)}{\partial x} \\
\bar{v}_i(x,y,z,t) &= v_i(x,y,t) - z_i \frac{\partial w_i(x,y,t)}{\partial y} \qquad i=1,3 \\
\bar{w}_i(x,y,z,t) &= w_i(x,y,t)
\end{aligned} \tag{1}$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the axial displacement components in *x, y, and z* directions, respectively, u, v and w are the axial displacement of the mid-plane layers in x, y, and z directions, and z_i is the distance from the mid-height of layer *i*.

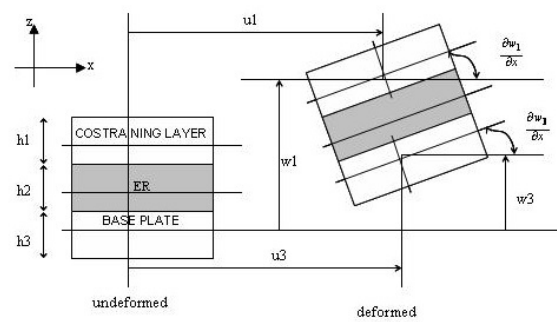


Fig. 2. Unreformed and deformed configurations of a sandwich plate.

The mid-plane layer displacement is expressed based on the geometric relationships and displacement components of the first and third elastic layers (w_1 ، w_3 ، u_1 ، u_3 ، v_1 and v_3).

$$\begin{aligned}
\bar{u}_2 &= \left(\frac{u_1+u_3}{2} + \frac{h_1\frac{\partial w_1}{\partial x} - h_3\frac{\partial w_3}{\partial x}}{4}\right) + \left(\frac{u_1-u_3}{h_2} + \frac{h_1\frac{\partial w_1}{\partial x} + h_3\frac{\partial w_3}{\partial x}}{2h_2}\right) z_2 \\
\bar{v}_2 &= \left(\frac{v_1+v_3}{2} + \frac{h_1\frac{\partial w_1}{\partial y} - h_3\frac{\partial w_3}{\partial y}}{4}\right) + \left(\frac{v_1-v_3}{h_2} + \frac{h_1\frac{\partial w_1}{\partial y} + h_3\frac{\partial w_3}{\partial y}}{2h_2}\right) z_2 \\
\bar{w}_2 &= \left(\frac{w_1+w_3}{2}\right) + \left(\frac{w_1-w_3}{h_2}\right) z_2
\end{aligned} \tag{2}$$

where z_2 is the distance from the mid-height of the second layer and w_2 is expressed according to the third assumption. In addition, the total kinetic energy of the sandwich plate, total strain energy, and damping energy of the second layer are obtained. Kinetic energy of the *i* the layer is obtained from the following equation:

$$T_i = \frac{1}{2}\int_0^a\int_0^b\int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \rho_i \left[\left(\frac{\partial \bar{u}_i}{\partial t}\right)^2 + \left(\frac{\partial \bar{v}_i}{\partial t}\right)^2 + \left(\frac{\partial \bar{w}_i}{\partial t}\right)^2 \right] dzdydx, \; i=1,2,3 \tag{3}$$

In the above equation, ρ_i is the density of each layer. By substituting velocity field of the first and third layers in the above equation and integrating over the thickness, the kinetic energy equation of layers is obtained as follows:

$$\begin{aligned}
T_i = \frac{1}{2}\int_0^a\int_0^b \Big\{ & \rho_i h_i (\dot{u}_i^2 + \dot{v}_i^2 + \dot{w}_i^2) + \\
& \rho_i \frac{h_i^3}{12}\left(\left(\frac{\partial \dot{w}_i}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_i}{\partial y}\right)^2 \right) \Big\} dydx, \; i=1,3
\end{aligned} \tag{4}$$

By substituting the second layer displacement field in Eq. (3) and integrating over the thickness, the kinetic energy of this layer is obtained as follows:

$$\begin{aligned}
T_2 = \int_0^a\int_0^b \rho_2 h_2 \Big[& \frac{1}{3}\dot{u}_1^2 + \frac{1}{3}\dot{u}_3^2 + \frac{h_1^2}{12}\left(\frac{\partial \dot{w}_1}{\partial x}\right)^2 + \frac{h_3^2}{12}\left(\frac{\partial \dot{w}_3}{\partial x}\right)^2 + \frac{1}{3}\dot{u}_1\dot{u}_3 + \\
& \frac{h_1}{3}\dot{u}_1 \frac{\partial \dot{w}_1}{\partial x} - \frac{h_3}{6}\dot{u}_1 \frac{\partial \dot{w}_3}{\partial x} + \frac{h_1}{6}\dot{u}_3 \frac{\partial \dot{w}_1}{\partial x} - \frac{h_3}{3}\dot{u}_3 \frac{\partial \dot{w}_3}{\partial x} - \\
& \frac{h_1 h_3}{12}\frac{\partial \dot{w}_1}{\partial x}\frac{\partial \dot{w}_3}{\partial x} + \frac{1}{3}\dot{v}_1^2 + \frac{1}{3}\dot{v}_3^2 + \frac{h_1^2}{12}\left(\frac{\partial \dot{w}_1}{\partial y}\right)^2 + \frac{h_3^2}{12}\left(\frac{\partial \dot{w}_3}{\partial y}\right)^2 + \\
& \frac{1}{3}\dot{v}_1\dot{v}_3 + \frac{h_1}{3}\dot{v}_1 \frac{\partial \dot{w}_1}{\partial y} - \frac{h_3}{6}\dot{v}_1 \frac{\partial \dot{w}_3}{\partial y} + \frac{h_1}{6}\dot{v}_3 \frac{\partial \dot{w}_1}{\partial y} - \frac{h_3}{3}\dot{v}_3 \frac{\partial \dot{w}_3}{\partial y} - \\
& \frac{h_1 h_3}{12}\frac{\partial \dot{w}_1}{\partial y}\frac{\partial \dot{w}_3}{\partial y} + \frac{1}{3}\dot{w}_1^2 + \frac{1}{3}\dot{w}_3^2 + \frac{1}{3}\dot{w}_1\dot{w}_3 \Big] dxdy
\end{aligned} \tag{5}$$

Total kinetic energy is obtained from the sum of kinetic energy of the first, second, and third layers.

$$T = T_1 + T_2 + T_3 \tag{6}$$

The elastic strain energy of the first and third layers, with vertical and shear stresses is obtained as follows:

$$U_i = \frac{E_i}{2(1+\upsilon_i)} \int_0^a \int_0^b \int_{\frac{h_i}{2}}^{\frac{h_i}{2}} \left[\frac{\upsilon_i(\varepsilon_{ixx}+\varepsilon_{iyy})^2}{1-2\upsilon_i} + \varepsilon_{ixx}^2 + \varepsilon_{iyy}^2 + \frac{\varepsilon_{ixy}^2}{2} \right] dzdydx i=1,3 \tag{7}$$

where υ_i , E_i denote Poisson's ratio and Young's modulus of these layers, respectively.The vertical and shear strains are:

$$\begin{aligned}
\varepsilon_{ixx} &= \frac{\partial \overline{u}_i}{\partial x} = \frac{\partial u_i}{\partial x} - z\frac{\partial^2 w_i}{\partial x^2} \\
\varepsilon_{iyy} &= \frac{\partial \overline{v}_i}{\partial y} = \frac{\partial v_i}{\partial y} - z\frac{\partial^2 w_i}{\partial y^2} \qquad i = 1,3 \\
\varepsilon_{ixy} &= \frac{\partial \overline{u}_i}{\partial y} + \frac{\partial \overline{v}_i}{\partial x} = \frac{\partial u_i}{\partial y} + \frac{\partial \upsilon_i}{\partial x} - 2z\frac{\partial^2 w_i}{\partial x \partial y}
\end{aligned} \tag{8}$$

By substituting Eq. (8) in Eq. (7) and integrating over the thickness, the strain energy of the first and third layers is obtained as follows:

$$\begin{aligned}
U_i = \frac{E_i}{2(1+\upsilon_i)} \int_0^a \int_0^b \Bigg\{ & \frac{1-\upsilon_i}{1-2\upsilon_i} \frac{h_i^3}{12} \left((\frac{\partial^2 w_i}{\partial x^2})^2 + (\frac{\partial^2 w_i}{\partial y^2})^2 + 2(\frac{\partial^4 w_i}{\partial x^2 \partial y^2}) \right) \\
& + \frac{1-\upsilon_i}{1-2\upsilon_i} h_i (\frac{\partial u_i}{\partial x})^2 + \frac{h_i}{2} (\frac{\partial u_i}{\partial y})^2 + \frac{1-\upsilon_i}{1-2\upsilon_i} h_i (\frac{\partial v_i}{\partial y})^2 + \frac{h_i}{2} (\frac{\partial v_i}{\partial x}) \\
& + \frac{\upsilon_i}{1-2\upsilon_i} 2h_i (\frac{\partial u_i}{\partial x} \frac{\partial v_i}{\partial y}) + h_i (\frac{\partial u_i}{\partial y} \frac{\partial v_i}{\partial x}) \Bigg\} dxdy, \quad i=1,3
\end{aligned} \tag{9}$$

Kelvin's model is used to describe the shear stress of the viscoelastic material (ER fluid) [34].

$$\tau = G'\varepsilon + G''\dot{\varepsilon} \tag{10}$$

where τ is the shear stress of ER fluid, ε is shear strain, $\dot{\varepsilon}$ is first differentiation of shear strain from time, G' is shear modulus, and G'' is viscous coefficient, where dependence of G'', G' on electric field can be obtained by experiments [22, 23]. Based on the fourth assumption in ER fluid, no normal stress needs to be considered; only the shear stresses on the z-direction plate due to the formation of dielectric particle chains under the electric field are important. Then, the strain energy of the second layer is in the following manner:

$$U_2 = \frac{1}{2} \int_0^a \int_0^b \int_{-\frac{h_2}{2}}^{\frac{h_2}{2}} G' (\varepsilon_{2xz} + \varepsilon_{2yz}) dzdydx \tag{11}$$

Shear strains ε_{2xz} , ε_{2yz} are expressed as follows:

$$\begin{aligned}
\varepsilon_{2xz} &= \frac{1}{h_2} \left[u_1 - u_3 + \frac{\partial w_1}{\partial x} (\frac{h_1+h_2}{2}) + \frac{\partial w_3}{\partial x} (\frac{h_3+h_2}{2}) + z(\frac{\partial w_1}{\partial x} - \frac{\partial w_3}{\partial x}) \right] \\
\varepsilon_{2yz} &= \frac{1}{h_2} \left[v_1 - v_3 + \frac{\partial w_1}{\partial y} (\frac{h_1+h_2}{2}) + \frac{\partial w_3}{\partial y} (\frac{h_3+h_2}{2}) + z(\frac{\partial w_1}{\partial y} - \frac{\partial w_3}{\partial y}) \right]
\end{aligned} \tag{12}$$

By substituting Eq. (12) in Eq. (11) and integrating over the thickness, the ER fluid's strain energy in terms of the displacement parameters of the first and third layer and the geometric parameters of the structure can be expressed as:

$$\begin{aligned}
U_2 = \frac{1}{2} \int_0^a \int_0^b \{ G \frac{1}{h_2} \Bigg[& u_1^2 + u_3^2 + \left(\frac{\partial w_1}{\partial x}\right)^2 \left(\frac{h_1+h_2}{2}\right)^2 + \left(\frac{\partial w_3}{\partial x}\right)^2 \left(\frac{h_3+h_2}{2}\right)^2 - \\
& 2u_1u_3 + 2u_1 \frac{\partial w_1}{\partial x}\left(\frac{h_1+h_2}{2}\right) + 2u_1 \frac{\partial w_3}{\partial x}\left(\frac{h_3+h_2}{2}\right) - 2u_3 \frac{\partial w_1}{\partial x}\left(\frac{h_1+h_2}{2}\right) - \\
& 2u_3 \frac{\partial w_3}{\partial x}\left(\frac{h_3+h_2}{2}\right) + 2\frac{\partial w_1}{\partial x}\frac{\partial w_3}{\partial x}\left(\frac{h_1+h_2}{2}\right)\left(\frac{h_3+h_2}{2}\right) + v_1^2 + v_3^2 + \\
& \left(\frac{\partial w_1}{\partial y}\right)^2 \left(\frac{h_1+h_2}{2}\right)^2 + \left(\frac{\partial w_3}{\partial y}\right)^2 \left(\frac{h_3+h_2}{2}\right)^2 - 2v_1v_3 + 2v_1 \frac{\partial w_1}{\partial y}\left(\frac{h_1+h_2}{2}\right) + \\
& 2v_1 \frac{\partial w_3}{\partial y}\left(\frac{h_3+h_2}{2}\right) - 2v_3 \frac{\partial w_1}{\partial y}\left(\frac{h_1+h_2}{2}\right) - 2v_3 \frac{\partial w_3}{\partial y}\left(\frac{h_3+h_2}{2}\right) + \\
& 2\frac{\partial w_1}{\partial y}\frac{\partial w_3}{\partial y}\left(\frac{h_1+h_2}{2}\right)\left(\frac{h_3+h_2}{2}\right) \Bigg] + \frac{Gh_2}{12} \Bigg[\left(\frac{\partial w_1}{\partial x}\right)^2 + \left(\frac{\partial w_1}{\partial y}\right)^2 + \left(\frac{\partial w_3}{\partial x}\right)^2 + \\
& \left(\frac{\partial w_3}{\partial y}\right)^2 - 2\left(\frac{\partial w_1}{\partial x}\frac{\partial w_3}{\partial x}\right) - 2\left(\frac{\partial w_1}{\partial y}\frac{\partial w_3}{\partial y}\right) \Bigg] \} dxdy
\end{aligned} \tag{13}$$

The total strain energy of the first, second and third layers is obtained as follows:

$$U = U_1 + U_2 + U_3 \tag{14}$$

Damping energy of ER fluids is obtained from the following equation:

$$D_2 = \frac{1}{2} \int_0^a \int_0^b \int_{-\frac{h_2}{2}}^{\frac{h_2}{2}} G'' (\dot{\varepsilon}_{2xz} + \dot{\varepsilon}_{2yz}) dzdydx \tag{15}$$

The damping energy of the second layer is obtained from the derivation of Eq. (12), substitution in Eq. (15), and integration over thickness. By choosing the vibrational modes and according to the boundary conditions of the problem and using the Navier analysis, the longitudinal displacements u_1 , u_3 , v_1 , v_3 and transverse displacements w_1 , w_3 of the sandwich structures are expressed as follows:

$$\begin{aligned}
w_1(x,y,t) &= \sum_m \sum_n {}^1W_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \\
w_3(x,y,t) &= \sum_m \sum_n {}^3W_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \\
u_1(x,y,t) &= \sum_m \sum_n {}^1U_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \\
u_3(x,y,t) &= \sum_m \sum_n {}^3U_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \\
v_1(x,y,t) &= \sum_m \sum_n {}^1V_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \\
v_3(x,y,t) &= \sum_m \sum_n {}^3V_{mn}(t) \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b})
\end{aligned} \tag{16}$$

The ${}^1U_{mn}(t), {}^3U_{mn}(t), {}^1V_{mn}(t), {}^3V_{mn}(t), {}^1W_{mn}(t), {}^3W_{mn}(t)$ phrases are the time function generalized coordinates of the displacement components and $\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})$ is assumed the place components of displacement.

Equation (16) has to be substituted in the equation of kinetic, strain layers energy, and loss energy of the second layer. By integrating over the surface, the final equation is obtained. For obtaining the displacement equations, the Lagrange principle is used.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}_i}\right) - \frac{\partial T}{\partial X_i} + \frac{\partial D_2}{\partial \dot{X}_i} + \frac{\partial U}{\partial X_i} = 0 \quad (17)$$

where X_i are the generalized coordinates ${}^1U_{mn}, {}^3U_{mn}, {}^1V_{mn}, {}^3V_{mn}, {}^1W_{mn}$ and ${}^3W_{mn}$.

By substituting the total kinetic energy, total strain energy, and damping energy in the Lagrange's equation of the ER fluid core, sandwich plate motion and constrained layer are obtained as follows:

$$[M]\{\ddot{X}_{mn}(t)\} + [C]\{\dot{X}_{mn}(t)\} + [K]\{X_{mn}(t)\} = \{0\} \quad (18)$$

where $[M]$, $[C]$, and $[K]$ are mass matrix, damping matrix, and stiffness matrix of structure, respectively. Considering the substitution of $X(t) = \chi e^{i\omega_* t}$ in Eq. (18):

$$\left(-\omega_*^2[M] + \omega_*[C] + [K]\right)\{\chi\} = \{0\} \quad (19)$$

where ω_* is the complex natural frequency of the system. By solving the relative equations and calculating the natural complex frequency of the system, natural frequencies (f_n) and loss factor (ξ) are obtained as follows:

$$\omega_* = -\xi f_n \pm j\sqrt{1-\xi^2}\, f_n \quad (20)$$

3. Results and discussion

Vibrations of a three-layered sandwich plate with an ER fluid core, constrained layer, and four simply-supported end conditions using analytical Navier's method were studied in this paper. ER fluid viscosity and elasticity properties were changed by the applied electric field, and natural frequencies and loss factor for electric fields and various thickness ratios were calculated. To validate the proposed algorithm and calculations, comparisons between the present results and those of the existing models were made first. The numerical results that were compared with them are displayed in Table 1. A good agreement was observed between the above results and the numerical results in [27]. The geometrical and physical parameters of the sandwich plate were as follows:

$$\begin{aligned} &a = 0.3\ m, \quad b = 0.25\ m, \\ &h_1 = 0.05\ mm, \ h_2 = h_3 = 0.5\ mm \\ &E_1 = E_3 = 68.9*10^9\ N/m^2, \ \upsilon_1 = \upsilon_3 = 0.29 \\ &\rho_1 = \rho_3 = 2737\ kg/m^3, \ \rho_2 = 1700\ kg/m^3 \end{aligned} \quad (21)$$

Upper and lower layers were elastic and aluminum.

Table 1. Comparisons of natural frequency and loss factor.

Mode	Ref. [27]		Present	
	f_n	ξ	f_n	ξ
1	58.7	0.201	62.1	0.186
2	113.8	0.211	116.9	0.198
3	129.2	0.208	131.6	0.190
4	175.5	0.189	178.5	0.181

Based on the existing information on the ER material's pre-yield rheology, only the electric field dependence of ER material in the pre-yield regime needed to be considered. The complex modulus of the used ER fluid was experimentally measured by Don [22] and can be expressed as follows; they are called fluid type (A) and fluid type (B) and their relations were presented by Yalcintas and Coulter [23].

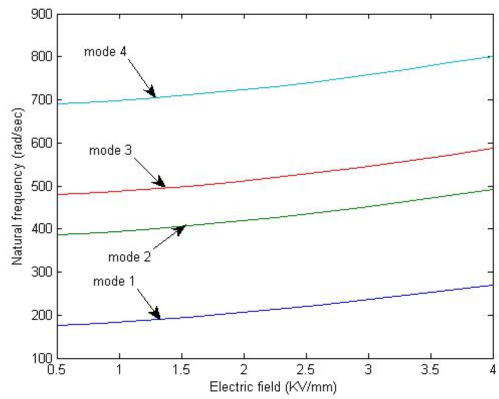
$$\begin{aligned} &G_d = G'_d + G''_d \\ &G'_d = 15000 E_*^2, G''_d = 6900 \end{aligned} \quad (22\text{-A})$$

$$\begin{aligned} &G_y = G'_y + G''_y \\ &G'_y = 50000 E_*^2, G''_y = 2600 E_* + 1700 \end{aligned} \quad (22\text{-B})$$

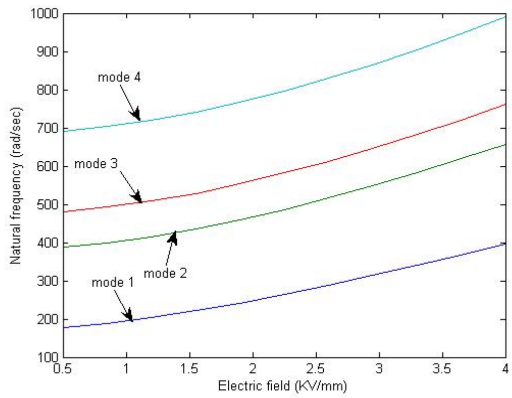
where E_* is the electric field in terms of kV/mm.

The natural frequencies of a sandwich plate with different electric fields are shown in Fig. 3. Effect of the electric field on the vibration response of the ER sandwich plate can be seen for the electric field levels from 0.5 to

$4\,kV/mm$, respectively. As is clear from the figures, the natural frequency of the sandwich plate also increased with increasing the electric field. It can be seen that the higher electric field strength increased the natural frequencies of the sandwich plate.



Fluid type (A)



Fluid type (B)

Fig. 3. Dependence of the first four natural frequency modes on the electric field, fluid type (A) & (B), $h_1 = 0.05mm, h_2 = h_3 = 0.5mm$.

The modal loss factor of the sandwich plate had a big effect on the strength of the damping structures. Fig. 4 shows the variations in the modal loss factor as a function of electric field. We can see that the modal loss factor decreased as the electric field increased. Also, a relative decrease in the modal loss factor can be observed by increasing the mode number.

Definitely, thickness of the middle layer had an important effect on the sandwich structure damping. The effects of thickness ratio h_2/h_3 on type (A) of ER sandwich plate's natural frequency and two applied electric field levels of 1.5 and $3.5\,kV/mm$ are presented in Fig. 5. This figure shows the vibration mode of the first four natural frequencies with simply-supported end conditions and $h_1 = 0.05\,mm$ and $h_3 = 0.5\,mm$; the mid-layer thickness is changed from one-fifth to the twice of the main layer thickness and the natural frequencies decrease with the increasing thickness ratio of the ER layer. Like the previous figure, due to thickness ratio h_2/h_3 on type (B) of sandwich plate with an ER fluid core, natural frequency and two applied electric field levels of 1.5 and $3.5\,kV/mm$ are presented in Fig. 6. It can be seen that, by increasing the thickness ratio of the fluid layer compared to the main layer, the natural frequency decreased.

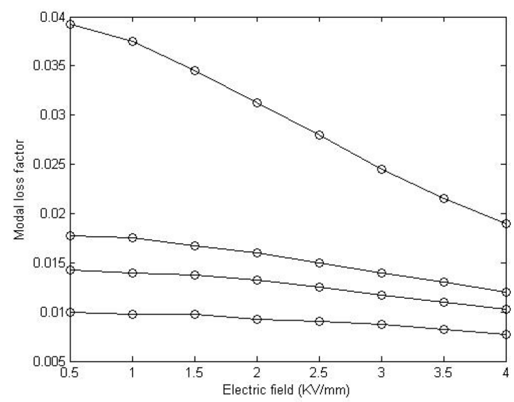
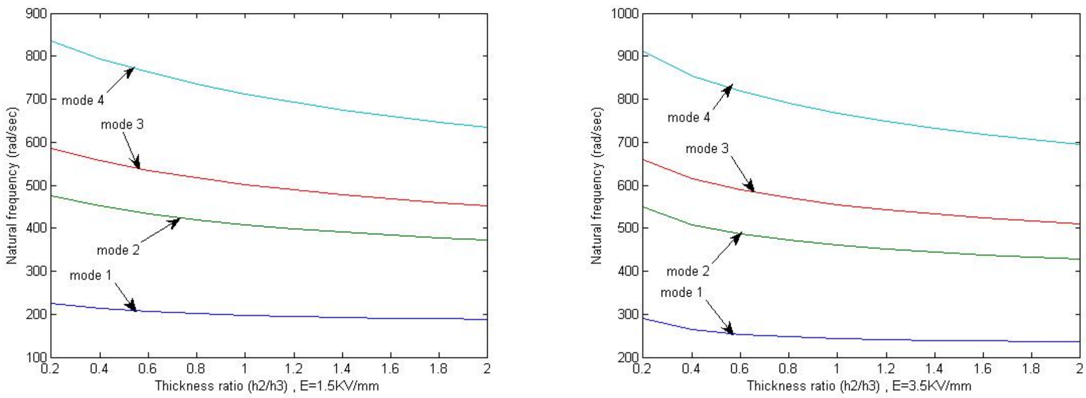


Fig. 4. Dependence of modal loss factor on the electric field, fluid type (A), $h_1 = 0.05mm, h_2 = h_3 = 0.5mm$.

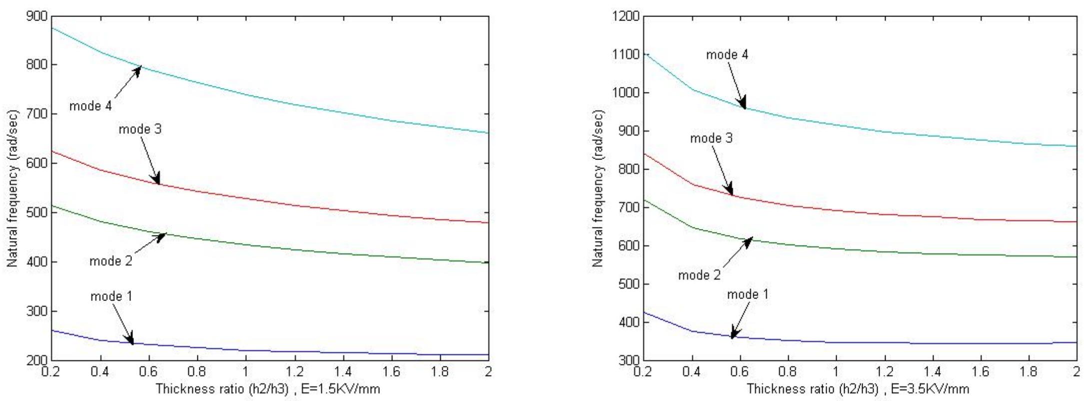
Effects of thickness ratio h_1/h_3 on the natural frequency of the sandwich plate structure type (A) and two applied electric field levels of $1.5\,kV/mm$ and $3.5\,kV/mm$ on the ER layer are presented in Fig. 7. The natural frequencies were decreased at first with the increasing thickness ratio of h_1/h_3 and, thus, the increment of the mass matrix exceeded the stiffness matrix. But, the effect was adverse when the thickness ratio h_1/h_3 of the constrained layer exceeded 0.5. When 3.5 kV/mm electric field was applied, the natural frequency of the sandwich plate increased continuously as the thickness ratio of the ER layer increased; therefore, the shear modulus of the ER fluid core was big enough when 3.5 kV/mm electric field was applied. These studies were done for the first four vibrational modes with simply-supported end conditions and $h_2 = h_3 = 0.5\,mm$. So, it can be concluded that the natural frequency of the sandwich plate changes when the thickness of the constrained layer increases or decreases.

Effect of the thickness ratio h_2/h_3 on the modal loss factors of ER fluid core is shown in Fig. 8. This figure demonstrates the change toward modal loss for various thicknesses of the ER fluid core (A) of the electric field $1.5\,kV/mm$ and $3.5\,kV/mm$. As shown in this figure, the modal loss factor increases as the thickness ratio of ER fluid increases. It can be seen that, at low modes, the modal loss factor increases as the thickness ratio h_2/h_3 is increased. The higher mode of the modal loss factor of the sandwich plate is larger than the lower mode when the thickness ratio h_2/h_3 of the ER layer is almost smaller than 0.5. Then, applying the higher electric fields, the natural frequencies always decrease and the modal loss factors always increase with the increase of the thickness ratio h_2/h_3 .



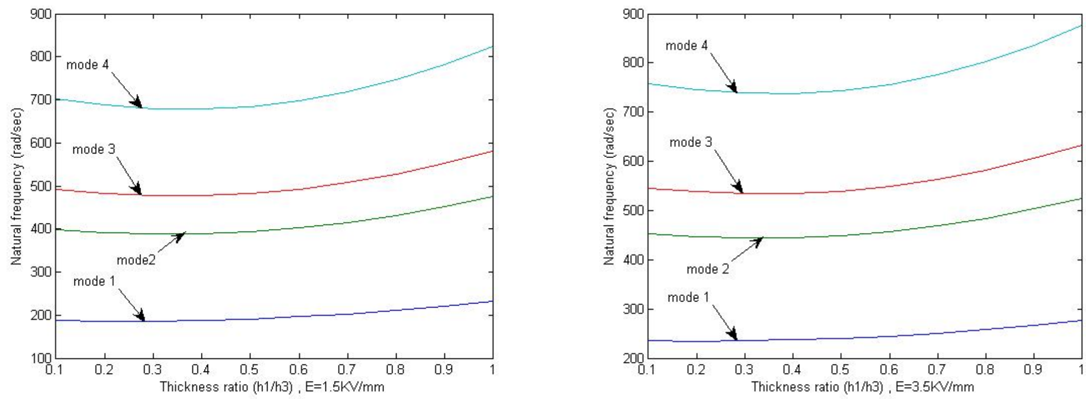
$E=1.5kV/mm$ $E=3.5kV/mm$

Fig. 5. Dependence of the first four modes natural frequencies on the thickness of the ER layer, fluid type (A), $h_1 = 0.05\,mm, h_3 = 0.5\,mm, E = 1.5kV/mm \,\&\, E = 3.5kV/mm$.



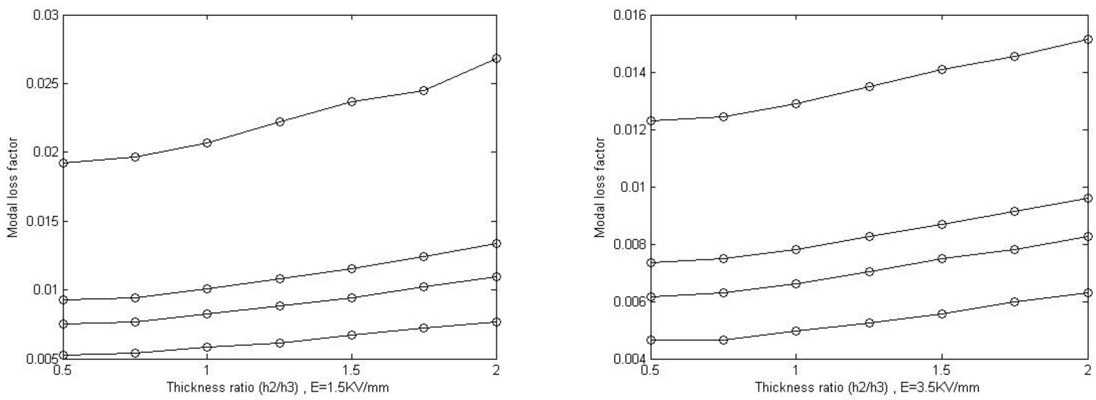
$E=1.5kV/mm$ $E=3.5kV/mm$

Fig. 6. Dependence of the first four modes natural frequencies on the thickness of the ER layer, fluid type (B), $h_1 = 0.05\,mm, h_3 = 0.5\,mm, E = 1.5kV/mm \,\&\, E = 3.5kV/mm$.



$E=1.5kV/mm$ $E=3.5kV/mm$

Fig. 7. Dependence of the first four natural frequencies on the thickness of the constrained layer, fluid type (A), $h_2=h_3=0.5\ mm, E=1.5kV/mm\ \&\ E=3.5kV/mm$.



$E=1.5kV/mm$ $E=3.5kV/mm$

Fig. 8. Dependence of modal loss factor at different thickness of the ER layer, fluid type (A), $h_1=0.05\ mm, h_3=0.5mm, E=1.5kV/mm\ \&\ E=3.5kV/mm$.

4. Conclusions

In this study, vibration of a sandwich plate with an ER fluid core, constrained layer, and four simply-supported edges was investigated. The two outer layers of this sandwich structure were elastic and the behavior of the core, subjected to the electric field for the small strain, was expressed by Kelvin's viscoelastic material model. The Navier's solution method was used for the vibration analysis of sandwich structures. In this paper, two kinds of ER materials were discussed. The similar viscoelastic properties can be observed. The viscoelastic characteristics, which can be changed by applying different electric fields, of the ER materials were shown to have significant effects on the free vibration of the sandwich structures. Electric field changed the stiffness of the sandwich plate. The property of the ER fluid was a function of the electric fields and changed the material properties when subjected to the different electric fields. As the applied electric field increased, the natural frequency of the sandwich plate increased.

The increase in the thickness of the ER layer decreases the natural frequencies of the sandwich plate. On the other hand, the modal loss factor of the sandwich plate played an important role in the stability of the damped structures. The modal loss factor was changed when subjected to different electric fields. The modal loss factor decreased as the applied electric field increased and a relative increment in the modal loss factor was observed by increasing the thickness ratio. The thickness of the constrained layer also had some effects on the stability of the sandwich plate. The natural frequency of the sandwich plate was changed when the thickness of the constrained layer increased or decreased. Change of the thickness of the ER layer also could have a significant effect on the modal loss factor of a sandwich plate structure.

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