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Resonant frequency of bimorph triangular V-shaped piezoelectric cantilever energy harvester

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Abstract

The concept of "energy harvesting" is to design smart systems to capture the ambient energy and to convert it to usable electrical power for supplying small electronics devices and sensors. The goal is to develop autonomous and self-powered devices that do not need any replacement of traditional electrochemical batteries. Now piezoelectric cantilever structures are being used to harvest vibration energy for self-powered devices. However, the geometry of a piezoelectric cantilever beam will greatly affect its vibration energy harvesting ability. This paper deduces a remarkably precise analytical formula for calculating the fundamental resonant frequency of bimorph V-shaped cantilevers using Rayleigh method. This analytical formula, which is convenient for mechanical energy harvester design based on Piezoelectric effect, is then validated by ABAQUS simulation. This formula raises a new perspective that, among all the bimorph V-shaped cantilevers and in comparison with rectangular one, the simplest tapered cantilever beam can lead to maximum resonant frequency and highest sensitivity. The derived formula can be commonly used as a relatively precise rule of thumb in such systems.

1. Introduction

Energy harvesting has been around for decades. To feed the world's needs for energy, macro scale energy harvesting technologies have successfully established. On the other hand, for low powered electronics devices, harvesting energy from the ambient vibrations seems to be an ideal solution due to the definite life span and high cost for replacement of the traditional batteries. Three mechanisms are available for vibration energy harvesting; using electrostatic devices, electromagnetic field and utilizing piezoelectric based materials. The performance of piezoelectric vibration energy harvesters is more often than other methods. Compared to other structural forms of beams, a cantilever beam can obtain the maximum deformation and strain under the same conditions. The larger deflection leads to more stress, strain, and consequently a higher output voltage and power. Therefore the vast majority of piezoelectric vibration energy harvesting devices use a cantilever beam structure. [1-4].

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A cantilever-type energy harvester has been intensively studied. The cantilever geometrical structure plays an important role in improving the harvester's efficiency and a triangular tapered cantilever has been found to be the optimum design [5], because it ensures a large constant strain in the piezoelectric layer resulting in higher power output compared with the rectangular beam with the width and length equal to the base and height of the corresponding triangular tapered cantilever beam.

Most of the previous research works focused on designing a linear vibration resonator, which has maximum output power when reaching resonance frequency. Therefore the practical applications of these devices are limited due to narrow bandwidth as well as small power density. If the excitation frequency slightly shifts, the performance of the harvester will dramatically decrease. Since in the majority of practical cases, the vibration in the environment is frequency-varying or totally random with the energy distributed in a wide spectrum, how to broaden the bandwidth of harvesters becomes one of the most challenging issues before their practical deployment [6].

In practice, the energy harvester is a multi-degree-of-freedom system or a distributed parameter system. Certain vibration mode can be excited when the driving frequency approaches one natural frequency of the harvester. To date, one of the most important strategies to widen the bandwidth, include using a generator array consisting of small generators with different resonant frequencies. If multiple vibration modes of the harvester structure are utilized, useful power can be harvested over multiple frequency spectra, that is, wider bandwidth can be covered for efficient energy harvesting. Rather than discrete bandwidth due to the multiple modes of a single beam, multiple cantilevers or cantilever array integrated in one energy harvesting device can provide continuous wide bandwidth, if the geometric parameters of the harvester are appropriately selected. Power spectrum of a generator array is a combination of the power spectra of each small generator [6-8]. Accordingly, by division of a triangular bimorph piezoelectric beam into some V-shaped bimorph beams with different dimensions and mass and hence different resonant frequencies, can be found in an array of beams that can cover a wider range of frequencies (Fig. 1). If the Δ in **Error! Reference source not found.**, assumed to be negligible, the V-shaped beams, will be a cantilever beam [9, 10].

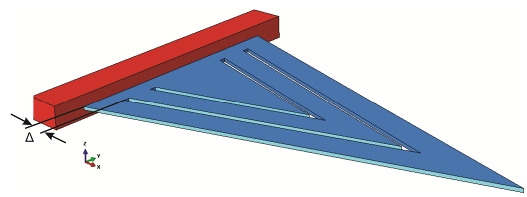

Fig. 1. Division of a triangular beam into some V-shaped beams [9].

The geometry of a piezoelectric cantilever beam will greatly affect its vibration energy harvesting ability. The sensitivity of resonant cantilever piezoelectric energy harvesters is directly proportional to the resonant frequency. So far, the calculation of resonant frequency of bimorph V-shaped cantilevers has not been reported in the literature and the calculation only for a simple V-shaped cantilever beam is done [11]. In order to calculate the resonant frequency of V-shaped cantilevers, this paper deduces a highly precise analytical formula using Rayleigh method, and then introduces the optimization method for enhancing the resonant frequency with this formula. This useful analytical formula, is confirmed by simulation results in ABAQUS 14.1 software, and presents a strong potential to be used in the design and optimization of triangular V-shaped cantilever bimorph piezoelectric energy harvesters. It is noteworthy that a cantilever beam can have many different modes of vibration, each with a different resonant frequency. The first mode of vibration has the lowest resonant frequency, and typically provides the most deflection and therefore electrical energy. Accordingly, energy harvesters are generally designed to operate in the first resonant mode [9, 10].

This research proposes a new design for a cantilever-type bimorph piezoelectric energy harvester called V-shaped cantilever and the main focus of this paper is to study the resonant frequency of the new design in piezoelectric mechanical energy harvester.

2. Theoretical analysis

2. 1. Deflection function of rectangular bimorph cantilevers

Equating the maximum total potential energy associated with vibration to the maximum kinetic energy associated with vibration results in an upper-bound estimate of the fundamental natural frequency, provided the dynamic displacement forms assumed are admissible. A displacement function is admissible if it does not violate any geometric constraints and can represent the displaced form of the system without any discontinuity.

Fig. 2 shows the structure of bimorph piezoelectric rectangular cantilever with length L, width W, density ρ_1 and ρ_2, thickness H_1 and H_2, cross-sectional area moment of inertia I_1 and I_2 and Young's modulus E_1 and E_2 for substrate and piezoelectric layers, respectively. When applying a normal force F at the free end of the cantilever, the differential equation of the cantilever can be expressed as [12]

$$\frac{d^2z(x)}{dx^2}=\frac{F(L-x)}{\overline{EI}}=\frac{F(L-x)}{E_1I_1+E_2I_2}=\frac{12F(L-x)}{EWH^3} \quad (1)$$

where x is the distance from the fixed end. It is notable that for a doubly symmetric section, $\overline{EI}=\sum_i E_iI_i$ [13].

As one end of the cantilever is fixed, the corresponding boundary conditions are;

$$z(0)=0 \quad (2)$$

and

$$\left.\frac{dz(x)}{dx}\right|_{x=0}=0 \quad (3)$$

The solution of (1) - (3) can be expressed as;

$$\begin{aligned} z(x)&=\frac{2Fx^2(3L-x)}{E_1WH_1^3+E_2W(8H_2^3+6H_1^2H^2+12H_1H_2^2)} \\ &=Ax^2(3L-x) \end{aligned} \quad (4)$$

This is the deflection function along the length direction where A is a constant.

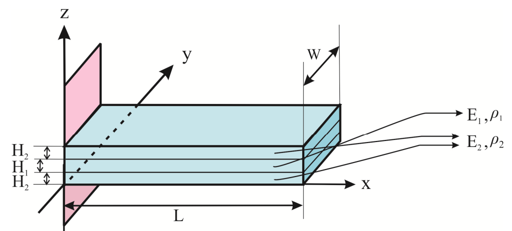

Fig. 2. Schematic drawing of a cantilever beam [10].

2. 2. Resonant frequency of cantilevers with arbitrary shapes

When considering the resonant behavior of a cantilever with an arbitrary shape whose width function is *W(x),* the deflection function of (4) can be used as the mode shape, and the vibration displacement at each position can be written as;

$$z(x,t)=Ax^2(3L-x)\sin(\omega t+\alpha) \quad (5)$$

where A and α are constants, t is the time, and $\omega=2\pi f$ is the angular frequency.

The kinetic energy of the system is [14];

$$\begin{aligned} T&=\int_0^L \frac{1}{2}(\rho_1H_1+2\rho_2H_2)W(x)dx\left(\frac{\partial z}{\partial t}\right)^2 \\ &=\frac{1}{2}(\rho_1H_1+2\rho_2H_2)\omega^2A^2\cos^2(\omega t+\alpha)\int_0^L W(x)x^4(3L-x)^2dx \end{aligned} \quad (6)$$

So the maximum kinetic energy of the system is;

$$T_{\max}=\frac{1}{2}(\rho_1H_1+2\rho_2H_2)\omega^2A^2\int_0^L W(x)x^4(3L-x)^2dx \quad (7)$$

The potential energy of the system is [14];

$$
\begin{aligned}
V &= \frac{1}{2}\iiint_V \sigma_{xx}\varepsilon_{xx} dV \\
&= \frac{1}{2}\int_0^L \int_0^W \int_{-(\frac{H_1}{2}+H_2)}^{\frac{H_1}{2}+H_2} \sigma_{xx}\varepsilon_{xx} dV \\
&= \int_0^L \frac{W(x)}{2}\left(\frac{\partial^2 z}{\partial x^2}\right)^2 \left[\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2\right] dx \\
&= \int_0^L \frac{W(x)}{2}\left[6A(L-x)\sin(\omega t+\alpha)\right]^2 \left[\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2\right] dx \\
&= 18A^2 \sin^2(\omega t+\alpha) \left[\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2\right] \int_0^L W(x)(L-x)^2 dx
\end{aligned}
\tag{8}
$$

Therefore, the maximum potential energy of the system is;

$$
V_{\max} = 18A^2 \sin^2(\omega t+\alpha) \left[\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2\right] \int_0^L W(x)(L-x)^2 dx \tag{9}
$$

According to conservation law of mechanical energy;

$$
T_{\max} = V_{\max} \tag{10}
$$

Hence, the resonant frequency can be obtained as;

$$
f(W(x)) = \frac{\omega}{2\pi} = \frac{3}{\pi}\sqrt{\frac{\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2}{\rho_1 H_1 + 2\rho_2 H_2}} \sqrt{\frac{\int_0^{L_1} W(x)(L-x)^2 dx}{\int_0^{L_1} W(x) x^4 (3L-x)^2 dx}} \tag{11}
$$

In particular, for the case of a rectangular cantilever with length L_1 and width W_1, the resonant frequency can be deduced from (11);

$$
\begin{aligned}
f_{rect} &= \frac{3}{\pi}\sqrt{\frac{\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2}{\rho_1 H_1 + 2\rho_2 H_2}} \sqrt{\frac{\int_0^{L_1} W_1 (L_1-x)^2 dx}{\int_0^{L_1} W_1 x^4 (3L_1-x)^2 dx}} \\
&= \frac{3}{\pi}\sqrt{\frac{\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2}{\rho_1 H_1 + 2\rho_2 H_2}} \frac{1}{L^2}\sqrt{\frac{35}{99}} \\
\Rightarrow f_{rect} &= \frac{\sqrt{385}}{11\pi L^2}\sqrt{\frac{\frac{E_1 H_1^3}{12} + \frac{2E_2 H_2^3}{3} + \frac{E_2 H_2 H_1^2}{2} + E_2 H_1 H_2^2}{\rho_1 H_1 + 2\rho_2 H_2}}
\end{aligned}
\tag{12}
$$

2. 3. Resonant frequency of bimorph triangular V-shaped cantilevers

Fig. 3(a) shows that a typical bimorph triangular V-shaped cantilever can be treated as the difference between two bimorph triangular cantilevers, with lengths L_0 and L_1, and with widths W_0 and W_1 respectively. It can be easily confirmed by (11), that due to the mirror symmetry of bimorph triangular V-shaped cantilever, we need only analyze half of it, which is a quadrilateral cantilever as shown in Fig. 3(b).

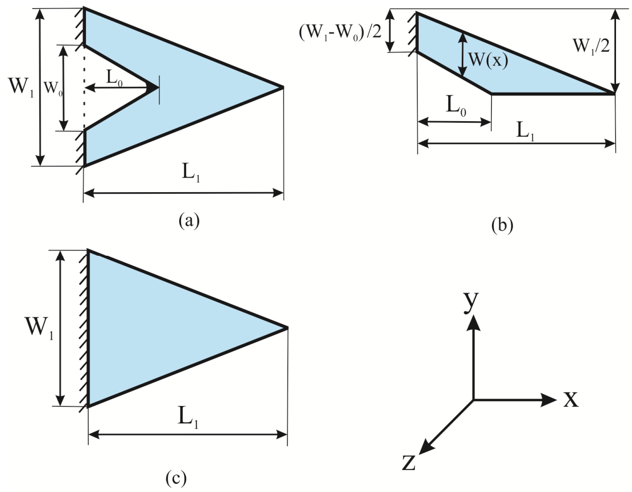

Fig. 3. Shape and dimension of (a) bimorph V-shaped cantilever (b) half of the bimorph V-shaped cantilever (c) triangular tapered cantilever.

Obviously, the width function of the quadrilateral cantilever is a piecewise-continuous function of x, that is;

$$W(x)=\begin{cases}\frac{W_1}{2}\left(1-\frac{x}{L_1}\right)-\frac{W_0}{2}\left(1-\frac{x}{L_0}\right), x\in[0,L_0]\\ \frac{W_1}{2}\left(1-\frac{x}{L_1}\right), x\in[L_0,L_1]\end{cases} \quad (13)$$

For calculation convenience, it is reasonable to define the width ratio u and the length ratio v of the two bimorph tapered cantilevers;

$$u=\frac{W_0}{W_1}, v=\frac{L_0}{L_1} \quad (14)$$

Substituting (13) and (14) into (11), the resonant frequency formula of the quadrilateral cantilever (just the resonant frequency of bimorph triangular V-shaped cantilever) is obtained.

$$\begin{aligned} f(W(x)) &= \frac{3}{\pi}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}\sqrt{\frac{\int_0^{L_1}W(x)(L_1-x)^2dx}{\int_0^{L_1}W(x)x^4(3L_1-x)^2dx}}\\ &=\frac{3}{\pi}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}\sqrt{\frac{70(3W_1L_1^3-6W_0L_1^2L_0+4W_0L_1L_0^2-W_0L_0^3)}{3(49W_1L_1^7-84W_0L_1^2L_0^5+40W_0L_1L_0^6-5W_0L_0^7)}}\\ &=\frac{\sqrt{210}}{\pi L_1^2}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}\sqrt{\frac{3-6uv+4uv^2-uv^3}{49-84uv^5+40uv^6-5uv^7}} \end{aligned} \quad (15)$$

In order to represent the relationship between the resonant frequency and the two ratios u and v, a characteristic function can be defined;

$$g(u,v)=\sqrt{\frac{3-6uv+4uv^2-uv^3}{49-84uv^5+40uv^6-5uv^7}} \quad (16)$$
$$u\in[0,1], v\in[0,1]$$

Thus, the resonant frequency of V-shaped cantilever is;

$$f(W(x))=\frac{\sqrt{210}}{\pi L_1^2}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}g(u,v) \quad (17)$$

As shown in Fig. 4, $g(u,v)$ reaches the maximum value $\frac{\sqrt{3}}{7}\approx 0.2474$, when $v=0$ or $v=1$ or $u=0$ [15]. That means bimorph V-shaped cantilever achieves maximum resonant frequency only when $L_0=0$ or $L_0=L_1$ or $W_0=0$. Apparently, when $L_0=0$ or $W_0=0$, the V-shaped cantilever turns into a tapered cantilever as shown in Fig. 3(c). When $L_0=L_1$, the bimorph V-shaped cantilever turns into two side by side bimorph triangular tapered cantilevers, however, this peculiar shape is difficult to carry out in practice.

Anyway, triangular tapered cantilever, a special kind of V-shaped cantilever and easy for micro-fabrication, can reach the maximum resonant frequency and thus the highest sensitivity.

3. Verification by simulation results

In order to assess the accuracy of (17), relative error δ is introduced to compare the calculation results using this formula with the corresponding simulation results.

$$\delta=\frac{f-f'}{f} \quad (18)$$

where f refers to the calculation results with (17), and f' refers to simulation results with ABAQUS modal analysis.

Consider a bimorph rectangular cantilever, assuming ρ_1=8740kg/m^3, ρ_2=7800kg/m^3, E_1=9.7×10^{10}Pa, E_2=6.6×10^{10}Pa, H_1=1mm, H_2=1mm, W_1=80mm and L_1=100mm. The frequency calculation according to (12) is 8.84 Hz and the corresponding simulation result with ABAQUS is 8.82 Hz. Hence the relative error is only 0.24% and an excellent agreement is obtained between the calculation results and the simulation results, yielding little relative error. The simulated shape is shown in Fig. 5.

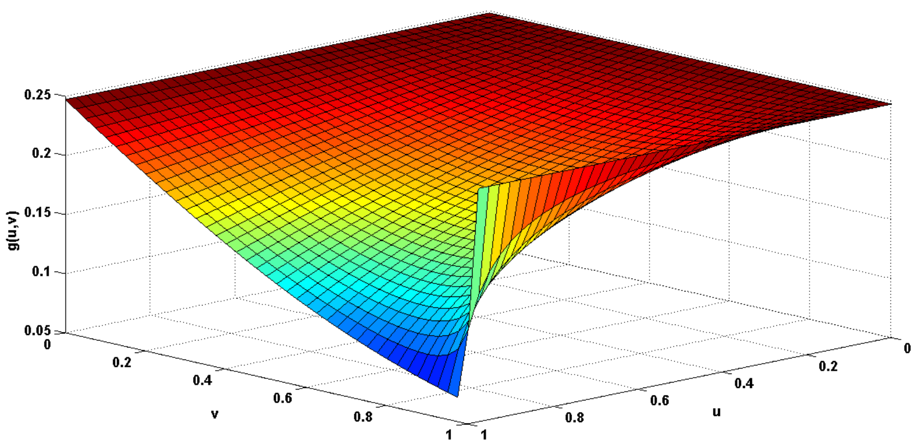

Fig. 4. The function image of $g(u,v)$.

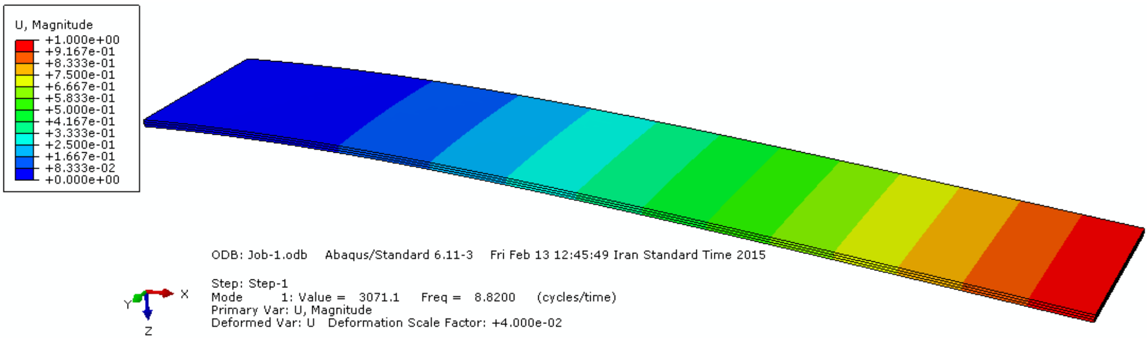

Fig. 5. Deformed shaped for the first vibration mode of bimorph piezoelectric cantilever [10].

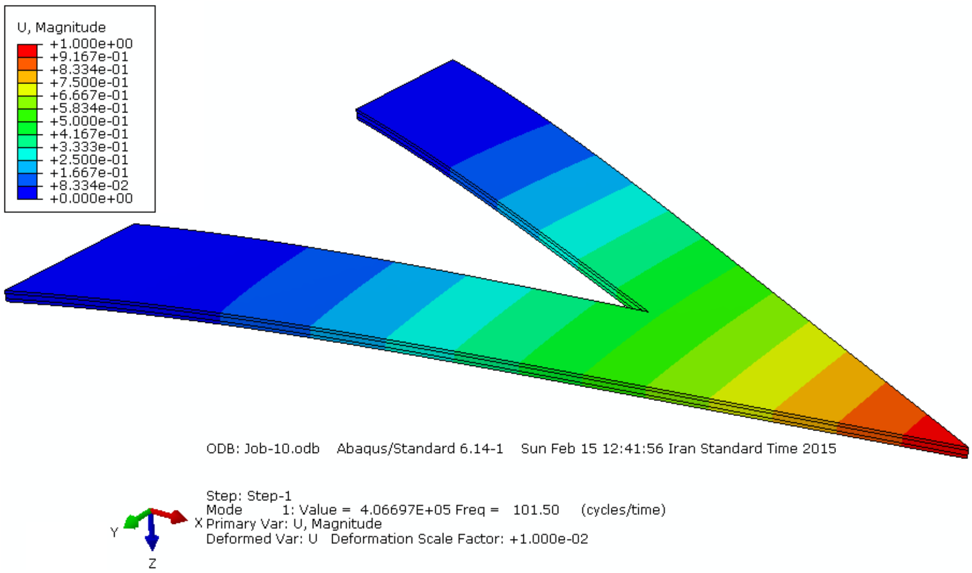

Fig. 6. Deformed shaped for the first vibration mode of bimorph piezoelectric cantilever.

Also the experimental results are achieved for another rectangular cantilever and it is observed that a good agreement is obtained between experimental, FEM and analytical methods [10].

Also consider a series of V-shaped cantilevers with different shapes, assuming, ρ_1=8740kg/m^3, ρ_2=7800kg/m^3, E_1=9.7×10^{10}Pa, E_2=6.6×10^{10}Pa, H_1=0.6mm, H_2=0.4mm, W_1=80mm , W_0=40mm, L_1=100mm and changing L_0, the calculation according to (17) and the corresponding simulation results with ABAQUS are listed in Table 1.

It can be seen from Table 1 that, a very good agreement is obtained between the calculation results and the simulation results, yielding little relative error (less than 6.2%). When L_0=60mm, the simulated shape is shown in Fig. 6.

Table 1. Comparison between the calculation results and the simulation results of the resonant frequencies of bimorph triangular V-shaped cantilevers.

L_0(mm)	f (Hz)	f' (Hz)	δ %
0	124.99	133.24	6.19
10	119.55	126.86	5.76
20	114.87	121.06	5.11
30	110.23	115.92	4.91
40	106.91	111.56	4.17
50	104.19	108.21	3.71
60	101.5	106.19	4.42
70	102.84	106	2.98
80	105.06	108.56	3.22
90	111.53	115.8	3.69
100	125.85	133.24	5.55

4. Application

The resonant frequency formula presented in this paper is useful for many applications. First, this simple formula can be effectively used to determine the resonant frequency of bimorph triangular V-shaped cantilevers of any dimensions and material properties. Another significant application is the optimization of bimorph V-shaped cantilever vibration energy harvesters. The sensitivity of resonant cantilever vibration energy harvesters is directly proportional to the resonant frequency, and the resonant frequency is a key parameter to design a mechanical energy harvester. As mentioned above, with given length L_1, given width W_1, given thickness H_1 *and* H_2 and given material properties E_1, E_2, ρ_1 and ρ_2, triangular tapered cantilever-a special kind of V-shaped cantilevers-can reach the maximum resonant frequency and highest sensitivity.

For a triangular tapered cantilever, substituting $v=0$ into (17), the maximum resonant frequency is obtained

$$f_{tap} = \frac{\sqrt{210}}{\pi L_1^2}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}g(u,0 = \frac{\sqrt{210}}{\pi L_1^2}\sqrt{\frac{\frac{E_1H_1^3}{12}+\frac{2E_2H_2^3}{3}+\frac{E_2H_2H_1^2}{2}+E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}\frac{\sqrt{3}}{7} \approx \frac{0.3295}{L_1^2}\sqrt{\frac{E_1H_1^3+8E_2H_2^3+6E_2H_2H_1^2+12E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}} \tag{19}$$

Apparently, the resonant frequency of a bimorph tapered cantilever is unrelated to its width W_1. It is necessary to point out that, for a tapered cantilever, when increasing W_1 and keeping other parameters fixed, its resonant frequency will remain constant. It is worth comparing (12) and (19), and we can get the resonant frequency ratio of bimorph tapered cantilever and bimorph rectangular cantilever.

$$\frac{f_{tap}}{f_{rect}} = \frac{\frac{0.3295}{L_1^2}\sqrt{\frac{E_1H_1^3+8E_2H_2^3+6E_2H_2H_1^2+12E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}}{\frac{0.1639}{L_1^2}\sqrt{\frac{E_1H_1^3+8E_2H_2^3+6E_2H_2H_1^2+12E_2H_1H_2^2}{\rho_1H_1+2\rho_2H_2}}} = 2.0104 > 2 \tag{20}$$

Hence, the bimorph tapered cantilevers can lead to much higher resonant frequency and higher sensitivity than that of bimorph rectangular cantilevers.

5. Conclusions

This paper deduces a highly precise explicit formula to calculate the fundamental resonant frequency of bimorph V-shaped cantilevers based on Rayleigh method. It is clear that the results obtained using the Rayleigh's principle is dependent on the type of admissible function used. If the function used resembles the fundamental mode, the resulting estimate of the frequency is likely to be close to the exact fundamental natural frequency. It may also be noted that this approach gives only one value for the frequency. However, energy harvesters are generally designed to operate in the first resonant mode.

With this analytical formula, the calculation results are in perfect agreement with the simulation results, yielding little relative error (less than 6.2%). This error for a bimorph rectangular cantilever reduces to only 0.24%. In the first mode of vibration, the exact shape of the cantilever is not identical to the static deflection profile. Accordingly the velocity distribution is not exactly proportional to the static deflection profile. This is why the natural frequency estimates are slightly different from the simulation values.

It is clear that with the same material properties and given length L_1, given width W_1, given thickness H_1 *and* H_2, triangular tapered cantilever can reach the maximum resonant frequency. Also width increasing in the base of triangular tapered cantilever has no effect on the resonant frequency of the structure.

Because of simplicity of the derived formula, it is an easily learned and easily applied procedure for approximately calculating or recalling some value, or for making some determination. Finally, an application for calculating frequency of bimorph V-shaped cantilever energy harvesters is presented with this formula in order to achieve a Multi-Modal energy harvester. This formula can be commonly used in the design and optimization of vibration energy harvesters.

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