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The effect of small scale and intermolecular forces on the nonlinear pull-in instability behavior of nano-switche susing differential quadrature method

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Abstract

Using differential quadrature method, this study investigated pull-in instability of beam-type nano-switches under the effects of small-scale and intermolecular forces including the van der Waals and the Casimir forces. In these nano-switches, electrostatic forces were served as the driving force, and von-Karman type nonlinear strain was used to examine nonlinear geometric effects. To derive nonlinear governing equations as well as the related boundary conditions for the nano-beam, variation method was used. Besides, to study the influence of size effect, the nonlocal elasticity theory was employed and the resulting governing equations were solved using differential quadrature method. Finally, the pull-in parameters were studied using the nonlocal theory and the results were compared with the numerical results of the classical continuum theory as well as experimental results contained in the references. Results demonstrated that taking into consideration the von-Karman type nonlinear strain increases the beam stiffness and hence, the pull-in voltage. Besides, use of the small scale, compared with the classical theory of elasticity, yields results much closer to experimental results.

1. Introduction

Micro-electro-mechanical systems (MEMS) is a process technology used to create small integrated devices or systems that combine mechanical and electrical components. They are fabricated through a mix of integrated circuit manufacturing and micro-machining process and can range in size from a few micrometers to millimeters. These devices have the ability to sense, control and actuate on the micro scale. Nano-electro-mechanical systems (NEMS) are structures and mechanisms with nano-dimensions which serve as nano-switches to convert electrical energy into mechanical energy and as sensors to convert mechanical energy into electrical energy. The development of such nanostructure in sciences such as communication, electronics, medicine, aerospace, military, robotics, chemistry, and optics has resulted in new achievements. On the other hand, the need to investigate and predict the mechanical behavior of these structures has opened a new window for researchers in the field of mechanics [1,2]. The simplest nano-electro-mechanical actuator is a beam-type mechanism consisting of two conductive electrodes in the nanoscale of which one is usually fixed and the other

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is movable [3, 4]. This simple system can be found in systems such as nano-switches and nano-relays [5]. Applying opposite voltages to two electrodes causes electrostatic attraction between them. This attraction causes the fixed electrode to strain toward the movable electrode [6, 7]. If the bending moment resulting from the electrostatic force is higher than the one the nano-beam can withstand, the movable electrode collapses on the fixed one. This phenomenon is known as pull-in instability. Prediction and simulation of the pull-in instability of MEMS/NEMS are very crucial for reliable design and fabrication of nano-devices. Hence, it is highly important to investigate and understand this phenomenon and factors involved in it. It has received great attention by the researchers [8,9].

It should be noted that in nanoscale, in comparison with the macro scale, other phenomena appear which must be taken into consideration in modeling. Given the fact that nano-switches have nano dimensions, this study models the effects of two new phenomena, i.e. the small scale effect and intermolecular forces. The study also investigates the nonlinear geometrical effects which have a great impact on the results of studies of instability in nanostructures.

The first phenomenon which substantially influences the mechanical behavior of nanobeams is small scale effect. The effect of small scale in the nanoscale has been proven by different researchers using laboratory experiments. Also, in recent years, in addition to laboratory experiments, some methods such as molecular dynamic (MD) have been used to simulate and examine size effects. Besides, given the fact that methods such as MD are costly and include lengthy calculations, in recent years, in order to examine the mechanical behavior in nanostructure, researchers have used non-classical continuum theories such as the nonlocal [9-11], strain gradient [12-16], and couple stress [17-21] theories, which have the ability to model size effects. This paper uses the nonlocal theory to investigate the pull-in instability in which the small scale effect is included. The second phenomenon which drastically influences the mechanical behavior of nanobeams is the presence of intermolecular forces.Intermolecular forces in nanoscale are the van der Waals and Casimir forces. The van der Waals force gains significance when the gap between the two electrodes is narrower than a few tens of nanometers [22]. This force changes with the inverse cube of the gap between the two electrodes. The Casimir force is the most famous mechanical effect of vacuum fluctuations. An important physical quantity related to the Casimir force is field radiation pressure. Each field, even vacuum, contains energy. All magnetic fields are capable of being released into space and put pressure on surfaces. This radiation pressure has a direct relationship with energy and, as a result, with the frequency of the magnetic field. In the hole's resonance frequency, the radiation pressure is stronger inside than outside; hence, the surfaces repel each other. Conversely, in non-resonance conditions, the radiation pressure outside the hole is stronger, and the electrodes are attracted to each other. In a state of balance, the repulsion components are rather stronger than the attraction components. Therefore, for two fully parallel flat electrodes, the Casimir force is an attraction and the electrodes attract each other. This force is proportionate to the cross-sectional area of the electrodes and the inverse fourth power of the gap between the electrodes. Except for geometrical quantities, this force only depends upon the basic values of Planck's constant and the speed of light [23]. Considering the effects of intermolecular forces in the nano scale, different researchers have investigated the effects of these forces on nanostructures [24-28].

Finally, it should be noted that many researchers have studied the small-scale effect phenomena and intermolecular forces for nanostructures. This paper, however, examines small-scale effects and intermolecular forces in beam-type nano-switches. In so doing, the non-classical, non-local theory is used. In this paper, besides the two phenomena made in nanoscale, the nonlinear geometric effects and the nonlocal theory are also used to investigate the pull-in instability in beam-type nano-switches.No study has so far examined nano-switches using the nonlocal elasticity theory and DQM while considering the nonlinear geometric effect. Hence, this paper attempts to address that issue. For this purpose, variations method and minimum potential energy are used. Equations of motion, as well as boundary conditions,are derived and, finally, DQM is employed to solve the equations. The findings revealed that taking nonlinear displacements into consideration causes an increase in pull-in instability. The significant result of the study is that using a small scale and nonlinear nonlocal theory leads to the results very consistent with experimental results regarding the classical theory of elasticity, which shows the high efficiency of the nonlinear model used with DQM.

2. Preliminaries

2.1. Nonlocal continuum theory

According to the classical continuum theory, stress at one point of an object is simply a function of the strain at that point. In 1983,Eringen and Edelen published papers and put forth a new theory, demonstrating that stress at one point of an object may be a function of strain all over the object [29]. Accordingly, they formulated the nonlocal continuum theory. In fact, in this theory, the small scale effect, which was hitherto ignored in continuum theories, was introduced as a parameter effective in the stress field (for more information see Appendix). In the following years and concurrent with the emergence of micro/nano-electro-mechanical systems in different branches of science, the nonlocal theory, as a continuum theory able to predict the behavior of these systems by considering the aforementioned factor, appealed to researchers and scientists. According to this theory, the nonlocal stress tensor at an arbitrary point of an object is as follows [30]:

$$\sigma = \int_V K\left(|x' - x|, \tau\right) S\left(x'\right) dx' \tag{1}$$

where $\boldsymbol{\sigma}$ is the nonlocal stress tensor at point x, $K|x' - x|$ is the kernel function, τ is the material constant which is dependent on length, internal and external characteristic and $\mathbf{S}(x')$ is the classical stress tensor [12]. It is known that the relationship between stress and strain in a Hookean solid with Hook's law is as follows:

$$\mathbf{S}(x) = \mathbf{C}(x) : \boldsymbol{\varepsilon}(x) \tag{2}$$

where $(:)$ stands for the double-dot product and $\boldsymbol{C}$ represents the fourth-order elasticity tensor. Eq. (1) has a simpler form which is much easier to use than the integrated form and is as follows:

$$\left(1 - \tau^2 L^2 \Delta^2\right)\sigma = S \ , \ \tau = \frac{e_0 a}{L} \tag{3}$$

In this equation, e_0, a and Lrepresent the material constant, the internal characteristic length and the external characteristic length, respectively. Assuming the material to be homogeneous and isotropic, by substituting Eq. (2) into Eq. (3), one can derive the stress-strain equation in general, and, for the one-dimensional case, the constitutive equation based on the nonlocal theory is as follows:

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx} \qquad (\mu = e_0^2 a^2) \tag{4}$$

where E represents Young's modulus [31].

2.2. Euler–Bernoulli beam theory

There are different theories for modeling nano-beams. Here, the Euler-Bernoulli beam theory is used. In this theory, the displacement field is expressed as:

$$\begin{aligned} \bar{u}(x,z) &= u(x) - z\frac{\partial w}{\partial x}, \\ \bar{v} &= 0, \\ \bar{w}(x,z) &= w(x) \end{aligned} \tag{5}$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the components of the displacement field of a point in distance $z$from the middle surface of the beam in each beam section, and u and w respectively stand for the values of axial displacement and traverse displacement in the middle surface of the beam. By considering von-Karman-type nonlinear strain and dispensing with the Poisson's effect, one can obtain the components of the strain tensor for the Euler-Bernoulli beam where the only non-zero strain component is defined by non-linear geometrical effects (which are along the x-axis of the beam) as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{6}$$

In the above equation, $\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$ represents von-Karman-type nonlinear strain [32].

3. Governing equations of motion

Figure1 illustrates a nano-switch modeled with a clamped-free nano-beam. This system is made up of a fixed electrode called the ground plane and a movable electrode with a rectangular section with length L, height h and width$b$. These two electrodes are separated by a dielectric spacer and an initial gap g.

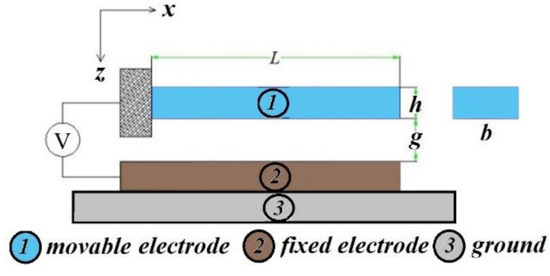

Fig. 1. Schematic model of a beam-type nano-switch.

As mentioned, in this paper, to derive the governing equations, variations method and the principle of minimum potential energy are used as follows:

$$\delta\left(U - V\right) = 0 \tag{7}$$

In the above equation, $U$and Vrepresent strain energy and the work of external forces, respectively. Considering the Euler-Bernoulli model, the strain energy in the nano-beam is expressed as:

$$\begin{aligned} U &= \frac{1}{2}\int_0^L\int_A \sigma_{xx}\varepsilon_{xx} dA\, dx \\ &= \frac{1}{2}\int_0^L\int_A \sigma_{xx}\left[\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] dA\, dx \end{aligned} \tag{8}$$

The above equation can be expressed as follows:

$$U = \frac{1}{2}\int_0^L\left[N_x\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) - M_x\frac{\partial^2 w}{\partial x^2}\right]dx \tag{9}$$

Here the values of parameters $N_x = \int_A \sigma_{xx}\, dA$ and $M_x = \int_A \sigma_{xx}\, zdA$ are determined. On the other hand, the work of external force acting on the beam is expressed using the following integral:

$$V = \int_0^L q w dx \tag{10}$$

In Eq. (10), $q$is the sum of the forces applied to the beam per unit length.

By substituting Eqs. (10) and (8) into Eq. (7) , performing variation operations, and setting δu and δw to zero, the equations of motion governing the system are derived as follows:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} = 0 \\ &\frac{\partial}{\partial x}(N_x\frac{\partial w}{\partial x}) + \frac{\partial^2 M_x}{\partial x^2} + q = 0 \end{aligned} \tag{11}$$

Also, according to Fig. 1, the boundary conditions for the clamped-free beam are derived from the following equations:

$$\begin{aligned} &w\left(x = 0\right) = 0\ , \frac{\partial w}{\partial x}\left(x = 0\right) = 0 \\ &Q\left(x = L\right) = M\left(x = L\right) = 0 \end{aligned} \tag{12}$$

In the above equation, M and Qrepresent the bending moment and the shear force, respectively.

From Eqs. (4) and (6), and by defining parameters $M_x$and N_x, one can express the normal resultant force and the bending moment as:

$$\begin{aligned} N_x &= (e_0 a)^2\frac{\partial^2 N_x}{\partial x^2} + EA\left[\frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2\right] \\ M_x &= -EI\frac{\partial^2 w}{\partial x^2} + (e_0 a)^2\left(\frac{\partial^2 M_x}{\partial x^2}\right) \end{aligned} \tag{13}$$

By substituting Eqs. (11) into Eqs. (13), the value of the bending moment is obtained as:

$$M_x = -EI\frac{\partial^2 w}{\partial x^2} - (e_0 a)^2\left[q + \frac{\partial}{\partial x}(N_x\frac{\partial w}{\partial x})\right] \tag{14}$$

By substituting Eq. (14) into the boundary conditions (11), the following equations are obtained:

$$\begin{aligned} &EA\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right) = 0 \\ &EI\frac{\partial^4 w}{\partial x^4} - EA\left[\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial^2 w}{v x^2} + \frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}\right] \\ &+(e_0 a)^2 EA\left[\frac{\partial^4 u}{\partial x^4}\frac{\partial w}{\partial x} + 9\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3} + \right. \\ &\frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^4 w}{\partial x^4} + 3\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 u}{\partial x^3} + 3\left(\frac{\partial^2 w}{\partial x^2}\right)^3 + \\ &\left. 3\frac{\partial^3 w}{\partial x^3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 w}{\partial x^4}\frac{\partial u}{\partial x}\right] \\ &= q - \left(e_0 a\right)^2\frac{\partial^2 q}{\partial x^2} \end{aligned} \tag{15}$$

The equations of motion in Eq. (15) can be reduced to a single equation by eliminating$u$. For this purpose, by integration and double differentiation of the first equation of motion in Eq. (15), the following equations are derived [32,33].

$$\begin{aligned} &\frac{\partial u}{\partial x} = -\frac{1}{2}(\frac{\partial w}{\partial x})^2 + \frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2 dx \\ &\frac{\partial^4 u}{\partial x^4} = -3\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3} - \frac{\partial w}{\partial x}\frac{\partial^4 w}{\partial x^4} \end{aligned} \tag{16}$$

Now, by substituting Eqs. (16) into Eq. (15), u can be eliminated and hence, the nonlinear equation of motion governing the nano-beam is derived as:

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} - EA\left[\frac{\partial^2 w}{\partial x^2}\frac{1}{2L}\int_0^L(\frac{\partial w}{\partial x})^2 dx\right] + \\ &(e_0 a)^2 EA\frac{1}{2L}\frac{\partial^4 w}{\partial x^4}\int_0^L(\frac{\partial w}{\partial x})^2 dx = q - (e_0 a)^2\frac{\partial^2 q}{\partial x^2} \end{aligned} \tag{17}$$

The boundary conditions for the clamped-free nano-beam are as follows:

$$
\begin{aligned}
& w\big|_{x=0}=0\ ,\ \frac{\partial w}{\partial x}\bigg|_{x=0}=0 \\
& Q\big|_{x=L}= \\
& \left\{-EI\,\frac{\partial^3 w}{\partial x^3}-\left(e_0 a\right)^2\left[\frac{\partial q}{\partial x}+EA\left(\frac{1}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2 dx\right)\frac{\partial^3 w}{\partial x^3}\right]\right\}\Bigg|_{x=L}=0 \\
& M\big|_{x=L}= \\
& \left\{-EI\,\frac{\partial^2 w}{\partial x^2}-\left(e_0 a\right)^2\left[q+EA\left(\frac{1}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2 dx\right)\frac{\partial^2 w}{\partial x^2}\right]\right\}\Bigg|_{x=L}=0
\end{aligned}
\tag{18}
$$

It should be noted that by setting the value of e_0 to zero, the above equation is reduced to the governing equation of classical nonlinear beam, and, the integral term of the equation of motion has appeared due to the von-Karman-type nonlinear strain. Therefore, in order to investigate the results of linear displacements, by setting the integral term to zero, one can obtain the equation of motion in the linear case.

In Eq. (19), $q$is the sum of intermolecular forces, Casimir or van der Waals or electrostatic forces applied to the nano-beam, which are expressed as:

$$
q=q_{ele}+q_{dis}\ \ ,\ q_{dis}=q_{cas}\ \ or\ \ q_{van} \tag{19}
$$

The electrostatic forces with fringing field effects, and Casimir and van der Waals forces per unit length are expressed as [33].

$$
\begin{aligned}
q_{elec} &= \frac{\varepsilon_0 b V^2}{2\left(g-w\right)^2}\left(1+0.65\frac{\left(g-w\right)}{b}\right) \\
q_{cas} &= \frac{\pi^2 \hbar c b}{240\left(g-w\right)^4} \\
q_{van} &= \frac{\bar{A} b}{6\pi\left(g-w\right)^3}
\end{aligned}
\tag{20}
$$

where ε_0, V, w, $\hbar$, $\bar{A}$ and c are permittivity of vacuum, applied voltage, beam transverse deflection, reduced Planck's constant, Hamaker constant and light speed respectively.

4. Solution method (DQM)

Along with the growing advancement of faster computing machines, the research is going to develop the new methods for numerical solution of problems in engineering and physical sciences. The numerical methods for the solution of initial or boundary-value problems, in general, seek to transform, either through a differential or an integral formulation into an analogous set of first-order or algebraic equations in terms of the discrete values of the field variable at some specified discrete points of the solution domain. The differential quadrature method is a numerical solution technique has been successfully employed in a variety of problems in engineering and physical sciences. The method has been projected by its proponents as a potential alternative to the conventional numerical solution techniques such as the finite difference and finite element methods. In order to solve the equation using DQM, first, the equation of motion and governing boundary conditions are made dimensionless. To do this, the following dimensionless parameters are defined:

$$
\begin{aligned}
& \zeta=\frac{w}{g},\ \eta=\frac{x}{L},\ \alpha_3=\frac{\bar{A} b L^4}{6\pi g^4 EI}, \\
& \alpha_4=\frac{\pi^2 \hbar c b L^4}{240 g^5 EI},\ \ \beta=\frac{\varepsilon_0 b V^2 L^4}{2 g^3 EI}, \\
& \gamma=0.65\,\frac{g}{b},\ e=\frac{e_0 a}{L},\ f=6\left(\frac{g}{H}\right)^2
\end{aligned}
\tag{21}
$$

By substituting the dimensionless parameters from Eq. (21) into Eqs. (17) and (18), the dimensionless type of the nonlinear equation of motion and the boundary conditions of the nano-beam behavior are obtained as:

$$
\begin{aligned}
& \zeta^{(4)}-\left(1-e^2\frac{d^2}{d\eta^2}\right)\left[f\int_0^1\left(\frac{\partial \zeta}{\partial \eta}\right)^2 d\eta\right]\frac{\partial^2 \zeta}{\partial \eta^2}= \\
& \left(1-e^2\frac{d^2}{d\eta^2}\right)\left(\frac{\alpha_n}{\left(1-\zeta\right)^n}+\frac{\beta}{\left(1-\zeta\right)^2}+\frac{\gamma\beta}{\left(1-\zeta\right)}\right)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
& \zeta\big|_{\eta=0}=0 \\
& \frac{\partial \zeta}{\partial \eta}\bigg|_{\eta=0}=0 \\
& -\frac{\partial^3 \zeta}{\partial \eta^3}-e^2\left[\frac{\partial \bar{q}}{\partial \eta}+f\left(\int_0^1\left(\frac{\partial \zeta}{\partial \eta}\right)^2 d\eta\right)\frac{\partial^3 \zeta}{\partial \eta^3}\right]\Bigg|_{\eta=1}=0 \\
& -\frac{\partial^2 \zeta}{\partial \eta^2}-e^2\left[\bar{q}+f\left(\int_0^1\left(\frac{\partial \zeta}{\partial \eta}\right)^2 d\eta\right)\frac{\partial^2 \zeta}{\partial \eta^2}\right]\Bigg|_{\eta=1}=0
\end{aligned}
\tag{23}
$$

In Eq. (22), for the Casimir and van der Waals forces, n assumes the values 4 and 3, respectively. Then, after obtaining a dimensionless form of governing equations of the nano-beam, the DQM is used and the pull-in parameters are calculated.
The main idea of the DQM is that the derivative of a function at a point in the domain can be approximated as the weighted summation of the function value at all points in the domain. Using this approximation, the differential equation is reduced to a series of algebraic equations. The number of algebraic equations depends on the number of sample points. In the DQM, the m-th derivative of the dimensionless transverse displacement ζ at each point such as η_i is approximated as follows [17,34]:

$$\frac{d^m\zeta}{d\eta^m} = \sum_{i=1}^{N} C_{ij}^{m}\zeta(\eta_j) \tag{24}$$

where C_{ij}^{m} is the weighted coefficient and N is the number of nodes. The values of η_i are obtained as follows:

$$\eta_j = \frac{1}{2}\left[1-\cos\frac{\pi(j-1)}{N-1}\right], \qquad \eta_1 = 0.0 \ , \quad \eta_N = 1.0 \tag{25}$$

Using Lagrange polynomial as the basic polynomial, the weighted coefficients for the first order derivative are obtained as:

$$\begin{aligned} &C_{ij}^{(1)} = \frac{\tilde{L}(\eta_i)}{(\eta_i - \eta_j)\tilde{L}(\eta_j)}, \\ &(i, j = 1, 2, ..., N \ , \ i \neq j) \\ &C_{ij}^{(1)} = C_{ii}^{(1)} = -\sum_{k=1}^{N} C_{ik}^{(1)}, \\ &(i, j = 1, ..., N; i \neq k \ , i = j) \end{aligned} \tag{26}$$

Where

$$\tilde{L}(\eta_i) = \prod_{j=1}^{N}(\eta_i - \eta_j), \ (\mathrm{i} \neq \mathrm{j}). \tag{27}$$

And, finally, the weighted coefficients for the higher order derivatives are defined as follows [33,34]:

$$C_{ij}^{(p+1)} = \sum_{k=1}^{N} C_{ik}^{(1)} C_{kj}^{(p)}, \ (\mathrm{p}=1,2,...) \tag{28}$$

Now, by substituting the above equations into Eq. (22), the equation of motion is changed into the following form through the DQM:

$$\begin{aligned} &C_{ij}^{(4)}\zeta_j - \left(\delta_{ij} - e^2 C_{ij}^{(2)}\right) \\ &\left\{\overline{q}_j + \left[f\sum_{k=1}^{N} C_k\left[C_{ij}^{(1)}\zeta_j(\eta_k)\right] O \left[C_{ij}^{(1)}\zeta_j(\eta_k)\right]\right]\left(C_{ij}^{(2)}\zeta_j\right)\right\} = 0 \end{aligned} \tag{29}$$

where $\mathbf{O}$ is the Hadamard Matrix product. Also, coefficients C_k are calculated using Newton-Cotes integration as follows [35,36]:

$$C_k = \int_0^1 \prod_{\substack{i=1 \\ i\neq k}}^{N} \frac{\eta - \eta_i}{\eta_k - \eta_i} d\eta \tag{30}$$

Here, to apply the boundary conditions of the clamped-free beam, the CBCGE method, initially proposed by Shu and Du [37], is used. According to this method, first, using Eq. (28), the boundary conditions in Eq. (23) are rewritten as follows:

$$\begin{aligned} &\zeta_1 = 0 \\ &C_{1,1}^{(1)}\zeta_1 + C_{1,2}^{(1)}\zeta_2 + C_{1,3}^{(1)}\zeta_3 + \\ &....+C_{1,N}^{(1)}\zeta_N = 0 \\ &C_{N,1}^{(3)}\zeta_1 + C_{N,2}^{(3)}\zeta_2 + C_{N,3}^{(3)}\zeta_3 + ...+C_{N,N}^{(3)}\zeta_N + \\ &e^2\left(C_{N,1}^{(1)}\overline{q}_1 + C_{N,2}^{(1)}\overline{q}_2 + ...+C_{N,N}^{(1)}\overline{q}_N\right) \\ &+\left(e^2 \ s \ f\right)\left[C_{N,1}^{(3)}\zeta_1 + C_{N,2}^{(3)}\zeta_2 + C_{N,3}^{(3)}\zeta_3 + \right. \\ &\left. ....+C_{N,N}^{(3)}\zeta_N\right] = 0 \\ \\ &C_{N,1}^{(2)}\zeta_1 + C_{N,2}^{(2)}\zeta_2 + C_{N,3}^{(2)}\zeta_3 + ...+ \\ &C_{N,N}^{(2)}\zeta_N + e^2\overline{q}_N \\ &+\left(e^2 \ s \ f\right)\left[C_{N,1}^{(2)}\zeta_1 + C_{N,2}^{(2)}\zeta_2 + C_{N,3}^{(2)}\zeta_3 + \right. \\ &\left. ....+C_{N,N}^{(2)}\zeta_N\right] = 0 \end{aligned} \tag{31}$$

By resolving the dimensionless displacement vector ζ, the Eqs. (31) can be rewritten in the following matrix form:

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ C_{1,1}^{(1)} & C_{1,2}^{(1)} & C_{1,N-1}^{(1)} & C_{1,N}^{(1)} \\ C_{N,1}^{(2)} & C_{N,2}^{(2)} & C_{N,N-1}^{(2)} & C_{N,N}^{(2)} \\ C_{N,1}^{(3)} & C_{N,2}^{(3)} & C_{N,N-1}^{(3)} & C_{N,N}^{(3)} \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_{N-1} \\ \zeta_N \end{bmatrix} +
\begin{bmatrix} 0 & 0 & \dots & 0 \\ C_{1,3}^{(1)} & C_{1,4}^{(1)} & \dots & C_{1,N-2}^{(1)} \\ C_{N,3}^{(2)} & C_{N,4}^{(2)} & \dots & C_{N,N-2}^{(2)} \\ C_{N,3}^{(3)} & C_{N,4}^{(3)} & \dots & C_{N,N-2}^{(3)} \end{bmatrix} \begin{bmatrix} \zeta_3 \\ \zeta_4 \\ \vdots \\ \zeta_{N-2} \end{bmatrix} +
$$

$$
\left(e^2\ s\ f\right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C_{N,1}^{(2)} & C_{N,2}^{(2)} & C_{N,N-1}^{(2)} & C_{N,N}^{(2)} \\ C_{N,1}^{(3)} & C_{N,2}^{(3)} & C_{N,N-1}^{(3)} & C_{N,N}^{(3)} \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_{N-1} \\ \zeta_N \end{bmatrix} +
e^2 \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ C_{N,1}^{(1)} & C_{N,2}^{(1)} & \dots & C_{N,N}^{(1)} \end{bmatrix} \begin{bmatrix} \bar{q}_1 \\ \bar{q}_2 \\ \vdots \\ \bar{q}_N \end{bmatrix} +
$$

$$
\left(e^2\ s\ f\right) \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ C_{N,3}^{(2)} & C_{N,4}^{(2)} & \dots & C_{N,N-2}^{(2)} \\ C_{N,3}^{(3)} & C_{N,4}^{(3)} & \dots & C_{N,N-2}^{(3)} \end{bmatrix} \begin{bmatrix} \zeta_3 \\ \zeta_4 \\ \vdots \\ \zeta_{N-2} \end{bmatrix} = 0 \qquad (32)
$$

where

$$
s = \int_0^1 \left(\frac{\partial \zeta}{\partial \eta}\right)^2 d\eta = \sum_{k=1}^{N} C_k \left[C_{ij}^{(1)} \zeta_j(\eta_k)\right] \mathrm{O} \left[C_{ij}^{(1)} \zeta_j(\eta_k)\right] \qquad (33)
$$

By defining matrices of coefficients B_A, B_B, B_C and B_D, Eq. (32) is written as follows:

$$
B_A \Xi^{(A)} + B_B \Xi^{(B)} + \left(e^2\ s\ f\right) B_C \Xi^{(A)} + \left(e^2\ s\ f\right) B_D \Xi^{(B)} = -E\{\bar{q}\} \qquad (34)
$$

where B_A and B_B are 4×4 order matrices, and B_B and B_D are (N-4)×4 order matrices. Also, vectors $\Xi^{(A)}$ and $\Xi^{(B)}$ are as follows:

$$
\Xi^{(A)} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_{N-1} \\ \zeta_N \end{bmatrix}, \ \Xi^{(B)} = \begin{bmatrix} \zeta_3 \\ \zeta_4 \\ M \\ \zeta_{N-2} \end{bmatrix} \qquad (35)
$$

Now, the equation of motion (29) can be written as a matrix form, yielding the following equation:

$$
C^{(4)} \Xi - T(\bar{q} + H) = 0 \qquad (36)
$$

where

$$
T_{ij} = \delta_{ij} - e^2 C_{ij}^{(2)}\ , \ H_i = f \times s \times \left[C_{ij}^{(2)} \zeta_j\right], \quad \Xi_i = \zeta_i \qquad (37)
$$

Like the equations of boundary conditions, the equation of motion in Eq. (36) can be rewritten based on the two displacement vectors $\Xi^{(A)}$ and $\Xi^{(B)}$ as follows:

$$
D_A \Xi^{(A)} + D_B \Xi^{(B)} - T(\bar{q} + H) = 0 \qquad (38)
$$

where D_A is a 4×4 matrix and D_B is a N×(N-4) matrix. By substituting the equations of boundary conditions into Eq. (34) and equations of motion in Eq. (38), and using linear algebraic theorems, the governing equations of the beam as well as the clamped-free boundary conditions are obtained through the DQM as follows:

$$
\left\{D_D - D_B\left[B_A + \left(e^2\ s\ f\right)B_C\right]^{-1}\left[B_B + \left(e^2\ s\ f\right)B_D\right]\right\}\Xi^{(B)} - \left\{T + D_B\left[B_A + \left(e^2\ s\ f\right)B_C\right]^{-1} E\right\}\bar{q} - TH = 0 \qquad (39)
$$

The above equation can be summarized as:

$$
A\Xi^{(B)} = G(\bar{q}) + M \quad , \quad M = TH \qquad (40)
$$

where A and G are modified matrices of the coefficients considering clamped-free boundary conditions.

To solve Eq. (40), one must solve a system of two equations and two unknowns. Given that vector $\{\bar{q}\}$ is a function of the displacement vector Ξ, one has to use an iteration algorithm such as the Newton-Raphson algorithm in programming software. The following is a summary of the process used in the present research to solve the nonlinear pull-in instability problem.

1. For a certain value of β, with an initial assumption for vector Ξ, the values of vector $\{\bar{q}\}$ and M are determined. Then, the nonlinear Eq. (40) becomes a linear equation, and by solving this equation, the new displacement vector Ξ is calculated.
2. By substituting the displacement vector resulting in stage 1 at the beginning of the cycle, vectors $\{\bar{q}\}$ and M are modified and hence, at each point of the cycle, the displacement of points is updated.

3. Stages 1 and 2 are repeated until the following convergence condition is satisfied[33].

$$\sqrt{\frac{\Sigma\left((\Delta\Xi_i)^{(k)}\right)^2}{\Sigma\left((\Xi_i)^{(k+1)}\right)^2}} \leq 0.0001 \qquad (41)$$

where $k$represents the number of iteration of the Newton-Raphson method and Δ stands for the degree of variation of the components of the new displacement vector compared with the displacement vector of the previous stage.

4. The value of the electrostatic force βapplied is increased and stages 1 to 3 are repeated for new input values. The increase of β is repeated until the slope of the nano-beam displacement curve in relation to the dimensionless voltage β approaches infinity, i.e. $\frac{d\zeta}{d\beta} \to \infty$. The last value of the electrostatic force represents voltage β^{PI} and the related displacement represents ζ^{PI}.

5. Results and discussion

This section is devoted to the investigation of different parameters affecting the nonlinear pull-in instability of the clamped-free beam-type nano-beam. First, the effects of geometrical dimensions are examined and then, the small scale or the nonlocal theory is investigated. Afterward, the effects of Casimir and van der Waals forces on the pull-in instability of the nano-beam are evaluated. After that, to consider the intermolecular forces, the freestanding behavior in the nano-beam is studied and finally, the validity of the results of the present research and other studies, as well as experimental results,are evaluated. It should be noted that in the diagrams and figures presented in the following sections, considering Eq. (21), the dimensionless parameters ζ, η, α_3, α_4, β, and e stand for nano-beam deflection, the length of nano-beam, van der Waals force, Casimir force, applied voltage, and small scale in the nonlocal theory, respectively.

5.1. Effect of geometrical dimensions

In Fig. 2, the nano-beam deformation is displayed based on the different values of the applied voltage β. As the voltage increases, the values of electrostatic force and beam displacement increase, too. It can be seen from Fig. 2 that as the applied voltage reaches up to $\beta=1.118$ if the linear model is used, the movable electrode collapses on the fixed electrode and the pull-in modification takes place, whereas if the same voltage is applied in the nonlinear model, the pull-in modification does not occur. In other words, it could be argued that considering the nonlinear geometric effects leads to an increase in the pull-in voltage. It should be noted that in the nonlinear case, the value of $f$is taken as 37.5.

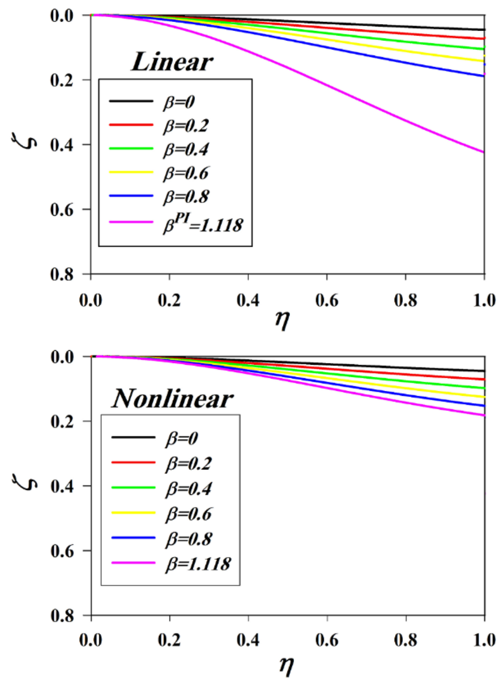

Fig. 2. Beam deformation for different values of β for g/b=α_3=0.5, e=0.3.

As regards the equation of motion, two geometrical parameters appear in this equation, showing the geometrical conditions governing the problem. The first parameter is the ratio of separation gap to the beam width, i.e. g/b. This ratio is the second term coefficient of the electrostatic force, known as fringing field effect. In fact, this parameter is a criterion which can be altered to compare the instability behavior of the narrow beam with that of the wide beam. The second parameter is ‘f’ which, as mentioned before, is entered the equation of motion due to the consideration of von-Karman-type nonlinear strain. Figure 3 simultaneously illustrates the effects of these two parameters on beam deformation.As displayed by Fig. 3, as the value of g/b increases or the beam narrows, the force resulting from the fringing fields increases, and consequently, the nano-beam develops a greater tendency for deformation. In other words, the beam stiffness has a reverse relationship with the value of this ratio. However, by contrast, the value of ‘f’ reduces beam deformation; therefore, this value has a direct relationship with the beam stiffness.

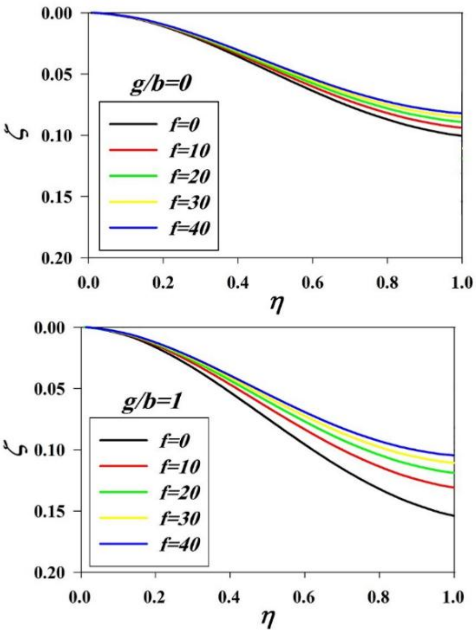

Fig. 3. Effects of fringing fields and the coefficient of nonlinear effects on beam deformation, assuming α_4=β=0.7 and e=0.4.

In Fig. 4 and 5, the effects of two geometrical parameters on the pull-in voltage are illustrated by considering the Casimir and van der Waals forces, respectively. As illustrated by Fig. 4, as the g/b value increases or the beam narrows, the pull-in voltage decrease and beam deformation increases. Also, as can be seen in Fig. 4, the higher the value of the small scale, the higher the pull-in voltage. As illustrated in Fig. 5, in the presence of the van der Waals force, the increase in ‘f’ leads to an increase in the pull-in voltage, and, this increase is lower in narrow beams.

5.2. Effect of small scale

Results of experiments carried out by different researchers demonstrate that in nanoscale, the mechanical properties of materials are size-dependent. Therefore, this section is devoted to the effect of the small scale on the pull-in parameters in beam-type nano-switches. Figure6 shows the effect of the size parameter on nano-beam deformation in the two linear and nonlinear models. As illustrated, an increase in the size parameter leads to a decrease in beam deformation. In other words, the size parameter has a direct relationship with beam stiffness. Comparison of linear and nonlinear displacements in Fig. 6 reveals that in the nonlinear case, the beam has less displacement and stiffer behavior than in the linear case.

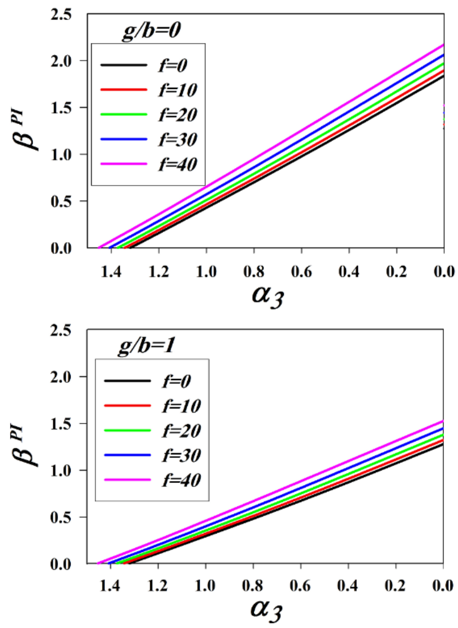

Fig. 5. Comparison of nonlinear β^{PI} for different values of g/b and ‘f’ with the assumption of n=3, e=0.2.

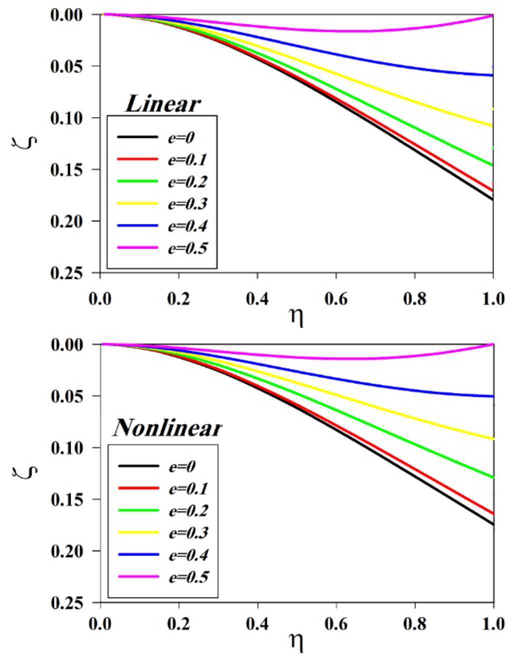

Fig. 6. Nano-beam deformation for different values of ‘e’ for α_4=0.3, g/b=β=0.5.

Figure 7 displays the effect of the size parameter on the pull-in voltage in linear and nonlinear models. As illustrated, nonlinear displacements cause the nano-beam to store more strain energy in itself and to have higher pull-in voltage. Besides, Fig. 7 shows that in both linear and nonlinear cases, an increase in the small scale is accompanied by an increase in the pull-in voltage, and, an increase in the value of the van der Walls force leads to a decrease in the pull-in voltage.

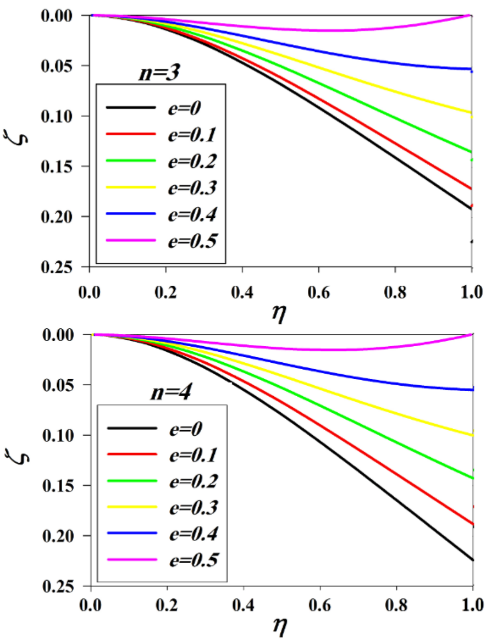

Fig. 7. Effect of size parameter on β^{PI} for differentvalues of ‘e’ with the assumption of g/b=0.

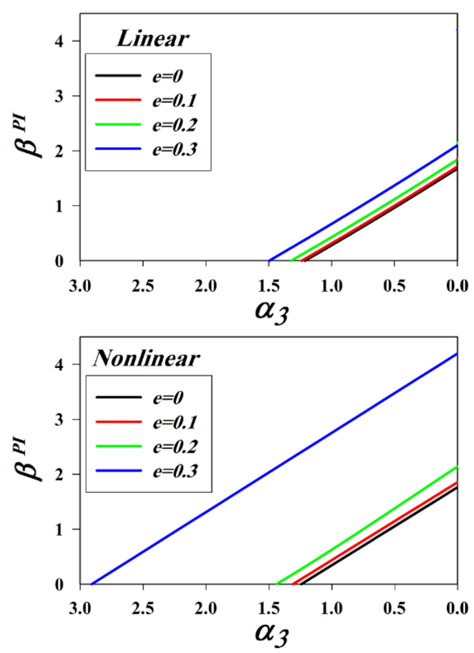

Fig. 8. Comparison of nonlinear deformation of nano-beam under the influence of Casimir and van der Waals intermolecular forces for different values of ‘e’ with the assumption of α=0.3, β=g/b=0.5 and f=37.5.

5.3. Effect of intermolecular forces

As mentioned before, in the nanoscale, the two Casimir and van der Waals forces influence the nano-beam behavior. However, depending on the separation gap size, one of the two forces gains significant. Researchers usually consider the van der Waals force as the dominant force in distances below 10 nm [38-40] and the Casimir force in distances above 20 nm. The attraction resulting from these forces causes the pull-in phenomenon to occur at a lower voltage.As can be seen in Fig. 8, when the van der Waals force is considered as the intermolecular force, the nano-beam undergoes a lower deformation than the case where the Casimir force is considered as the intermolecular force.The effect of intermolecular forces on β^{PI} is very significant. Intermolecular forces decrease the pull-in voltage of the nano-switch, as shown in Fig. 9.

It can be understood from this illustration that given the same amount of α_3 and α_4, the Casimir force decreases the pull-in voltage more than the van der Walls force and has a greater effect than the van der Walls force.

Figure 10 show the deformation at the nano-beam end ζ_{Tip}as a function of the dimensionless voltage β for different values of intermolecular forces in linear and nonlinear cases. Two important points can be implied from this illustration. First, the nano-beam experiences greater displacement in the presence of the Casimir force than in the presence of the van der Waals force. In other words, the effect of the Casimir force is greater than that of the van der Waals force. Second, to consider the nonlinear effects in the equations causes the decrease of the highest nano-beam deflection, compared to the linear case.

5.4. The freestanding behavior

Of the significant measures in the design of nano-sensors and nano-switches is the investigation of their freestanding behavior. Given the fact that in the nanoscale, forces such as Casimir and van der Waals forces possess a magnitude comparable to the electrostatic force, there is the possibility that the nano-beam undergoes pull-in instability even in the absence of the electrostatic force. This takes place when either the gap between the fixed and the movable electrodes is narrow, or the nano-beam length is longer than a certain limit. Thus, the intermolecular force is high enough to cause the movable electrode to collapse on the fixed electrodes. This is known as the investigation of freestanding behavior in nano-switches which shows in figure 11. By knowing the value of the intermolecular force causing this condition, one can determine the gap between the electrodes and the critical length of the nano-beam.

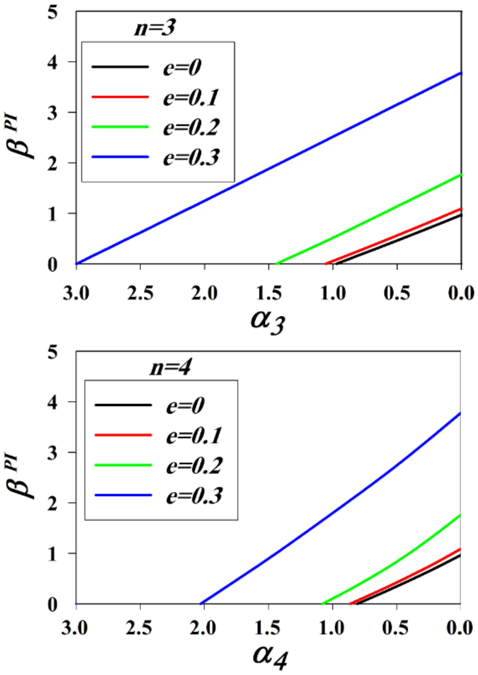

Fig. 9. Effect of intermolecular forces on the pull-in voltage of the nano-beam for g/b=0.5 and f=37.5.

5.5. Validation of results

In order to verify the validity of the results of the present research, they are compared with those reported by other researchers in two sections. In the first section, the results obtained through the DQM are compared with those obtained by other researchers using other methods. In the second section, a comparison is made between the results of this study and experimental results. Given the fact that previous researchers have investigated the pull-in and beam-type nano-switches usually through the classical model and linear model and without the use of the effects of intermolecular forces, here the results are compared in three consecutive, distinct tables.

Table 1 displays the geometrical parameters used to analyze wide and narrow nano-beams. These parameters are used in the analysis of the following tables.

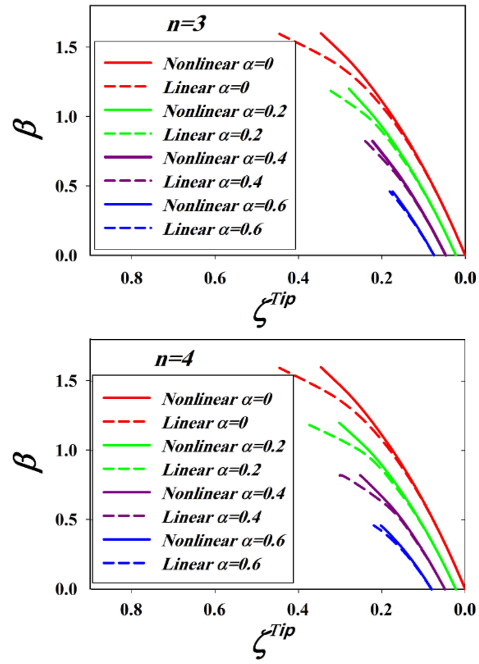

Fig. 10. Effect of intermolecular forces on displacement at the nano-beam end in linear and nonlinear cases.

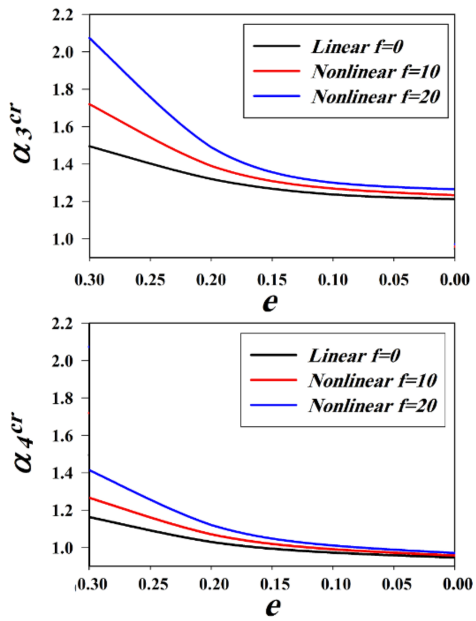

Fig. 11. Values of α_{cr} according to the size parameter for g/b=1 and different values of 'f'.

Table 1. Geometric parameter of nano-beam.

Case	Dimensions (μm)			
	L	b	h	g
Wide beam	300	50	1	2.5
Narrow beam	300	0.5	1	2.5

Table2. Comparison of values of pull-in voltage β^{PI} using different methods.

Model	β^{PI}	
	Narrow beam	Wide beam
DQM (present linear model)	1.2	2.15
Numerical	1.24	2.27
M-test[41]	1.23	2.25
Closed-form Model [42]	1.21	2.27
HPM[23]	1.21	2.16
MAD [6]	1.27	2.31

Given the fact that in some of the studies mentioned in the references, the value of dimensionless deformation at the nano-beam end in the pull-in moment is presented, in Table 3, like in Table 2, the results for the values of $\zeta^{PI}$are compared when the effects of the intermolecular forces and the size parameter are ignored and the linear model is used. As can be seen in Table 3, the results of the DQM have appropriate consistency with those of other methods. On the other hand, in the case of the nonlinear model, the nano-beam has a lower maximum deformation than in the case of the linear model, which is consistent with the results obtained in previous sections.

Table 3. Comparison of values of dimensionless deformation at the nano-beam tip (ζ^{PI}) in different methods.

Model	ζ^{PI}	
	g/b=0	g/b=1
DQM (present linear model)	0.429	0.463
DQM (present nonlinear model)	0.418	0.451
DQM [43]	0.436	0.478
LDL [22]	0.333	0.369
Numerical	0.436	0.478

Table 4. Comparison of values of pull-in voltage (β^{PI}) with the presence of Casmir force.

Model	Linear		Nonlinear	
	e=0.1	e=0.2	e=0.1	e=0.2
DQM (present model)	0.492	0.738	0.532	0.843
DQM [43]	0.497	0.742	-	-
Numerical	0.497	0.742	-	-
LDL [22]	0.452	0.637	-	-

In Table 4, the results obtained for the pull-in voltage (β^{PI}) through the DQM in this paper are compared with the results obtained in the references in the presence of the Casimir force. As can be seen, the results of the DQM have appropriate consistency with the results of other methods with the presence of the Casmir force; and, applying the linear model enhances the prediction of the pull-in voltage.

Finally, in order to evaluate the results of the DQM using the nonlocal theory as well as the linear model used in the present research, the results of this study are compared to the experimental results included in the references [41]. The geometrical properties of the nano-beam are displayed in Table 5 according to this reference.

Table 5. Geometric parameter of nano-beam [41].

Material	Dimentions(μm)			
	L	h	b	g
Silicon-110 direction	250	2.94	50	1.05

Figure12 compares the variations of the pull-in voltage with the experimental results according to the nano-beam length and based on the classical elasticity theory and the nonlocal theory. As is clear from the illustration, by assuming e=0.3037, the results of the nonlocal theory have the best consistency with the experimental results. It can be argued that by considering the nonlocal theory and size effects in the nano-switch, the gap between the classical theory and experimental results is filled, and this can prove the effectiveness of the use of non-classical theories in nanostructures.

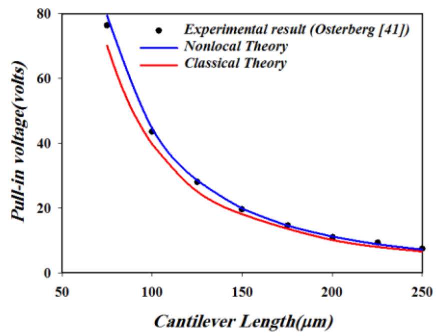

Fig. 12. Comparing the theoretical and experimental pull-in voltages for Silicon-110.

6. Conclusions

This paper investigated the static instability of a clamped-free nano-switch under the influence of electrostatic forces and intermolecular forces such as Casimir and van der Waals attractions based on Euler-Bernoulli’s model and the nonlocal elasticity theory by considering von-Karman-type nonlinear strain. The principle of minimum potential energy was used to derive the governing equations of motion of the system, and the governing equations derived through the DQM were solved so as to derive significant pull-in parameters such as the pull-in voltage and displacement of the beam tip, and to investigate the effect of intermolecular forces, geometrical dimensions, small effect, fringing field, and von-Karman-type nonlinear strain on the static instability behavior of nano-switches.

The results demonstrate that the presence of intermolecular forces and fringing fields causes a reduction in the pull-in voltage, and, among the intermolecular forces in the nanoscale, the Casimir force plays a more significant role than the van der Waals force in reducing the pull-in voltage. In comparison with linear analysis, von-Karman-type nonlinear strain reduces the displacement of the beam tip and increase the pull-in voltage, particularly for higher *(g/H)* values. In other words, the nano-beam has stiffer behavior in the non-linear case than in the linear case. Finally, it can be argued that by taking into consideration the nonlocal theory or the size effect in the nano-switch, the gap between the classical theory and experimental results is filled. This can prove the effectiveness of the use of non-classical theories in nanostructures.

Appendix: the material length scale parameters in nonclassical continuum theory

It is well-established that mechanical behaviors of micro/nanostructure are size dependent. Experiments reveal an increase in materials characteristics with decreasing the size at the ultra-small scales. All these experiments imply that when the characteristic size (thickness, diameter, etc.) of a micro/nanoelement is in the order of its intrinsic the material length scales (typically sub-micron), the material elastic constants highly depend on the element dimensions.

The source of difference between the mechanical properties of ultra-small and bulk materials with the same composition can be attributed to several physical phenomena such as differences in structure, deformation, or fracture mechanisms [44, 45]. The differences typically occur when the material dimensions reach characteristic length scales that are associated with defect dimensions such as dislocation, spacing and grain size [46]. At nanoscale level, the gradient deformations vary sharply, hence the microscopic stresses and strains are not constant and depend on the shrinking length scale of the nanostructures: the smaller the structure, the more rapid the microscopic fields vary, and they do so in a way that leads to either stiffening or softening of the material [47]. In order to model these gradient effects, higher-order continuum theories such as strain gradient theory, couple stress theory, surface elasticity [48,49] and nonlocal theory [50,51] are introduced with length scale parameters. It should be noted that there is no comparative study on the properties of nanostructure based on classical, nonlocal, couple stress and strain gradient elasticity theories. But some study shows that the material properties predicted by the nonlocal elasticity theory are smaller than those by the classical elasticity theory and an opposite trend is observed for the predicted by the couple stress or strain-gradient theories. All theories converge to the classical elasticity theory as the nanostructure global dimension increases.

The material length scale parameters also might be determined via molecular dynamic simulation or experiments. Previous researchers used atomistic simulations and molecular dynamics to determine the size effect parameters [52]. Maranganti and Sharma [52] used an atomistic approach to determine strain gradient elasticity constants of structures. They present mathematical derivations that relate the strain-gradient material constants to atomic displacement correlations in a molecular dynamics computational ensemble. The elastic constants are explicitly determined for some representative semiconductor, metallic, amorphous and polymeric materials. Moreover, these parameters can also be determined using mechanical tests [52]. As mentioned above, several methods such as atomistic approach and experimental used to determine the microscale parameters in nonclassical continuum theories.

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