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Analytical study of nano-bioconvective flow in a horizontal channel using Adomian decomposition method

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Abstract

In this paper, the bioconvective nanofluid flow in a horizontal channel was considered. Using the appropriate similarity functions, the partial differential equations of the studied problem resulting from mathematical modeling are reduced to a set of non-linear differential equations. Thereafter, these equations are solved numerically using the fourth order Runge-Kutta method featuring shooting technique and analytically via the Adomian decomposition method (ADM). This study mainly focuses on the effects of several physical parameters such as Reynolds number ($\boldsymbol{Re}$), thermal parameter ($\boldsymbol{\delta_\theta}$), microorganisms density parameter ($\boldsymbol{\delta_s}$) and nanoparticles concentration ($\boldsymbol{\delta_\phi}$) on the velocity, temperature, nanoparticle volume fraction and density of motile microorganisms. It is also demonstrated that the analytical ADM results are in excellent agreement with the numerical solution and those reported in literature, thus justifying the robustness of the adopted Adomian Decomposition Method.

Nomenclature

a and b	Constants
$a_0,.a_7$	Constants
A_n	Adomian polynomials
C	Volumetric fraction of nanoparticles
D_b	Brownian diffusion coefficient
D_n	Diffusivity of microorganisms
D_T	Thermo-phoretic diffusion coefficient
ψ	Stream function
F	Dimensionless velocity
θ	Dimensionless temperature

Greek symbols

η	Non-dimensional angle
α	Thermal diffusivity
ζ	Vorticity function
γ	Constant
$\delta_\theta, \delta_\phi, \delta_\phi, \delta_s$	Constants
ρ_f	Fluid density
ρ_s	Solid nanoparticles density
ϕ	Dimensionless nanoparticle volume fraction
S	Dimensionless density of motile microorganisms

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P	Pressure
N	density of motile microorganisms
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
Pr	Prandtl number
Pe_b	Péclet number in bioconvection application
Re	Reynolds number
ℓ	Distance
L_e	Lewis number
L_i	Derivative operator
${L_i}^{-1}$	Inverse derivative operator
u, v	Velocity components along x- and y-direction
S_c	Schmidt number
$V,$	Velocity
$\hat{V}$	Velocity vector
T	Temperature
T_0	Reference temperature
W_c	Maximum cell swimming speed
μ_{nf}	Dynamic viscosity of nanofluid
ν	Kinematic viscosity
∂	Derivative operator
Subscript	
$\blacksquare_{nf}$	Nanofluid
$\blacksquare_{f}$	Base fluid
$\blacksquare_{s}$	Solid nanoparticles
Abbreviation	
$RK4$	Fourth order Runge-Kutta Method
ADM	Adomian Decomposition Method
$MLSM$	Modified Least Square Method

1. Introduction

Nowadays, it is well established that the nano-fluids play an important role in many domestic and industrial applications [1]. This novel category of fluids is created by the dispersion of the particles of nanometric size such as: Cu, Al_2O_3, SiC, ... in a base fluid like water. Nano-fluids are very useful for thermal systems due to the higher thermal conductivity of solid nanoparticles when compared to that of the base fluid. The "nano-fluid" term was firstly proposed by Choi [2-3] since 1995. Subsequently, nano-fluids were characterized by several researchers experimentally [4-6] and theoretically [7-9]. Due to their superior thermal properties, Huminicet al. [10] have given an interesting review on the applicability of nano-fluids in thermal systems, especially in heat exchangers. Cheng and Minkowycz [11] investigated the problem of natural convection around a vertical flat plate drowned in a highly saturated porous medium. Nield and Kyznestov [12] proposed a mathematical model that characterizes the Cheng-Minkowycz problem for natural convective boundary-layer flow in a porous medium under the effect of a nano-fluid. Pourmehran et al. [13] mathematically characterized the convective hydromagnetic nano-fluid flux on a vertical plate under simultaneous effect of thermal radiation and buoyancy via differential equations. In their study, the governing equations were solved numerically by the fourth-order Runge-Kutta method with the firing technique. Furthermore, the study shows the effect of various physical parameters such as magnetic parameter, nanoparticle size, nanoparticle concentration and radiation parameter on dynamic and thermal profiles, as well as on the Skin friction and Nusselt number. Buongiorno [14] attemped to develop a mathematical model that characterizes the convective transport in nanofluids and . The study gives momentum and heat and mass transfer equations. Kuznetsov [15] considers the mobility effect of microorganisms. The obtained mathematical model was solved numerically by the Galerkin method. In another work, Hang Xu et al. [16] analyzed the bio-convection flow in a horizontal channel under the effect of microorganisms' mobility. The problem was solved analytically using an improved homotopy analysis technique. Das et al. [17] were interested in the bioconvection nanofluid flow in a porous medium. The effect of various physical parameters such as Brownian motion, thermophoresis, bio-convection of gyrotactic microorganisms and chemical reaction was investigated. The study done by by Ghorai et al. [18] employed the finite difference method to solve the mathematical model provided by the combination of the Navier-Stokes equation and microorganisms' conservation equation. Kuznestov [19] reviewed the new developments in bio-convection in a fluid-saturated porous medium caused by either gyrotactic or oxytactic microorganisms. The Galerkin method was used to solve a linear stability of bio-thermal convection that generates correspondence between the value of the bio-convection

Rayleigh number and the traditional thermal Rayleigh number. Mosayebidorcheh etal [20] studied the convective flow of a nano-fluid in a horizontal channel with the presence of gyrotactic microorganisms analytically. The mathematical model proposed by Nield and Kuznetsov was solved analytically by the Modified Least Square Method (MLSM). Particular attention was paid to the effects of various physical parameters on the evolution of velocity, temperature, nano-particles volume fraction and the density of motile microorganisms. Ramly et al. [21] studied the axisymmetric thermal radiative boundary layer flow of nanofluid over a stretched sheet. They investigated the effects of zero and nonzero fluxes on the thermal distribution and volumetric fraction of nanoparticles. Ramly et al. [22] also investigated the natural convection flow of nanofluid in Cheng-Minkowycz problem along a vertical plate. Rizwan Ul Haq et al. [23] investigated the fully developed squeezing flow of water functionalized magnetite nanoparticles between two parallel disks numerically. Three types of nanoparticles having better thermal conductivity: Magnetite (Fe3O4), Cobalt ferrite ($CoFe_2O_4$) and Mn–Zn ferrite ($Mn\text{–}ZnFe_2O_4$) are added to the water base fluid.

In recent decades, several methods were developed in order to solve the nonlinear initial or boundary values problems analytically, such as the Adomian Decomposition Method (ADM), the Homotopy Analysis Method (HAM) and the Variational Iteration Method (VIM). The concept of decomposition method pioneered by George Adomian [24] has been efficiently used by several researchers [25-27]. Also, the Adomian decomposition method coupled with Padé approximants is employed by Noor [28-30] for the resolution of linear and nonlinear differential equations. Generally, Adomian Decomposition Method gives the solution in the form of a polynomial series and can be accurately applied without linearization, discretization or digital processing.

In the current study, Adomian decomposition method (ADM) is successfully applied to solve the nonlinear problem of nano bio-convective flow between two parallel plates. In fact, we were particularly interested on the evolution of velocity F (η), temperature θ (η), nano-particle volume fraction ϕ (η) and density of motile microorganisms s (η) under the effects of several physical parameters such as thermal parameter (δ_θ), nanoparticles concentration (δ_ϕ), microorganismas density parameter (δ_s) and "N_t/N_b" ratio.

2. Formulation of the problem

The heat transfer by convection in a fully developed flow of a nano-fluid between two parallel planar plates separated by a distance $2\boldsymbol{\ell}$ was considered. Fig. 1 shows the geometry of the investigated flow.

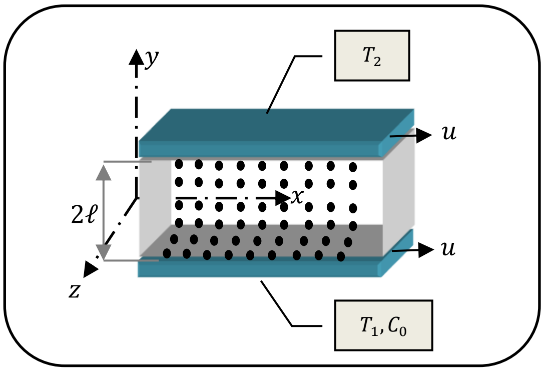

Fig. 1. The geometry of the flow channel.

As drawn in Fig. 1, the "Oy" axis is perpendicular to the walls, while the center of the channel is directed along the "Ox" axis. The two walls (lower and upper) move (are stretched) at a speed of the form ($u = ax$). The temperature at the walls is assumed to be constant. T_1 and T_2 represent the temperatures of the lower and upper walls respectively. Moreover, the distribution of the nanoparticles at the base of the channel (lower wall) is assumed to have a constant C_0 value. For the considered nano-fluid, the basic fluid is water. A stable suspension of non-accumulating nanoparticles was considered.

Taking into account the above assumptions, the continuity equation, the momentum equation, the energy equation, the nanoparticle volume fraction equation and diffusion equation, as suggested by Kuznestov and Nield [31], can be expressed as follows :

$$\nabla V = 0 \tag{1}$$

$$\rho_f (V\, \nabla) V = -\nabla P + \mu \nabla^2 V \tag{2}$$

$$V \nabla T = \alpha \nabla^2 T + \tau \left[D_B \nabla T \nabla C + \left({D_T}/{T_0} \right) \nabla T \nabla T \right] \tag{3}$$

$$(V\,\nabla)C = D_B\nabla^2 C + \left(D_T/T_0\right)\nabla^2 T \quad (4)$$

$$\nabla . j = 0, \quad (5)$$

where :

V is the velocity of the flow (function of u in the direction Ox and v in the direction Oy). The terms P and T represent the pressure and the temperature respectively. The constant C characterizes the volumetric fraction of the nanoparticles, D_B is the Brownian diffusion coefficient and D_T is the thermo-phoretic diffusion coefficient. T_0 is a reference temperature.

The density of a nanofluid is estimated by the parameter ρ_f and the dynamic viscosity of nanofluid suspension is characterized by the term μ. The parameter τ characterizes the ratio $((\rho c)_p/(\rho c)_f)$ of thermal capacities of nanoparticles and base fluid. The term α represents the thermal diffusivity of the nano-fluid.

Brownian motion is a random motion of particles suspended in the fluid as a consequence of quick atoms or molecules collision [32]. Thermophores refers to the transport of particles resulting from the temperature gradient [33]. j is another parameter defined according to the fluid convection, self-propelled swimming, and diffusion.

$$j = Nv + N\hat{v} - Dn\nabla N, \quad (6)$$

Now, by introducing the parameters $\hat{v} = (\frac{bW_c}{\Delta C})\nabla C$ into the Eq. (6),we obtain:

$$j = Nv + N(\frac{bW_c}{\Delta C})\nabla C - Dn\nabla N \quad (7)$$

Where :

N : density of motile microorganisms.

$\hat{v}$: velocity vector related to the cell swimming in nano-fluids.

Dn : diffusivity of microorganisms.

b and W_c : represent the constant of chemotaxis and the maximum cell swimming speed respectively.

For a two dimensional flow, the Eq. (2) in Cartesian coordinates can be expressed as follows :

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (8)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (9)$$

The parameter $\nu = \frac{\mu}{\rho_f}$ is the kinematic viscosity of the nano-fluid.

Moreover, to simplify the Eqs. (1-4), the following equation is used:

$$\zeta = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} = -\nabla^2\psi \quad (9)$$

Where ζ is the vorticity function

Taking into account Eqs. (10, 1, 2, 3 and 4) become:

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (11)$$

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \alpha\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right) \quad (12)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\nabla T\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \left(\frac{D_T}{T_0}\right)\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right)\right] \quad (13)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_0}\right)\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + \frac{\partial}{\partial y}(N\hat{v}) = D_N\left(\frac{\partial^2 N}{\partial x^2}\right)$$

With the relevant boundary conditions:

$$\frac{\partial u}{\partial y} = 0, v = 0 \text{ at } y = 0 \quad (16.a)$$

$$u = ax, v = 0, T = T_2, D_B\frac{dC}{dy} + \frac{D_T}{T_0}\frac{dT}{dy} = 0 \text{ at } y = \ell \quad (16.b)$$

It is very important to normalize the equations of the investigated flow. To achieve this goal, we consider the dimensionless variables $F(\eta), \theta(\eta), \phi(\eta)$ and $S(\eta)$ defined by:

$$\eta = y/\ell \;; \quad \phi(x,y) = ax\ell F(\eta)\;; \quad \theta(\eta) = \frac{T - T_0}{T_2 - T_0}\;; \quad \phi(\eta) = \frac{C - C_0}{C_0} \quad (17)$$

$$S(\eta) = \frac{N}{N_2}$$

Considering terms of Eq. (17), Eqs. (12-15) become:

$$F'''' + R_e(FF''' + 4\alpha^2 F'F'') = 0 \quad (18)$$

$$(\theta'' + R_e P_r F\theta' + N_b\theta'\phi' = 0 \quad (19)$$

$$\phi'' - \frac{N_t}{N_b}\theta'' - R_e L_e F\phi' = 0 \quad (20)$$

$$s'' - Pe_b(\phi' s' + s\phi'') + R_e S_c F s' = 0 \quad (21)$$

The Boundary conditions are:

At $\eta = -1 \rightarrow \theta(-1) = \delta_\theta$, $\phi(-1) = \delta_\phi$ and $s(-1) = \delta_s$ (22-a)

At $\eta = 0 \rightarrow F(0) = 0$ and $F''(0) = 0$ (22-b)

At $\eta = +1 \rightarrow F(1) = 0, F'(1) = 1, \theta(1) = 1, \phi'(1) + \gamma\theta'(1) = 0$ and $s(1) = 1$ (22-c)

The dimensionless numbers represented in Eqs. (18-21) are given as:

Reynolds number : $R_e = \frac{a.L^2}{\nu}$

Prandtlnumber : $P_r = \frac{\nu}{a}$

Parameter of Brownian motion : $N_b = \frac{\tau.D_B.C_0}{\alpha}$

Thermophoresis parameter: $N_t = \left(\frac{D_T}{T_0}\right)\frac{T_2 - T_0}{\alpha}$

Lewis number : $L_e = \frac{\nu}{D_B}$

Schmidt number : $S_c = \frac{\nu}{D_n}$

Péclet number in bioconvection application:

$$Pe_b = \frac{b\mathrm{Wc}}{D_n}$$

Constant $\gamma = \frac{N_t}{N_b}$

3. Adomian decomposition method

In this section, we present the basic principle of Adomian decomposition method. Consider the following nonlinear differential equation:

$$L(y) + N(y) = f(t) \quad (23)$$

Where:

$L = \frac{d^n \blacksquare}{dx^n}$ isthe n-order derivative operator, N is a nonlinear operator and f is a given function.

Assume that L^{-1} is an inverse operator that represents n-fold integration for an n-th order of the derivative operator L. Applying the inverse operator $\mathbf{L}^{-1}$ to both sides of (Eq. (23)) yields:

$$\begin{cases} L^{-1} = \iint \ldots\ldots \int \blacksquare dx^n \\ L^{-1}L(y) = L^{-1}f - L^{-1}N(y) \end{cases} \quad (24)$$

As a result, we obtain:

$$y = \beta + L^{-1}f - L^{-1}N(y) \quad (25)$$

Where β is a constant determined from the boundary or initial conditions.

Now, based on the Adomian decomposition procedure, the solution y of the Eq. (23) can be constructed by a sum of components defined by the following infinite series:

$$y = \sum_{n=0}^{\infty} y_n \quad (26)$$

Also, the nonlinear term is given as follows:

$$Ny = \sum_{n=0}^{+\infty} A_n(y_0, y_1, \ldots, y_n) \quad (27)$$

Where:

$$y_0 = \beta + L^{-1}f,\ y_{n+1} = -L^{-1}(A_n). \quad (28)$$

A_n'sare called the Adomian polynomials. The recursive formula that defines the Adomian polynomials [24] is given as follows:

$$A_n(y_0, y_1, \ldots, y_n) = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^i y_i\right)\right]\right]_{\lambda=0}, \quad n = 0,1,2, \ldots \quad (29)$$

Finally, after some iterations, the solution of the studied equations can be given as an infinite series by:

$$y \cong y_0 + y_1 + y_2 + y_3 + \cdots + y_n. \quad (30)$$

The Adomian decomposition method (ADM) is a powerful technique which provides efficient algorithms for several real applications in engineering and applied sciences. The main advantage of this method is to obtain the solution of both nonlinear initial value problems (IVPs) and boundary value problems (BVPs) as fast as convergent series with elegantly computable terms while it does not need linearization, discretization or any perturbation.

4. Implementing of ADM method

According to the principle of Adomian, Eqs. (18-21) can be written as:

$$L_1F = -R_e(FF''' + 4\alpha^2 F'F'') \quad (31)$$
$$L_2\theta = -R_eP_rF\theta' - N_b\theta'\emptyset' \quad (32)$$
$$L_3\emptyset = -(N_t/N_b)\theta'' - R_eL_ef\emptyset' \quad (33)$$
$$L_4s = +Pe_b(\emptyset's' + s\emptyset'') - (R_eS_c)fs' \quad (34)$$

Where differential operators (L_1, L_2, L_3 and L_4) Are given by: $L_1 = \frac{d^4F}{d\eta^4}$, $L_2 = \frac{d^2\theta}{d\eta^2}$, $L_3 = \frac{d^2\emptyset}{d\eta^2}$ and $L_4 = \frac{d^2S}{d\eta^2}$

The parameters L_1, L_2, L_3 and L_4 are the differential operators. The inverses of these operators are expressed as:

$$\begin{cases} L_1^{-1} = \iiint_0^{\eta}\int \boldsymbol{F d\eta d\eta d\eta d\eta} \\ L_2^{-1} = \iint_0^{\eta} \boldsymbol{\theta d\eta d\eta} \\ L_3^{-1} = \iint_0^{\eta} \boldsymbol{\emptyset d\eta d\eta} \\ L_4^{-1} = \iint_0^{\eta} \boldsymbol{s d\eta d\eta} \end{cases} \quad (35)$$

By applying $L_i^{-1}(i = 1,2,3,4)$ to the Eqs. (27-28) and considering boundary conditions (10), we get:

$$F(\eta) = F(0) + F'(0)\eta + \frac{1}{2}F''(0)\eta^2 + \frac{1}{6}F'''(0)\eta^3 + L_1^{-1}\left(-R_e(FF''' + 4\alpha^2F'F'')\right)$$

(36)

$$\theta(\eta) = \theta(0) + \theta'(0)\,\eta + L_2^{-1}(-R_eP_rF\theta' - N_b\theta'\emptyset')$$
$$\emptyset(\eta) = \emptyset(0) + \emptyset'(0)\,\eta + L_2^{-1}(-(N_t/N_b)\,\theta'' - R_eL_ef\emptyset')$$
$$s(\eta) = s(0) + s'(0)\,\eta + L_2^{-1}(+Pe_b(\emptyset's' + s\emptyset'') - (R_eS_c)fs')$$

Where:

$$NF = -R_e(FF''' + 4\alpha^2F'F'') \quad (40)$$
$$N\theta = -R_eP_rF\theta' - N_b\theta'\emptyset' \quad (41)$$
$$N\emptyset = -(N_t/N_b)\,\theta'' - R_eL_eF\emptyset' \quad (42)$$
$$Ns = +Pe_b(\emptyset's' + s\emptyset'') - (R_eS_c)Fs' \quad (43)$$

The values of $F(0), F'(0), F''(0), F'''(0), \theta(0), \theta'(0), \emptyset(0)$, $s(0)$ and $s'(0)$ mainly depend on the boundary conditions. In fact, by applying the boundary conditions (8, 9) and considering: $F'(0) = a_0, F'''(0) = a_1, \theta(0) = a_2, \theta'(0) = a_3, \emptyset(0) = a_4, \emptyset'(0) = a_5, s(0) = a_6, s'(0) = a_7$, we obtain:

$$F(\eta) = \sum_{n=0}^{\infty} F_n = F_0 + L^{-1}(NF) \quad (44)$$
$$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n = \theta_0 + L^{-1}(N\theta) \quad (45)$$
$$\emptyset(\eta) = \sum_{n=0}^{\infty} \emptyset_n = \emptyset_0 + L^{-1}(N\emptyset) \quad (46)$$
$$s(\eta) = \sum_{n=0}^{\infty} s_n = s_0 + L^{-1}(Ns) \quad (47)$$

Where : $F_0, \theta_0, \emptyset_0$ and s_0 are expressed as follows:

$$F_0 = a_0\eta + a_1\frac{\eta^3}{6} \quad (48)$$
$$\theta_0 = a_2 + a_3\eta \quad (49)$$
$$\emptyset_0 = a_4 + a_5\eta \quad (50)$$
$$s_0 = a_6 + a_7\eta \quad (51)$$

By the application of the algorithm (29), the polynomials $(A_0, A_1, \ldots\ldots A_n)$ are expressed in the following way:

- For velocity :

$A_0^F = \frac{1}{3} a_1^2 \mathrm{R_e} \eta^3$ (52-a)

$A_1^F = -\frac{1}{12} a_0 a_1^2 \mathrm{R_e}^2 \eta^5 - \frac{11}{1260} a_1^2 \mathrm{R_e}^2 \eta^7$ (52-b)

- For temperature

$A_0^\theta = -a_3 a_5 \mathrm{N_b} - a_0 a_3 \mathrm{P_r R_e} \eta - \frac{1}{6} a_1 a_3 \mathrm{P_r R_e} \eta^3$ (53-a)

$A_1^\theta = a_3 a_5^2 \mathrm{N_b}^2 \eta + \frac{1}{2} a_0 a_3 a_5 \mathrm{L_e N_b R_e} \eta^2 + \frac{3}{2} a_0 a_3 a_5 \mathrm{N_b P_r R_e} \eta^2 + \frac{1}{2} a_0^2 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta^3 + \frac{1}{24} a_1 a_3 a_5 \mathrm{L_e N_b R_e} \eta^4 + \frac{5}{24} a_1 a_3 a_5 \mathrm{N_b P_r R_e} \eta^4 + \frac{1}{8} a_0 a_1 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta^5 - \frac{1}{2520} a_1^2 a_3 \mathrm{P_r R_e}^2 \eta^7 + \frac{1}{144} a_1^2 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta$ (53-b)

- For nanoparticle volume fraction:

$A_0^\phi - a_0 a_5 \mathrm{LeR_e} \eta - \frac{1}{6} a_1 a_5 \mathrm{LeR_e} \eta^3$ (54-a)

$A_1^\phi = a_3 a_5 \mathrm{N_t} + \frac{1}{\mathrm{Nb}} a_0 a_3 \mathrm{N_t P_r R_e} \eta + \frac{1}{6\mathrm{Nb}} a_1 a_3 \mathrm{N_t P_r R_e} \eta^3 + \frac{1}{2} a_0^2 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^3 + \frac{1}{8} a_0 a_1 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^5 - \frac{1}{2520} a_1^2 a_5 \mathrm{L_e R_e}^2 \eta^7 + \frac{1}{144} a_1^2 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^7$ (54-b)

- For density of motile microorganisms:

$A_0^S = a_5 a_7 \mathrm{Pr_e} - a_0 a_7 \mathrm{R_e S_c} \eta - \frac{1}{6} a_1 a_7 \mathrm{R_e S_c} \eta^3$ (55-a)

$A_1^S = a_5^2 a_7 \mathrm{Pr_e}^2 \eta - a_0 a_5 a_6 \mathrm{L_e Pr_e R_e} \eta - \frac{3}{2} a_0 a_5 a_7 \mathrm{L_e Pr_e R_e} \eta^2 - \frac{3}{2} a_0 a_5 a_7 \mathrm{Pr_e R_e S_c} \eta^2 - \frac{1}{6} a_1 a_5 a_6 \mathrm{L_e Pr_e R_e} \eta^3 + \frac{1}{2} a_0^2 a_7 \mathrm{R_e}^2 \mathrm{S_c}^2 \eta^3 - \frac{5}{24} a_1 a_5 a_7 \mathrm{L_e Pr_e R_e} \eta^4 - \frac{5}{24} a_1 a_5 a_7 \mathrm{Pr_e R_e S_c} \eta^4 + \frac{1}{8} a_0 a_1 a_7 \mathrm{R_e}^2 \mathrm{S_c}^2 \eta^5 - \frac{1}{2520} a_1^2 a_7 \mathrm{R_e}^2 \mathrm{S_c} \eta^7 + \frac{1}{144} a_1^2 a_7 \mathrm{R_e}^2 \mathrm{S_c}^2 \boldsymbol{\eta}^{\mathbf{7}}$ (55-b)

The application of Adomian Decomposition Method leads to the following solutions terms:

- For velocity :

$F_1 = \frac{1}{2520} a_1^2 \mathrm{R_e} \eta^7$ (56-a)

$\mathrm{F}_2 = -\frac{1}{36288} a_0 a_1^2 \mathrm{R_e}^2 \eta^9 - \frac{1}{907200} a_1^3 \mathrm{R_e}^2 \eta^{11}$ (56-b)

- For temperature

$:\theta_1 = -\frac{1}{2} a_3 a_5 \mathrm{N_b} \eta^2 - \frac{1}{6} a_0 a_3 \mathrm{P_r R_e} \eta^3 - \frac{1}{120} a_1 a_3 \mathrm{P_r R_e} \boldsymbol{\eta}^{\mathbf{5}}$ (57-a)

$\theta_2 = \frac{1}{6} a_3 a_5^2 \mathrm{N_b}^2 \eta^3 + \frac{1}{24} a_0 a_3 a_5 \mathrm{L_e N_b R_e} \eta^4 + \frac{1}{8} a_0 a_3 a_5 \mathrm{N_b P_r R_e} \eta^4 + \frac{1}{40} a_0^2 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta^5 + \frac{1}{720} a_1 a_3 a_5 \mathrm{L_e N_b R_e} \eta^6 + \frac{1}{144} a_1 a_3 a_5 \mathrm{N_b P_r R_e} \eta^6 + \frac{1}{336} a_0 a_1 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta^7 - \frac{1}{181440} a_1^2 a_3 \mathrm{P_r R_e}^2 \eta^9 + \frac{1}{10368} a_1^2 a_3 \mathrm{P_r}^2 \mathrm{R_e}^2 \eta^9$ (57-b)

- For nanoparticle volume fraction:

$\emptyset_1 = -\frac{1}{6} a_0 a_5 \mathrm{L_e R_e} \eta^3 - \frac{1}{120} a_1 a_5 \mathrm{L_e R_e} \eta^5$ (58-a)

$\emptyset_2 = \frac{1}{2} a_3 a_5 \mathrm{N_t} \eta^2 + \frac{1}{6\mathrm{N_b}} a_0 a_3 \mathrm{N_t P_r R_e} \eta^3 + \frac{1}{120\mathrm{Nb}} a_1 a_3 \mathrm{N_t P_r R_e} \eta^5 + \frac{1}{40} a_0^2 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^5 + \frac{1}{336} a_0 a_1 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^7 - \frac{1}{181440} a_1^2 a_5 \mathrm{L_e R_e}^2 \eta^9 + \frac{1}{10368} a_1^2 a_5 \mathrm{L_e}^2 \mathrm{R_e}^2 \eta^9$ (58-b)

- For density of motile microorganisms
-

$s_1 = \frac{1}{2} a_5 a_7 \mathrm{Pr_e}\, \eta^2 - \frac{1}{6} a_0 a_7 \mathrm{R_e} S_c \eta^3 - \frac{1}{120} a_1 a_7 \mathrm{R_e} S_c\, \eta^5$ (59-a)

$$s_2 = \frac{1}{6}a_5^2a_7\mathrm{Pr_e}^2\eta^3 - \frac{1}{6}a_0a_5a_6\mathrm{L_e}\mathrm{Pr_e}\mathrm{R_e}\eta^3 - \frac{1}{8}a_0a_5a_7\mathrm{L_e}\mathrm{Pr_e}\mathrm{R_e}\eta^4 - \frac{1}{8}a_0a_5a_7\mathrm{Pr_e}\mathrm{R_e}S_c\eta^4 - \frac{1}{120}a_1a_5a_6\mathrm{L_e}\mathrm{Pr_e}\mathrm{R_e}\eta^5 + \frac{1}{40}a_0^2a_7\mathrm{R_e}^2S_c^2\eta^5 - \frac{1}{144}a_1a_5a_7\mathrm{L_e}\mathrm{Pr_e}\mathrm{R_e}\eta^6 - \frac{1}{144}a_1a_5a_7\mathrm{Pr_e}\mathrm{R_e}S_c\eta^6 + \frac{1}{336}a_0a_1a_7\mathrm{R_e}^2S_c^2\eta^7 - \frac{1}{181440}a_1^2a_7\mathrm{R_e}^2S_c\eta^9 + \frac{1}{10368}a_1^2a_7\mathrm{R_e}^2S_c^2\eta^9 \quad (59\text{-b})$$

Finally, the approximate solutions for the studied problem are expressed as:

- For velocity:

$$F(\eta) = F_0 + F_1 + \cdots \ldots \ldots \ldots .. + F_n \quad (60)$$

- For temperature:

$$\theta(\eta) = \theta_0 + \theta_1 + \cdots \ldots \ldots \ldots .. + \theta_n \quad (61)$$

- For nanoparticle volume fraction:

$$\emptyset(\eta) = \emptyset_0 + \emptyset_1 + \cdots \ldots \ldots \ldots .. + \emptyset_n \quad (62)$$

- For density of motile microorganisms:

$$s(\eta) = s_0 + s_1 + \cdots \ldots \ldots \ldots .. + s_n \quad (63)$$

where: n is the iteration number.
The constants a_0, a_1, a_2, a_3 ……., a_7 can be easily determined with the boundary conditions (Eqs. (22-a) - (22.c)).

5. Results and discussion

In this study, we were particularly interested in the evolution of velocity F(η), temperature θ(η), nano-particles volume fraction ∅(η) and density of motile microorganisms$s$(η). The set of nonlinear differential equations (Eqs. (18-21)) with the boundary conditions (Eqs. (22)) are solved numerically and analytically. Numerically, the fourth-order Runge-Kutta method was used. Analytically, the problem is treated via a powerful technique of computation called Adomian Decomposition Method.

Figs. 2-4 show the effect of δθ parameter on the temperature, the nanoparticle volume fraction and the density of motile microorganisms respectively. It can be clearly seen that the δθ parameter has a significant effect on the behavior of temperature and nanoparticle volume fraction. As depicted in Fig. 2, the temperature increases with increasing δθ parameter. One can also observe that the δθ parameter has more effect on temperature at the lower wall ($\eta = -1$) of the channel. In order to obtain a stable temperature profile along the channel, δθ parameter should be increased, which would result in higher temperature on the lower wall in comparison to the upper one. Furthermore, we can observe as displayed in Fig. 3 that the δθ parameter has more effect on the nanoparticle volume fraction at the lower wall ($\eta = +1$). This means that increasing temperature of the channel wall leads to the concentration of the nanoparticles in the vicinity of the upper wall; and to reach a more stable profile, a higher δθ value is required.

As drawn in Fig. 4, the behavior of motile microoganisms density s(η) as a function of δθ is approximately linear. The density s (η) increases as the δθ parameter increases.

when $\delta s = 1, \mathrm{N_b} = 0.2, \mathrm{N_t} = 0.4, \mathrm{Pr} = 1. \mathrm{L_e} = 2, \mathrm{Pe_b} = 1, \mathrm{S_c} = 3. \delta\phi = -0.5$ and $\mathrm{R_e} = 0.7$

Fig. 5 shows the effect of δφ parameter on the behavior of nanoparticle volume fraction. We notice that the nanoparticles volume fraction appears as an increasing function of δφ.

The effect of δs parameter on the evolution of microorganisms' density is shown in Fig. 6. As depicted, it is highly noticed that the microorganisms' density raises with the augment of δs parameter; although δs has a bigger influence on the density at the level of lower wall of the channel (when $\eta = -1$). Moreover, the density profile becomes stable in the middle of the channel for high δs values.

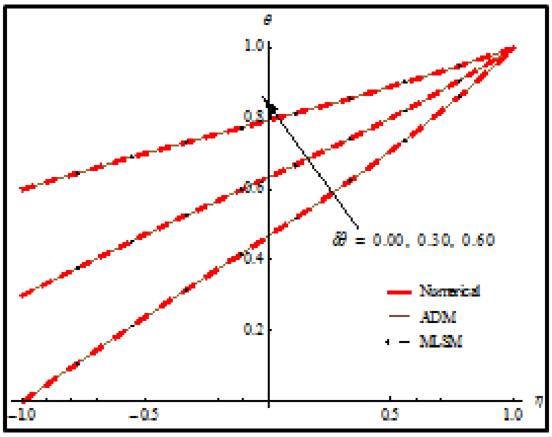

Fig. 2. Effect of δθ parameter on the temperature evolution.

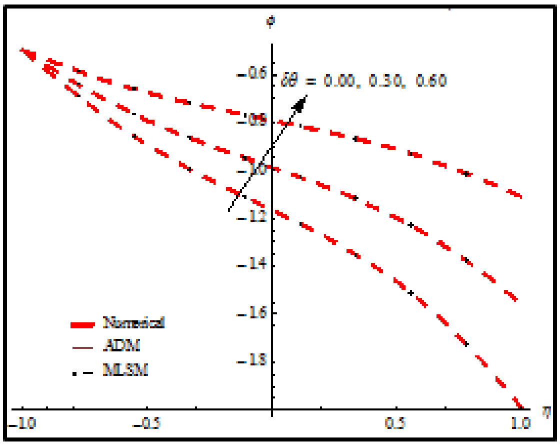

Fig. 3. Effect of δθ parameter on the evolution of motile microorganisms density when $\delta s = 1, N_b = 0.2, N_t = 0.4, P_r = 1. L_e = 2, Pe_b = 1, S_c = 3. \delta\phi = -0.5$ and $R_e = 0.7$.

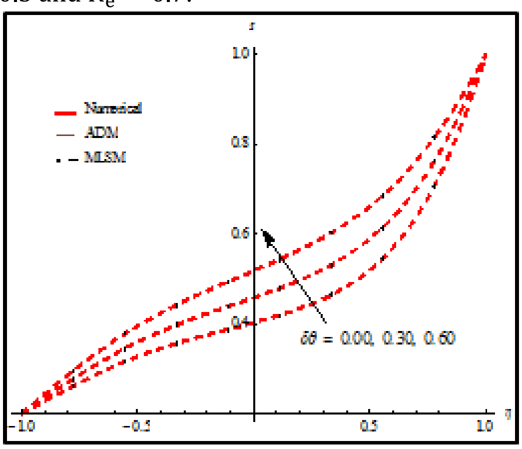

Fig. 4. Effect of δθ parameter on the evolution of nanoparticle volume fraction when: $\delta s = 1, N_b = 0.2, N_t = 0.4, P_r = 1. L_e = 2, Pe_b = 1, S_c = 3. \delta\phi = -0.5$ *and* $R_e = 0.7$.

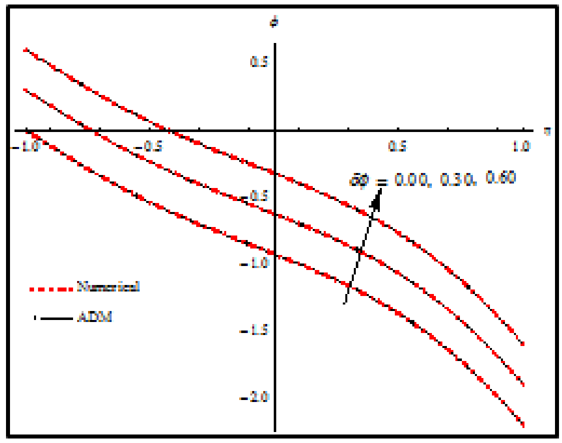

Fig. 5. Effect of δφ parameter on the evolution of nanoparticle volume fraction when $\delta s = 1, N_b = 0.2, N_t = 0.4, P_r = 1. L_e = 2, Pe_b = 1, S_c = 3. \delta\theta = -0.5$ and $R_e = 0.7$.

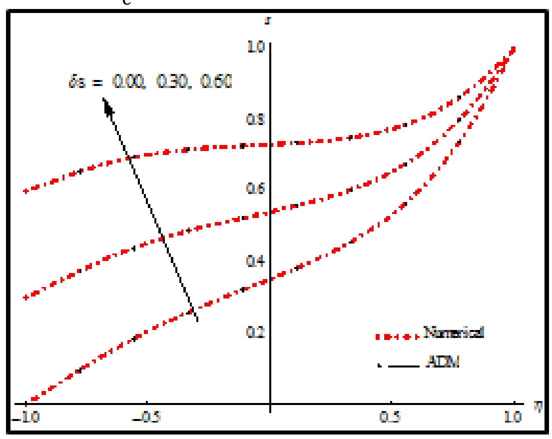

Fig. 6. Effect of δs parameter on the evolution of motile microorganisms density when $\delta\theta = 0.2, N_b = 0.2, N_t = 0.4, P_r = 1. L_e = 2, Pe_b = 1, S_c = 3. \delta\phi = -0.5$ and $R_e = 0.7$.

As can be seen from Fig. 7, the effect of the $Nt/_{Nb}$ ratio is visibly greater with high values at the level of upper wall ($\eta = +1$). By contrast, its effect on the density of microorganisms, as visualized in Fig. 8, is more pronounced along the axis of the channel ($\eta = 0$). Furthermore, the $Nt/_{Nb}$ ratio does not affect the density of the microorganisms at the level of walls ($\eta = \pm 1$). The heat transfer rate $\theta(-1)$ at the level of lower wall (when $\eta = -1$) is depicted in Fig. 9. It is clearly seen that with increasing $Nt/_{Nb}$ ratio, the temperature of the lower wall decreases, causing therefore a decrease in heat transfer rate $\theta(-1)$. Additionally, the heat transfer rate $\theta(-1)$ (or the Nusselt number) raises substantially with the augment of Reynolds number *Re*. In fact, the forced convection parameter *Re* has a drastic effect on the thermal behavior.

Consequently, with the increase of Reynolds number, the thermal layer becomes thin and concentrated near the wall. The greatest heat transfer rate is generally gained for the highest values of Reynolds number *Re*.

Table 1 represents a comparison between obtained numerical and analytical results. To highlight the effectiveness of the adopted analytical technique, a comparison with other works [16, 20] is reported in Figs. 10-11 and Table 2. Based on these comparisons, there is a clear evidence for a good agreement between analytical (ADM) and numerical (RK4) data, justifying the efficiency and the higher accuracy of the used Adomian decomposition method.

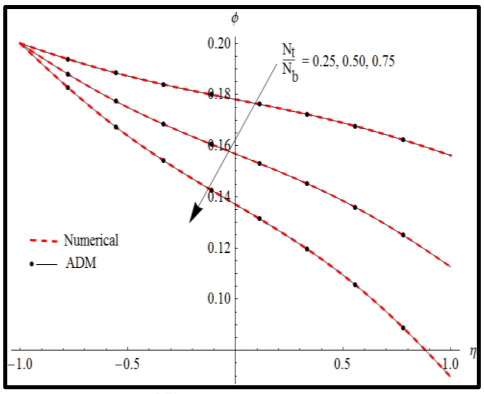

Fig. 7. Effect of $^{Nt}/_{Nb}$ ratio on the evolution of nanoparticle volume fraction when $\delta\theta = 0.8, \delta s = 0.3, P_r = 1. L_e = 1, Pe_b = 1, S_c = 1. \delta\phi = 0.2$ and $R_e = 1$.

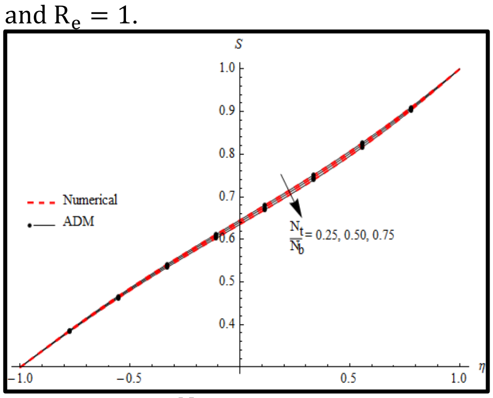

Fig. 8. Effect of $^{Nt}/_{Nb}$ ratio on the evolution of motile microorganisms density when $\delta\theta = 0.8, \delta s = 0.3, P_r = 1. Le = 1, Pe_b = 1, S_c = 1. \delta\phi = 0.2$ and $R_e = 1$.

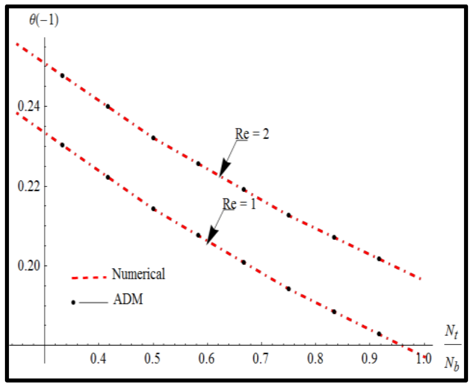

Fig. 9. Heat transfer rate $\theta(-1)$ as a function of the $^{Nt}/_{Nb}$ ratio when $\delta\theta = 0.5, \delta s = 1, P_r = 1. L_e = 1, Pe_b = 1, S_c = 1. \delta\phi = 0$ and $R_e = 5$

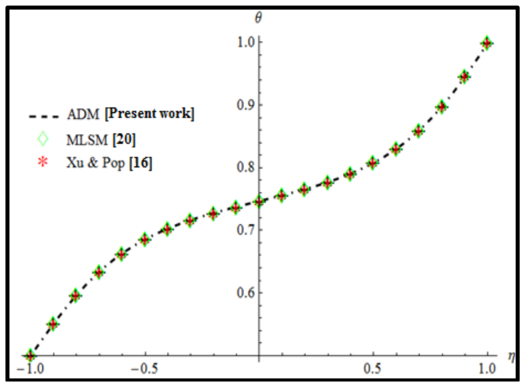

Fig. 10. Comparison between different results for temperature evolution when.$\delta s = 1. N_t = 0.2. P_r = 3. L_e = Pe_b = Sc = 1. \delta\theta = 0.5. \delta\phi = 0.$ and $R_e = 5$.

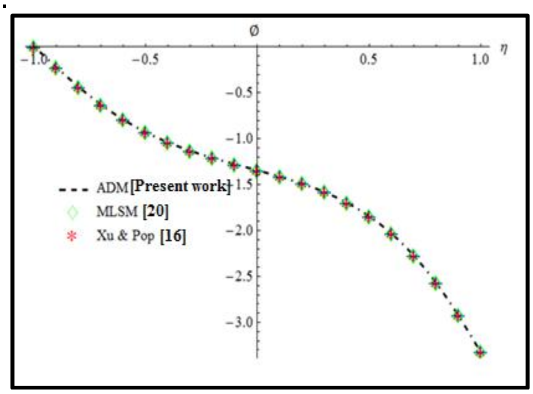

Fig. 11. Comparison between different results for nanoparticle volume fraction evolution when $.\delta s = 1. N_t = 1. P_r = 1. L_e = Pe_b = S_c = 1. \delta\theta = 0.5. \delta\phi = 0. and\ R_e = 5.$

Table 1. Comparison of numerical and analytical results when :$N_b = 0.2. N_t = 0.4. Pr = 1. L_e = 5. \delta\theta = 0.8, \delta\emptyset = 0.2, \delta s = 0.1.$ and $R_e = 2.$

	$F(\eta)$		$\theta(\eta)$		$\emptyset(\eta)$		$s(\eta)$	
η	$F(\eta)^{RK4}$	$F(\eta)^{ADM}$	$\theta(\eta)^{RK4}$	$\theta(\eta)^{ADM}$	$\emptyset(\eta)^{RK4}$	$\emptyset(\eta)^{ADM}$	$s(\eta)^{RK4}$	$s(\eta)^{ADM}$
-1.00	0.000000	0.000000	0.8000	0.8000	0.20000	0.20000	0.100000	0.100000
-0.75	0.161449	0.161446	0.827456	0.827454	0.150759	0.150755	0.229558	0.229556
-0.50	0.183032	0.183035	0.853145	0.853142	0.119607	0.119603	0.341845	0.341843
-0.25	0.113997	0.113995	0.876783	0.876787	0.102894	0.102896	0.433844	0.433842
0.00	0.0000	0.0000	0.899207	0.899204	0.0924287	0.0924285	0.514763	0.514766
+0.25	−0.113997	−0.113994	0.921676	0.921673	0.0818686	0.0818684	0.596634	0.596637
+0.50	−0.183032	−0.183036	0.945487	0.945485	0.0647272	0.0647274	0.693681	0.693683
+0.75	−0.161449	−0.161443	0.971610	0.971614	0.0322616	0.0322617	0.824178	0.824176
+1.00	0.000000	0.000000	1.000000	1.000000	−0.0200213	−0.0200211	1.000000	1.000000

Table 2. Comparison between the adopted techniques and other works [20] when : $R_e = 1, P_r = 1, N_b = 5, N_t = 0.1, L_e = 1, Pe_b = 2, S_c = 5, \delta\theta = 0.5, \delta\varphi = 0.25$ and $\delta s = 0.$

η	$\theta(\eta)^{Numerical}$	$\theta(\eta)^{MLSM}$	$\theta(\eta)^{ADM}$	$\emptyset(\eta)^{Numerica}$	$\emptyset(\eta)^{MLSM}$	$\emptyset(\eta)^{ADM}$	$s(\eta)^{Numerica}$	$s(\eta)^{MLSM}$	$s(\eta)^{ADM}$
-1.0	0.5	0.5	0.5	0.25	0.25	0.25	0	0	0
-0.8	0.553149	0.550491	0.551976	0.248943	0.248374	0.248969	0.133283	0.132346	0.133255
-0.6	0.604876	0.599782	0.602825	0.247942	0.246572	0.247989	0.249574	0.249358	0.249573
-0.4	0.65472	0.648522	0.65207	0.247014	0.24485	0.247075	0.34617	0.347755	0.346201
-0.2	0.702956	0.69722	0.699955	0.246147	0.243382	0.246216	0.428165	0.430306	0.428219
0.0	0.750277	0.746252	0.747158	0.245314	0.242265	0.245388	0.502742	0.503776	0.502824
0.2	0.797572	0.795857	0.794557	0.244482	0.241515	0.244555	0.5772	0.576945	0.577299
0.4	0.845728	0.846136	0.84305	0.243617	0.241068	0.243685	0.658768	0.658589	0.658875
0.6	0.895448	0.897056	0.893367	0.242692	0.240779	0.242748	0.754473	0.75548	0.750026
0.8	0.94702	0.948446	0.945825	0.241694	0.240424	0.241734	0.869169	0.870372	0.866466
1.0	1	1	1	0.24064	0.2397	1	1	1	1

6. Conclusions

In this paper, the dynamic and thermal problems of a nano-fluid flow in a horizontal channel are considered. As a first step, the equations governing the problems are described in detail. In the current stuyd, the model proposed by Kuznestov and Nield [31] was adopted. Thereafter, the set of differential equations arising from mathematical modeling (velocity *F*(η), temperature θ(η), nanoparticles volume fraction $\boldsymbol{\varphi}$(η) and motile microorganisms density *s*(η)) are solved numerically and analytically by the Runge-Kutta method featuring technique and the Adomian decomposition method (ADM) respectively. The effects of various physical parameters, namely the thermal constant " $\delta\theta$ ", the concentration constant "$\delta\varphi$" and "N_t/N_b'' ratio on the considered nano-fluid flow are visualized and discussed.

The main conclusions that may be drawn from this study are:

- The thermal behavior and nanoparticles volume fraction are affected by the δθ constant particularly in the vicinity of the walls. In this region, the effect of δθ is more significant.
- The nanoparticles volume fraction is an increasing function of δφ parameter.
- The nanoparticles volume fraction is significantly affected by the δs constant, especially in the vicinity of the lower wall ($\eta = -1$). However, for high δs values, the profile of nanoparticles volume fraction ϕ becomes stable.
- Heat transfer rate $\theta(-1)$ raises with the increase of Reynolds number.
- Heat transfer rate $\theta(-1)$ appears as a decreasing function of ${}^{Nt}/_{Nb}$ ratio.

- The obtained results highlight the robustness of the adopted analytical Adomian Decomposition Method (ADM) in comparison with numerical results and those of literature. Furthermore, the comparison reveals the applicability, reliability and simplicity of the used technique.

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