



# MHD thermal radiation and chemical reaction effects with peristaltic transport of the Eyring-Powell fluid through a porous medium

K. Venugopal Reddy<sup>a,\*</sup> and M. Gnaneswara Reddy<sup>b</sup>

<sup>a</sup> Department of Mathematics, Vignan Institute of Technology & Science, Deshmukhi village, Telangana - 508284, India

<sup>b</sup> Department of Mathematics, Acharya Nagarjuna University Campus, Ongole - 523 001, India

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**Abstract**

In this paper, thermal radiation and chemical reaction impacts on MHD peristaltic motion of the Eyring-Powell fluid are analyzed through a porous medium in a channel with compliant walls under slip conditions for velocity, temperature, and concentration. Assumptions of a long wavelength and low Reynolds number are considered. The modeled equations are computed by using the perturbation method. The resulting non-linear system is solved for the stream function, velocity, temperature, concentration, skin-friction coefficient, heat transfer coefficient, and mass transfer coefficient. The flow quantities are examined for various parameters. Temperature is depressed with an enhanced radiation parameter, while the opposite effect is observed for the concentration. The fluid concentration is enhanced and depressed with generative and destructive chemical reaction, respectively. The size of the trapped bolus reduces as the Powell-Eyring parameter increases while it increases as another Powell fluid parameter rises. Furthermore, the size of the trapped bolus grows when Darcy number increases.

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**Nomenclature**

$u$  and  $v$ : Velocities

$p$ : Pressure

$T$ : Temperature

$C$ : Concentration

$\rho$ : Density

$\sigma$ : Electrical conductivity

$B_o$ : Field with Magnetic

$c_p$ : Heat capacity

$k$ : Energy conductivity

$D$ : Mass diffusivity Coefficient

$T_m$ : Mean energy

$K_T$ : Diffusion ratio

$k_o$ : Permeability parameter

$k_l$ : Chemical reaction of the rate constant

$\sigma^*$ : Boltzmann-Stefan constant

$k^*$ : Absorption mean coefficient

$T_o$ : Energy at the wall

$C_o$ : Concentration at the wall

$M$ : Hartmann number

$\beta_1$ : Velocity slip

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\*Corresponding author

email address: venugopal.reddy1982@gmail.com

$\beta_2$  : Temperature slip

$\beta_3$  : Concentration slip

$A$  and  $B$  : Eyring-Powell fluid parameters

$\theta$  : Temperature

$\phi$  : Concentration

$Da$  : Darcy number

$Rd$  : Radiation parameter

$\gamma$  : Chemical reaction parameter

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## 1. Introduction

The mechanism of the pumping fluid in a channel/tube from a region of minor pressure to major one is known as peristalsis. The peristaltic phenomenon is unique in sensing the that fluid is transported by the action of a progressive sinusoidal wave due to area contraction or expansion, which propagates along an extensible tube/channel instead of using a piston. Peristaltic transporting of non-Newtonian and Newtonian fluids has been extensively recognized by several authors due to its numerous corporeal and engineering applications. In physiology, the peristalsis appears in esophagus over swallowing food, chyme movement in the gastrointestinal tract, urine movement from the kidney to bladder through the ureter, and vasomotion of blood vessels in capillaries and arterioles. The movement of corrosive fluids, sanitary fluid, slurries, and noxious fluids in the nuclear reactor. Furthermore, blood pumps in heart lung machine and roller and finger pumps also function on the principle of peristaltic pumping. The earliest study about the mechanism of peristaltic movement of a viscous fluid was carried out by Latham [1]. Shapiro et al. [2] presented a mathematical model for peristaltic pumping under the assumptions of a creeping flow approximation. He considered the transport of a viscous fluid in a two-dimensional channel. In that study, the authors analyzed the peristaltic pumping reflux and trapping. Afterward, several diagnostic, commutative, and empirical inspections related to peristaltic motions have been studied for different flow geometries and assumptions. Some useful attempts in this direction were

described in [3-6]. The analysis in all these attempts has been made by employing no-slip boundary conditions and one or more reduced assumptions of a low Reynolds number, small wave number, long wavelength, etc.

In many research fields, non-Newtonian fluids are considerably used in many corporeal and mechanical processes than Newtonian fluids. Consequently, the Powell-Eyring fluid exemplary is better than the fluid of power law in different ways. Its non-linear equations are proved from the kinetic theory of fluids rather than the experimental relations. Different researchers are analyzed the above statement [7-10]. Recently, few attempts have been investigated in the peristaltic information to analyze the combined impacts of energy and concentration transfer under different situations [11-14].

Radiation heat transfer plays a significant role in the cooling of electronics as well as conduction and convection heat transfer. Thermal radiation has bounteous applications such as freezing, heating, burners, reeking, and solar radiation systems. However, only a few researchers [15-18] have studied radiation in mechanism of peristalsis.

The aim of the current study is to analyze the impacts of thermal and synthetic reactions on MHD peristaltic flow of the Eyring-Powell fluid through a porous medium with wall properties. A mathematical model of the flow and heat and mass transfer, for the present problem, is constructed with imposing long wavelength and low Reynolds number approximations. Series solutions are matured for a small fluid parameter by a regular perturbation procedure. Numerical results for the emerging non-dimensional parameters on different parameters of interest are shown graphically and analyzed. Also, the trapping approach is presented for the pertinent parameters.

## 2. Modeling

The peristaltic motion of a viscous electrically conducting, radiating and reacting motion of Powell - Eyring

incompressible fluid filled with a porous medium in a two-dimensional channel of width  $2d_1$  is considered. The movement of the existing problem with geometry is conferred in Fig. 1.

The physical interpretation of the wall surface is given by:

$$y = \pm \eta(x, t) = \pm \left[ d_1 + a \sin \frac{2\pi}{\lambda} (x - ct) \right] \quad (1)$$

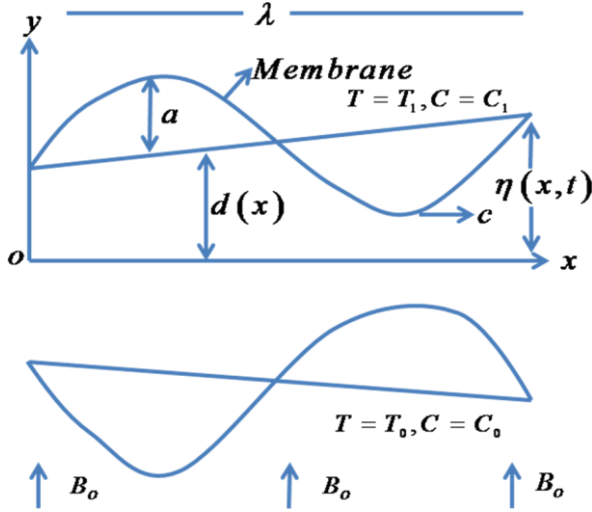


Fig.1. Flow configuration

The Powell-Eyring fluid exemplary is examined to investigate the non-Newtonian motion with shear. The Powell-Eyring fluid model with stress tensor is [13-18]:

$$\boldsymbol{\tau} = \left[ \mu + \frac{1}{\beta_{\dot{\gamma}}} \sinh^{-1} \left( \frac{1}{c_1} \dot{\gamma} \right) \right] \mathbf{A}_1 \quad (2)$$

where

$$\dot{\gamma} = \sqrt{\frac{1}{2} \text{tr}(\mathbf{A}_1)^2}, \quad \mathbf{A}_1 = (\text{grad } \mathbf{V})^T + \text{grad } \mathbf{V}$$

the  $\sinh^{-1}$  up as terms order of second is developed as:

$$\sinh^{-1} \left( \frac{1}{c_1} \dot{\gamma} \right) \cong \frac{\dot{\gamma}}{c_1} - \frac{\dot{\gamma}^3}{6c_1^3}, \quad \frac{\dot{\gamma}^5}{c_1^5} \ll 1 \quad (3)$$

The fundamental equations are [15-18]:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \quad (4)$$

$$\begin{aligned} \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \right) = & -\frac{\partial p}{\partial x} + \left( \mu + \frac{1}{\beta_{c_1}} \right) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma B_o^2 u - \frac{\mu}{k_o} u \\ & - \frac{1}{3\beta_{c_1}^3} \frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial x} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right] \\ & - \frac{1}{6\beta_{c_1}^3} \frac{\partial}{\partial y} \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right] \end{aligned} \quad (5)$$

$$\begin{aligned} \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = & -\frac{\partial p}{\partial y} + \left( \mu + \frac{1}{\beta_{c_1}} \right) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu}{k_o} v \\ & - \frac{1}{6\beta_{c_1}^3} \frac{\partial}{\partial x} \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right] \\ & - \frac{1}{3\beta_{c_1}^3} \frac{\partial}{\partial y} \left[ \frac{\partial v}{\partial y} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right] \end{aligned} \quad (6)$$

$$\begin{aligned} \rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} \right) = & k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \left( \mu + \frac{1}{\beta_{c_1}} \right) \left[ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \\ & - \frac{2}{3\beta_{c_1}^3} \left( \frac{\partial u}{\partial x} \right)^2 \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \\ & - \frac{1}{6\beta_{c_1}^3} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \\ & - \frac{1}{3\beta_{c_1}^3} \left( \frac{\partial v}{\partial y} \right)^2 \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] - \frac{\partial q_r}{\partial y} \end{aligned} \quad (7)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{DK_T}{T_m} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - k_l (C - C_o) \quad (8)$$

The heat flow of radiation in the direction of  $X$ -axis is studied as negligible compared to

the  $Y$ -axis. The heat flow of the radiation  $q_r$  is defined using the approximation of the Rosseland for thermic radiation as given by:

$$q_r = -\frac{16\sigma^* T_o^3}{3k^*} \frac{\partial T}{\partial Y} \quad (9)$$

The associated boundary conditions are:

$$\begin{aligned} \frac{\partial}{\partial x} \left( -\tau \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + d \frac{\partial}{\partial t} \right) \eta = & \left( \mu + \frac{1}{\beta c_1} \right) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma B_o^2 u - \\ & \frac{\mu}{k_o} u - \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \right) - \\ & \frac{1}{6\beta c_1^3} \frac{\partial}{\partial y} \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right] \\ & - \frac{1}{3\beta c_1^3} \frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial x} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \right. \\ & \left. + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \end{aligned} \quad (10)$$

$$u \pm \beta_1 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left[ \left( \mu + \frac{1}{\beta c_1} \right) - \frac{\mu}{k_o} v - \frac{1}{6\beta c_1^3} \left\{ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} \right. \\ \left. + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] = 0 \quad (11)$$

$$T \pm \beta_2 \frac{\partial T}{\partial y} = T_0, \quad C \pm \beta_3 \frac{\partial C}{\partial y} = C_0 \quad (12)$$

$$\text{at } y = \pm \eta$$

Proposing the stream function  $\psi(x, y, t)$

$$v = -\frac{\partial \psi}{\partial x}, u = \frac{\partial \psi}{\partial y} \quad (13)$$

and defining dimensionless quantities of:

$$\begin{aligned} x^* &= \frac{x}{\lambda}, y^* = \frac{y}{d_1}, \psi^* = \frac{\psi}{cd_1}, t^* = \frac{ct}{\lambda}, \eta^* = \frac{\eta}{d_1}, \\ p^* &= \frac{pd_1^2}{c\lambda\mu}, B = \frac{1}{\mu\beta c_1}, A = \frac{Bc^2}{2d_1^2 c_1^2}, \delta = \frac{d_1}{\lambda}, \theta = \frac{T - T_o}{T_o}, \\ \varphi &= \frac{C - C_o}{C_o}, \text{Re} = \frac{\rho cd_1}{\mu}, M = \sqrt{\frac{\sigma}{\mu}} B_o d_1, E_1 = \frac{-\tau d_1^3}{\lambda^3 \mu c}, \\ E_2 &= \frac{mcd_1^3}{\lambda^3 \mu}, E_3 = \frac{dd_1^3}{\lambda^2 \mu}, \beta_i^* = \frac{\beta_i}{d_1} (i=1-3), \\ \text{Pr} &= \frac{\mu c_p}{k}, Ec = \frac{c^2}{C_p T_o}, Sc = \frac{\mu}{D\rho}, Sr = \frac{\rho DK_T T_o}{\mu T_m C_o}, \\ \gamma &= k_l \frac{d_1^2}{\nu}, Da = \frac{k_o}{d_1^2}, Rd = \frac{16\sigma^* T_o^3}{3k^* \mu_o c_f}, Br = \text{Pr} Ec \end{aligned} \quad (14)$$

Using Eq. (13) and quantities (14) into Eqs. (5-12), and to apply the creeping flow approximations, it is accessed the following non-dimensional equations (after dropping asterisks):

$$(1+B) \frac{\partial^3 \psi}{\partial y^3} - \frac{A}{3} \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 - \left( M^2 + \frac{1}{Da} \right) \frac{\partial \psi}{\partial y} = \frac{\partial p}{\partial x} \quad (15)$$

$$\frac{\partial p}{\partial y} = 0 \quad (16)$$

$$\left( \frac{1}{\text{Pr}} + Rd \right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \left[ (1+B) - \frac{A}{3} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] = 0 \quad (17)$$

$$\frac{\partial^2 \phi}{\partial y^2} + Sc Sr \frac{\partial^2 \theta}{\partial y^2} - Sc \gamma \phi = 0 \quad (18)$$

The corresponding boundary conditions are:

$$\begin{aligned} \frac{\partial}{\partial x} \left( E_1 \frac{\partial^2}{\partial x^2} + E_2 \frac{\partial^2}{\partial t^2} + E_3 \frac{\partial}{\partial t} \right) \eta = \\ (1+B) \frac{\partial^3 \psi}{\partial y^3} - \frac{A}{3} \frac{\partial}{\partial y} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \quad \text{at } y = \pm \eta \\ - \left( M^2 + \frac{1}{Da} \right) \frac{\partial \psi}{\partial y} \end{aligned} \quad (19)$$

$$\frac{\partial \psi}{\partial y} \pm \beta_1 \left[ (1+B) \frac{\partial^2 \psi}{\partial y^2} - \frac{A}{3} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \right] = 0,$$

$$\theta \pm \beta_2 \frac{\partial \theta}{\partial y} = 0, \phi \pm \beta_3 \frac{\partial \phi}{\partial y} = 0 \quad (20)$$

Eliminating pressure by cross differentiation from Eqs. (15 and 16):

$$(1+B) \frac{\partial^4 \psi}{\partial y^4} - \frac{A}{3} \frac{\partial^2}{\partial y^2} \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 - \left( M^2 + \frac{1}{Da} \right) \frac{\partial^2 \psi}{\partial y^2} = 0 \quad (21)$$

### 3. Perturbation solution

The non-dimensional governing Eqs. (21, 17 and 18) are coupled, and extremely non-linear and their exact solutions may not be possible. Therefore, in order to obtain approximate series solutions, expressions for  $\psi$ ,  $\theta$  and  $\phi$  in small Eyring-Powell fluid parameter  $A$  are expanded as follows:

$$\begin{aligned} \psi &= A\psi_1 + \psi_o + O(A^2), \\ \theta &= A\theta_1 + \theta_o + O(A^2), \\ \phi &= A\phi_1 + \phi_o + O(A^2). \end{aligned} \quad (22)$$

Substituting Eq. (22) into Eqs. (21, 17 and 18) and the associated boundary conditions (19) and (20), the following systems of zeroth (Section 3.1) and first order (Section 3.2) are taken:

#### 3.1. Zeroth order system

$$(1+B) \frac{\partial^4 \psi_o}{\partial y^4} - \left( M^2 + \frac{1}{Da} \right) \frac{\partial^2 \psi_o}{\partial y^2} = 0 \quad (23)$$

$$\left( \frac{1}{Pr} + Rd \right) \frac{\partial^2 \theta_o}{\partial y^2} + Ec(1+B) \left( \frac{\partial^2 \psi_o}{\partial y^2} \right)^2 = 0 \quad (24)$$

$$\frac{\partial^4 \phi_o}{\partial y^4} + ScSr \frac{\partial^2 \theta_o}{\partial y^2} - Sc\gamma \phi_o = 0 \quad (25)$$

and the corresponding boundary conditions are:

$$\frac{\partial}{\partial x} \left( E_2 \frac{\partial^2}{\partial t^2} + E_1 \frac{\partial^2}{\partial x^2} + E_3 \frac{\partial}{\partial t} \right) \eta = (1+B) \frac{\partial^3 \psi_o}{\partial y^3} - \left( M^2 + \frac{1}{Da} \right) \frac{\partial \psi_o}{\partial y},$$

, at  $y = \pm \eta$  (26)

$$\begin{aligned} \frac{\partial \psi_o}{\partial y} \pm \beta_1 (1+B) \frac{\partial^2 \psi_o}{\partial y^2} &= 0, \beta_2 \frac{\partial \theta_o}{\partial y} \pm \theta_o = 0, \\ \beta_3 \frac{\partial \phi_o}{\partial y} \pm \phi_o &= 0 \text{ at } y = \pm \eta \end{aligned} \quad (27)$$

Solving the Eqs. (23, 24) and 25) with corresponding boundary conditions (26) and (27), the solutions are obtained as follows:

$$\psi_o = L_2 \sinh(Ny) + L_3 y \quad (28)$$

$$\theta_o = B_1 + B_2 y + L_{10} \cosh(2Ny) + L_{11} y^2 \quad (29)$$

$$\begin{aligned} \phi_o &= C_1 \cosh(N_2 y) + C_2 \sinh(N_2 y) \\ &+ L_{25} \cosh(2Ny) + L_{26} \end{aligned} \quad (30)$$

#### 3.2. First order system

$$(1+B) \frac{\partial^4 \psi_1}{\partial y^4} - \frac{1}{3} \frac{\partial^2}{\partial y^2} \left( \frac{\partial^2 \psi_o}{\partial y^2} \right)^3 - \left( M^2 + \frac{1}{Da} \right) \frac{\partial^2 \psi_1}{\partial y^2} = 0 \quad (31)$$

$$\begin{aligned} \left( \frac{1}{Pr} + Rd \right) \frac{\partial^2 \theta_1}{\partial y^2} + 2Ec(1+B) \left( \frac{\partial^2 \psi_o}{\partial y^2} \frac{\partial^2 \psi_1}{\partial y^2} \right) \\ - \frac{1}{3} Ec \left( \frac{\partial^2 \psi_o}{\partial y^2} \right)^4 = 0 \end{aligned} \quad (32)$$

$$\frac{\partial^2 \phi_1}{\partial y^2} + ScSr \frac{\partial^2 \theta_1}{\partial y^2} - Sc\gamma \phi_1 = 0 \quad (33)$$

and the boundary conditions associated with the above equations are:

$$(1+B)\frac{\partial^3\psi_1}{\partial y^3} - \frac{1}{3}\frac{\partial}{\partial y}\left(\frac{\partial^2\psi_o}{\partial y^2}\right)^3 - \left(M^2 + \frac{1}{Da}\right)\frac{\partial\psi_1}{\partial y} = 0$$

at  $y = \pm\eta$  (34)

$$\frac{\partial\psi_1}{\partial y} \pm \beta_1 \left[ (1+B)\frac{\partial^2\psi_1}{\partial y^2} - \frac{1}{3}\left(\frac{\partial^2\psi_o}{\partial y^2}\right)^3 \right] = 0,$$

$$\theta_1 \pm \beta_2 \frac{\partial\theta_1}{\partial y} = 0, \quad \phi_1 \pm \beta_3 \frac{\partial\phi_1}{\partial y} = 0 \quad \text{at } y = \pm\eta$$

(35)

Now using the solutions obtained at the zeroth order system into the first order system and then solving the resulting problem, the followings are finally attained:

$$\psi_1 = L_8 \sinh(Ny) + L_6 \sinh(3Ny) + L_7 y \cosh(Ny) \quad (36)$$

$$\theta_1 = B_3 + B_4 y + L_{14} \sinh(2Ny) + L_{20} \cosh(2Ny) + L_{21} \cosh(4Ny) + L_{22} y \sinh(2Ny) + L_{23} y^2$$

(37)

$$\varphi_1 = C_4 \sinh(N_2 y) + C_3 \cosh(N_2 y) + L_{28} \sinh(2Ny) + L_{29} \cosh(2Ny) + L_{30} \cosh(4Ny) + L_{31} y \sinh(2Ny) + L_{32}$$

(38)

where the constants are convoluted in Eqs. (28-30) and Eqs. (36-38) are defined in the Appendix.

#### 4. Results and discussions

This section aims to analyze the obtained results graphically. The variations in velocity, energy, mass particle, stream lines, heat and mass transfer coefficients caused by different physical parameters on the quantities of interest are discussed through Figs. 2-44. The following default fixed constants are adopted for numerical computation:

$$E_1 = 0.4, E_2 = 0.1, E_3 = 0.01, \varepsilon = 0.15, x = 0.2, \\ B = 2, M = 1.0, \beta_1 = 0.01, A = 0.1, Da = 0.3, \\ t = 0.1, Br = 2, \beta_2 = 0.02, Pr = 1, Rd = 1, \\ Sc = 1, \gamma = 1, Sr = 1, \beta_3 = 0.02, x = 0.2$$

##### 4.1. Flow characteristics

This section provides the variation of different physical parameters of interest on the axial velocity  $u$ , which is obtained using the

expression  $u = \frac{\partial\psi_o}{\partial y} + A \frac{\partial\psi_1}{\partial y}$ . Fig. 2 shows

that the axial velocity enhances as the effect of material parameter  $A$  increases. it involves the governing non-linear momentum while axial velocity retards with the rise in other material parameter  $B$  given in Fig. 3. Fig. 4 represents that with the rise of Hartmann number  $M$  the axial velocity is retarded. . It is because of the Lorentz force combined with the applied magnetic field along the transverse direction which resists against the flow. Fig. 5 depicts that the axial velocity enhances by increasing slip parameter  $\beta_1$  while the quite opposite behavior is observed on the axial velocity when a rise in Darcy number  $Da$  happens. This is because it leads to reduce the drag force and consequently the flow velocity increases as Fig. 6 demonstrates. Fig. 7 notices that the axial velocity rises with enhancing  $E_1$  or  $E_2$ , while the axial velocity diminishes by increasing wall damping parameter  $E_3$ .

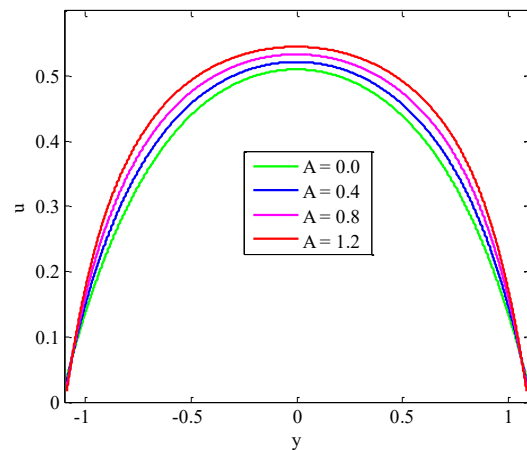
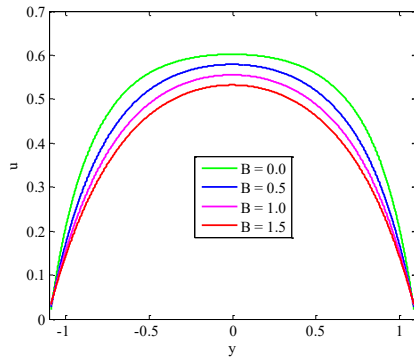
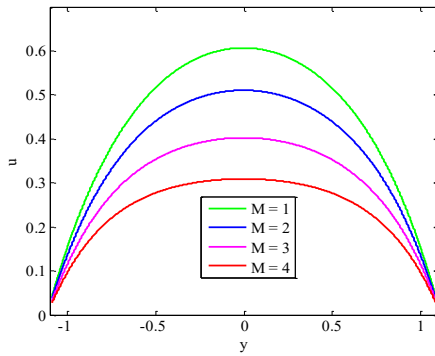
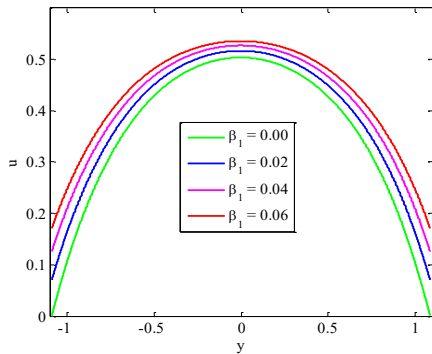
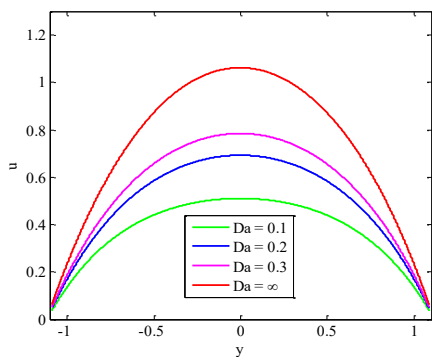
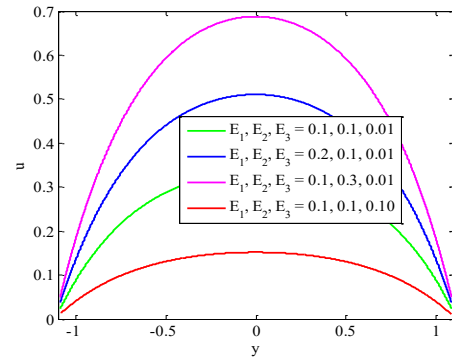
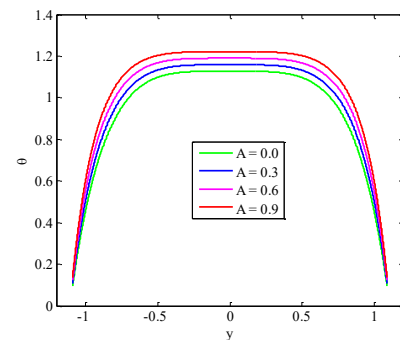


Fig. 2.  $u$  response for  $A$ .

Fig. 3.  $u$  response for  $B$ .Fig. 4.  $u$  response for  $M$ .Fig. 5.  $u$  response for  $\beta_1$ .Fig. 6.  $u$  response for  $Da$ .Fig. 7.  $u$  response for  $E_1, E_2$  and  $E_3$ .

#### 4.2. Heat characteristics

Here, the impact of different physical parameters of interest on the temperature distribution ( $\theta$ ), which is obtained using the expression  $\theta = \theta_0 + A\theta_1$  from Figs. 8 – 17, is investigated. It is found that in these figures the temperature is greater at the middlemost of the channel compared to the walls. This is due to the fact that the study of dissipation impacts on the temperature. The temperature increases by increasing Powell-Eyring fluid parameter  $A$  as shown in Fig. 8, while it has opposite behavior in the magnitude of another Eyring-Powell fluid parameter  $B$  (Fig. 9). The energy diminishes with an increase in  $M$ ,  $\beta_1$ ,  $Pr$ ,  $Da$  and  $Rd$  as shown in Figs. 10 – 14, while the quite opposite trend is observed with an increase of thermal slip parameter  $\beta_2$  and Brinkman number  $Br$ , based on Figs. 15 – 16. Fig. 17 illustrates that the energy enhances with an increase in  $E_1$  and  $E_2$ , while it depresses with an increase in  $E_3$ .

Fig. 8.  $\theta$  response for  $A$ .

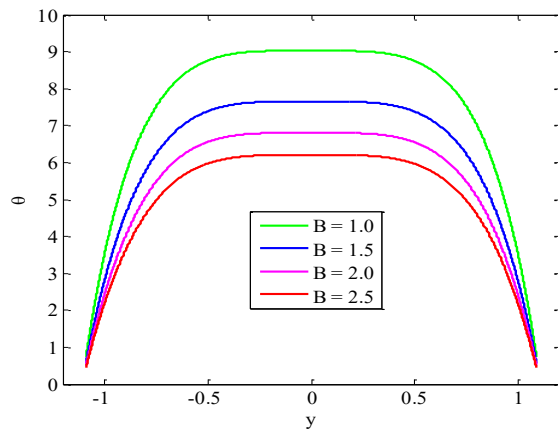


Fig. 9.  $\theta$  response for  $B$ .

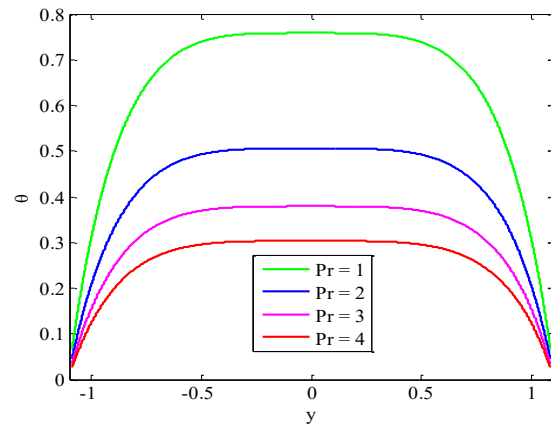


Fig. 12.  $\theta$  response for  $Pr$ .

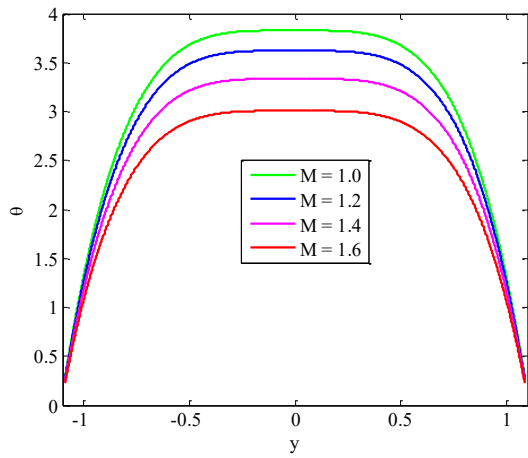


Fig. 10.  $\theta$  response for  $M$ .

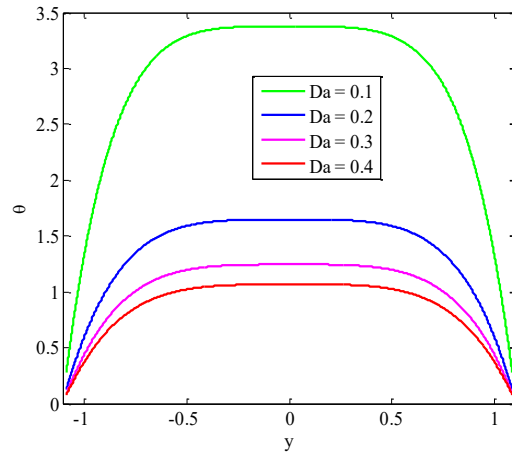


Fig. 13.  $\theta$  response for  $Da$ .

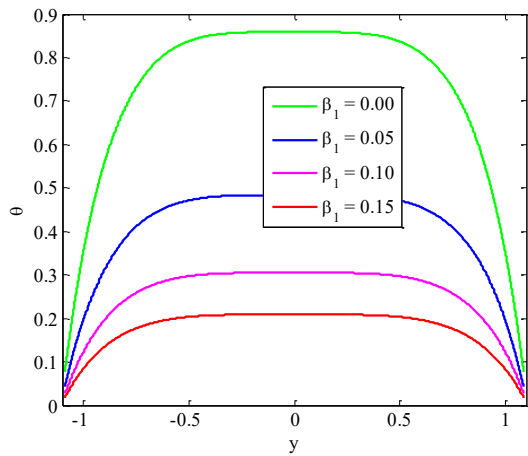


Fig. 11.  $\theta$  response for  $\beta_1$ .

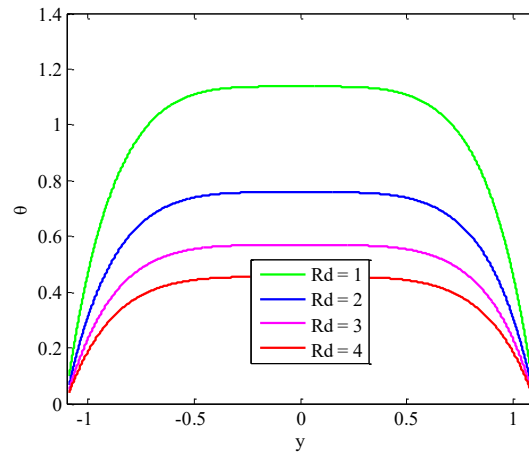


Fig. 14.  $\theta$  response for  $Rd$ .

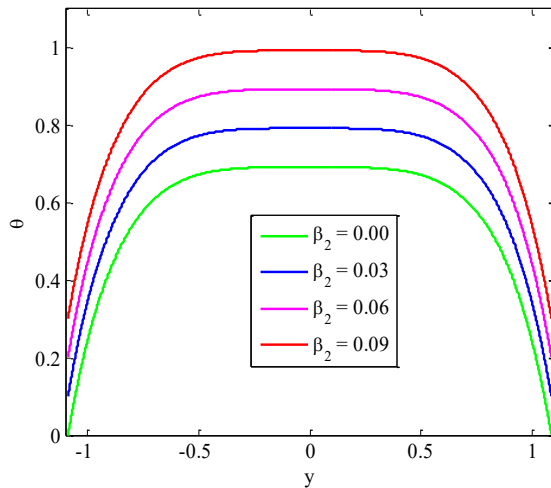


Fig. 15.  $\theta$  response for  $\beta_2$ .

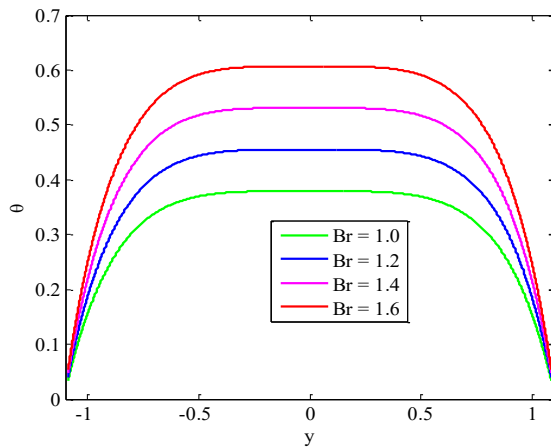


Fig. 16.  $\theta$  response for  $Br$ .

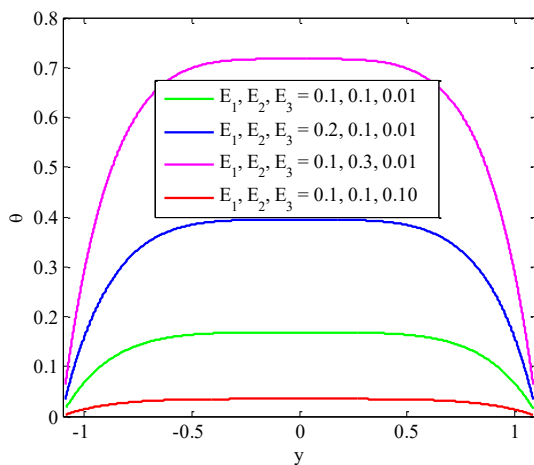


Fig. 17.  $\theta$  response for  $E_1, E_2$  and  $E_3$ .

#### 4.3. Mass characteristics

In this section, it is found that the effect of various parameters of interest on the mass distribution ( $\phi$ ) which is computed using the expression  $\phi = \phi_0 + A\phi_1$  through Figs. 18 - 27. The Figs. 18 - 19 show that the concentration distribution depresses as the larger values of Eyring – Powell parameter  $A$  from Fig. 18 while Fig. 19 exhibits opposite behavior as the higher values of another Eyring – Powell parameter  $B$ . The concentration distribution enhances with the effect of the higher values of Hartmann number  $M$  as well as  $Da$ , as shown in Figs. 20 and 21, while the concentration has opposite behavior with an increase in concentration slip parameter  $\beta_3$ , Brinkman number  $Br$ , and Schmidt number  $Sc$ , as illustrated in Figs. 22 - 24. Fig. 25 shows that the concentration increases with the rise in the value of the radiation parameter  $Rd$  while it follows the same behavior with the increase in chemical reaction parameter  $\gamma$  (Fig. 26). Fig. 27 depicts that the energy depresses with an enhance in  $E_1$  and  $E_2$ , while it intensifies with an increase in  $E_3$ .

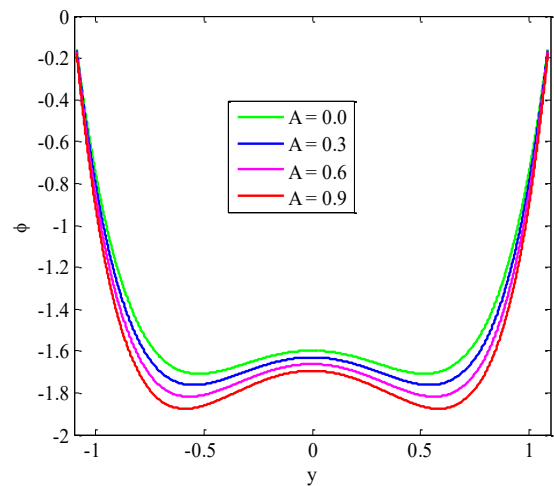


Fig. 18.  $\phi$  response for  $A$ .

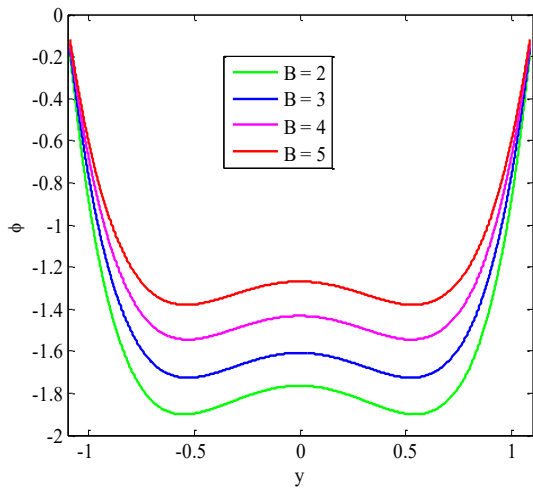


Fig. 19.  $\phi$  response for  $B$ .

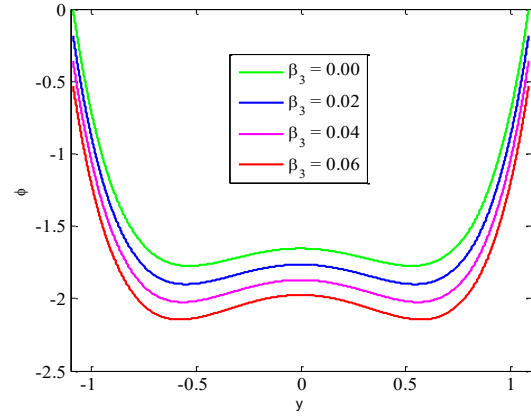


Fig. 22.  $\phi$  response for  $\beta_3$ .

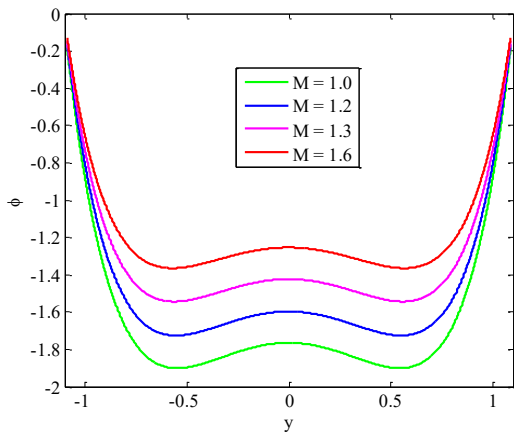


Fig. 20.  $\phi$  response for  $M$ .

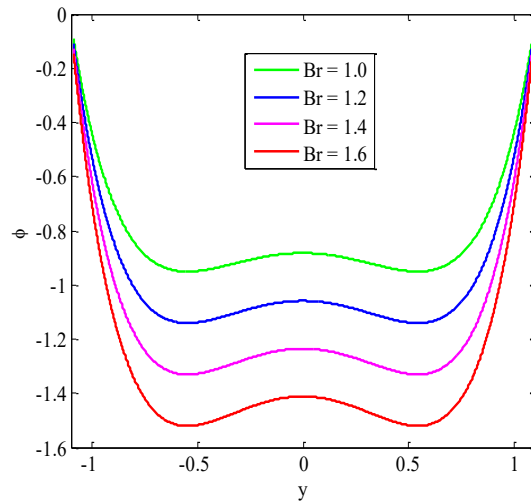


Fig. 23.  $\phi$  response for  $Br$ .

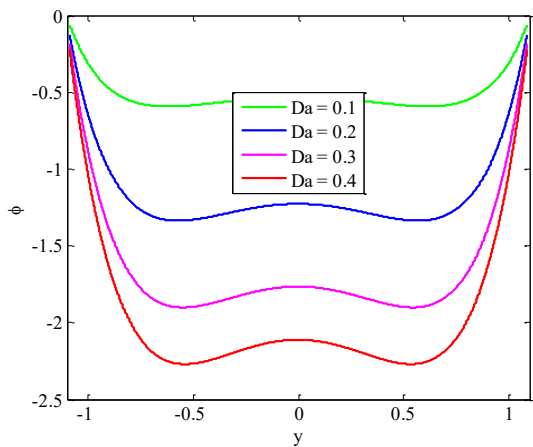


Fig. 21.  $\phi$  response for  $Da$ .

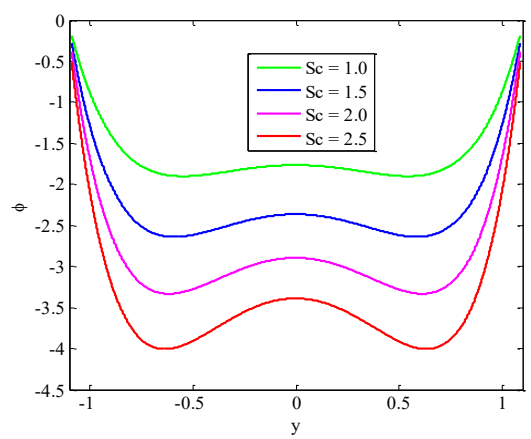
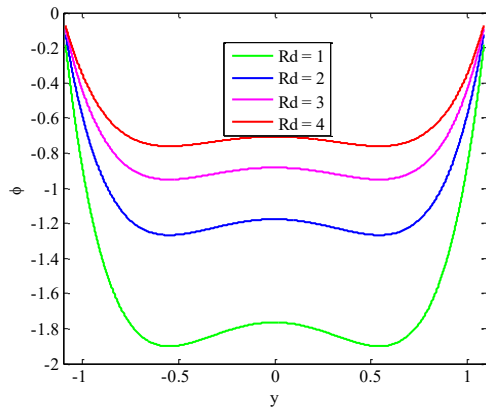
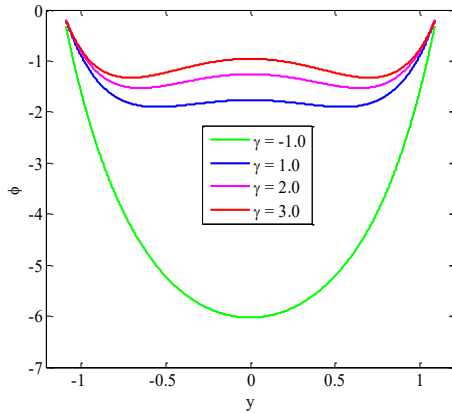
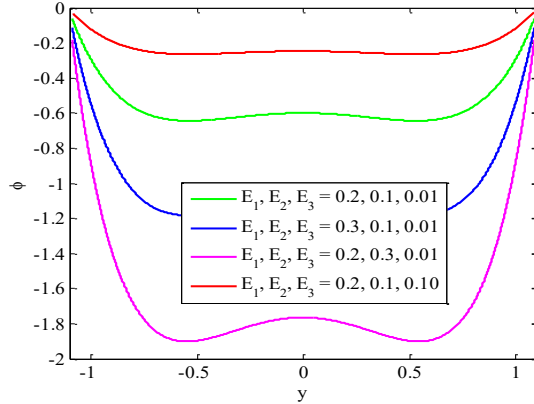


Fig. 24.  $\phi$  response for  $Sc$ .

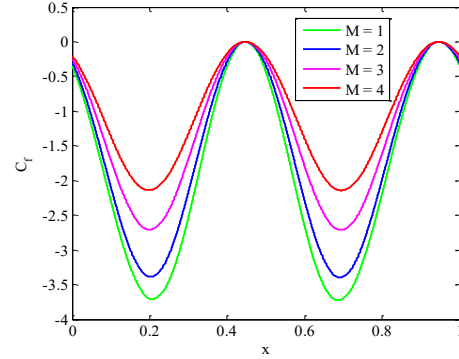
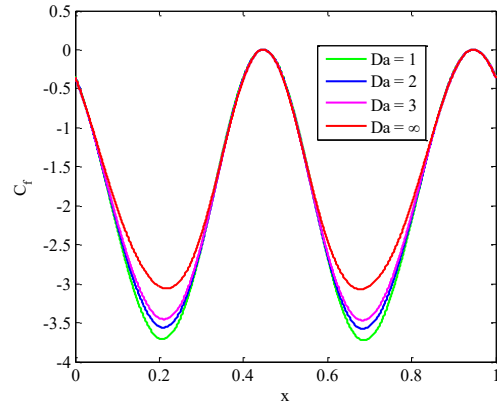
Fig. 25.  $\phi$  response for  $Rd$ .Fig. 26.  $\phi$  response for  $\gamma$ .Fig. 27.  $\phi$  response for  $E_1, E_2$  and  $E_3$ .

#### 4.4. Skin – friction coefficient

Here, the impact of different parameters of interest on  $c_f$ , which is simplified using the following expression, is achieved.

$$c_f = \eta_x \psi_{yy}(\eta) = \eta_x [\psi_{0yy}(\eta) + A\psi_{1yy}(\eta)].$$

Figs. 28 and 29 provide that the absolute value of skin - friction coefficient enhances with an increase in Hartmann number  $M$  and Darcy number  $Da$ .

Fig. 28.  $C_f$  response for  $M$ .Fig. 29.  $C_f$  response for  $Da$ .

#### 4.5. Nusselt number

Here, as Fig. 30-35 propose, the impact of different parameters of interest on Nusselt number  $Nu$ , which is evaluated using the following expression, is obtained.

$$Nu = \eta_x \theta_y(\eta) = \eta_x [\theta_{0y}(\eta) + A\theta_{1y}(\eta)]$$

The Nusselt number  $Nu$  rises at the minor wall followed by diminishing at the major wall with the increase in  $A$ , Hartmann number  $M$ , velocity slip parameter  $\beta_1$  and  $Rd$  as shown in Figs. 30 - 34, respectively. However, it has an opposite behavior with the effect of parameter  $Br$  (Fig. 35).

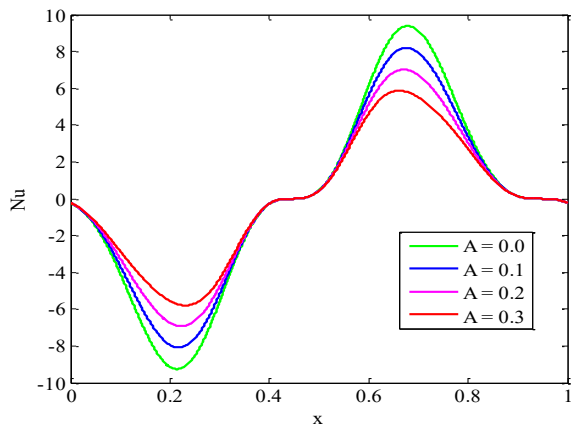


Fig. 30.  $Nu$  response for  $A$ .

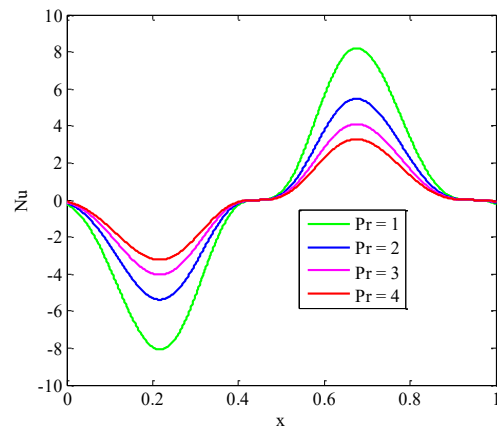


Fig. 33.  $Nu$  response for  $Pr$ .

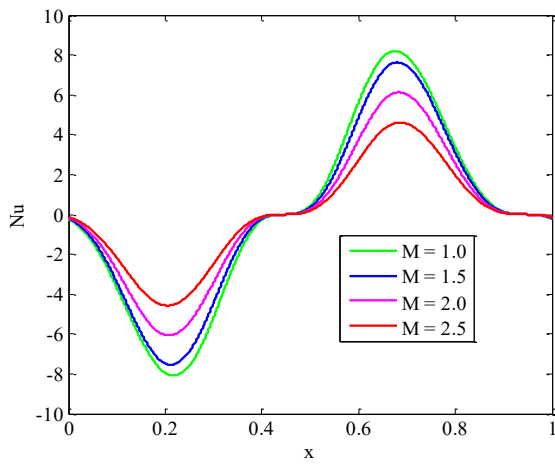


Fig. 31.  $Nu$  response for  $M$ .

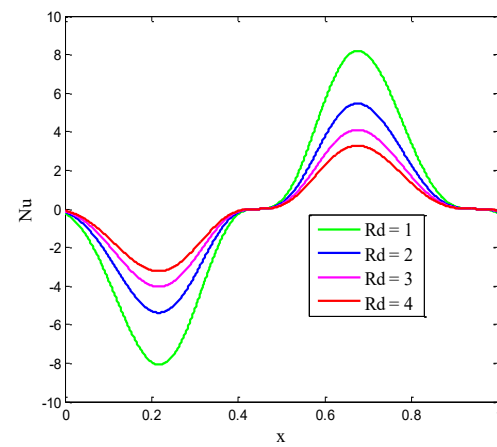


Fig. 34.  $Nu$  response for  $Rd$ .

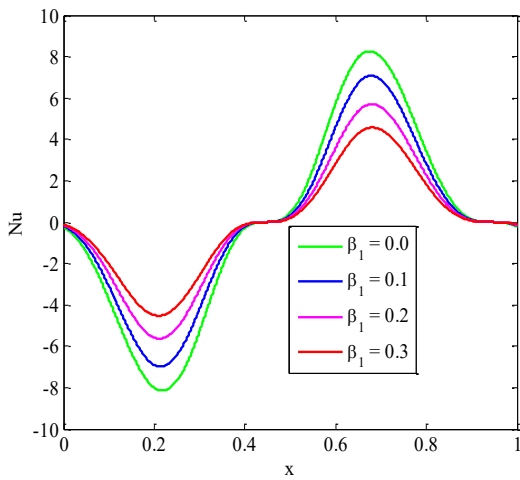


Fig. 32.  $Nu$  response for  $\beta_1$ .

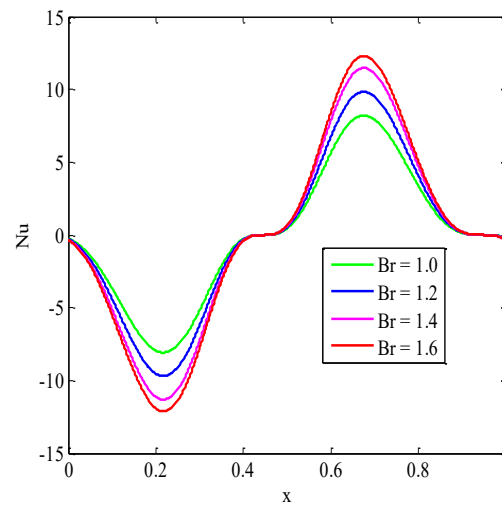


Fig. 35.  $Nu$  response for  $Br$ .

#### 4.6. Sherwood number

Here, the impact of different parameters of interest on Sherwood number  $Sh$ , which is simplified using the expression  $Sh = \eta_x \phi_y(\eta) = \eta_x [\phi_{0y}(\eta) + A \phi_{1y}(\eta)]$  is graphically provided in Figs. 36 - 38. It can be inferred that the Sherwood number  $Sh$  depresses at the minor wall and enhances at the major wall with increasing the Darcy number  $Da$  and chemical reaction parameter  $\gamma$  as shown in Figs. 36 and 37, respectively, while it has an opposite behavior with the effect of radiation parameter,  $Rd$  as shown in Fig. 38.

#### 4.7. Trapping phenomenon

Trapping is an important phenomenon in peristaltic motion. In a wave frame, the streamline under particular conditions split to trap a bolus which moves as a whole with the speed of the peristaltic wave. Figs. 39 - 44 show the stream lines for diverse values of adequate parameters of interest. In Figs. 39 - 42, the effect of  $A$  on the distribution of velocity is illustrated. In figures 39 and 40, the effect of Eyring-Powell fluid parameter  $A$  on the velocity distribution is portrayed. It is observed that the size of the trapped bolus number of circulations increases when Powell fluid parameter  $A$  increases. It is also found that size and shape of the boluses are similar in the upper and lower halves of the channel while it has quite opposite behavior for the effect of another Eyring-Powell fluid parameter  $B$  increases from Figs. 41-42. Figs. 43 and 44 illustrates that the shape and size of the boluses expand when the Darcy number  $Da$  increases.

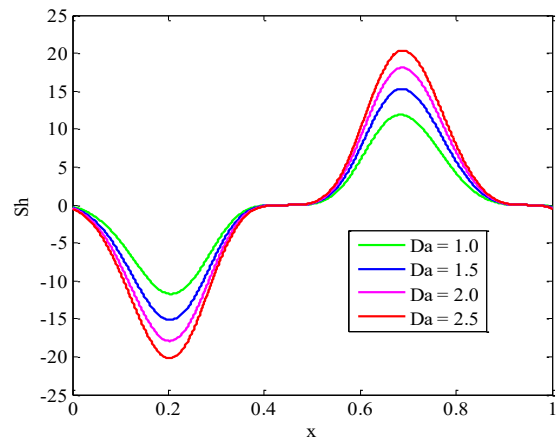


Fig. 36.  $Sh$  response for  $Da$ .

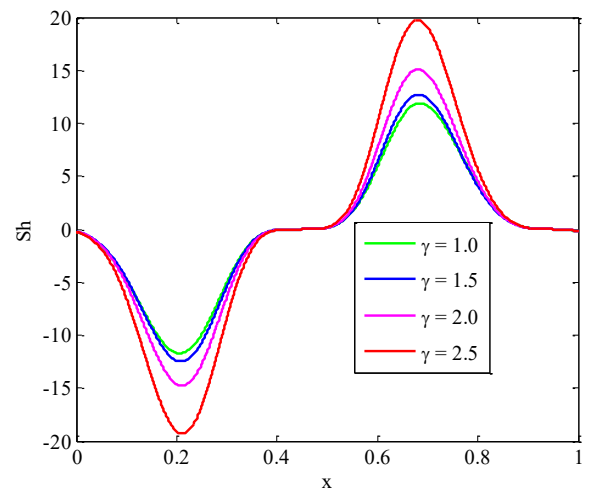


Fig. 37.  $Sh$  response for  $\gamma$ .

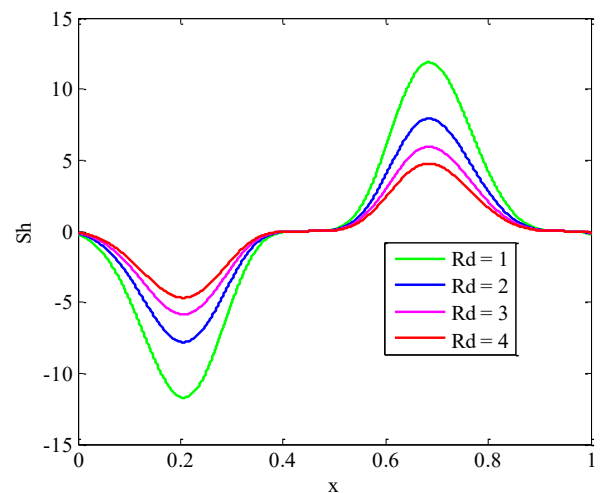
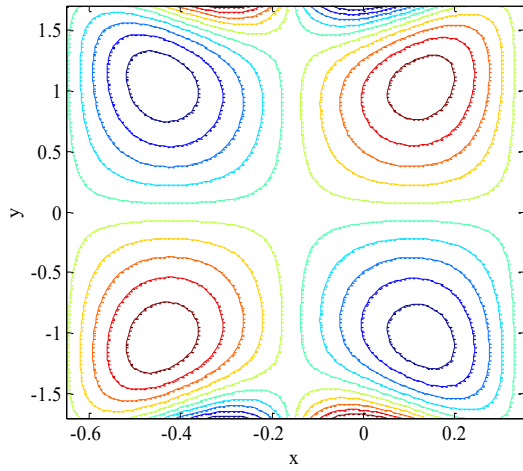
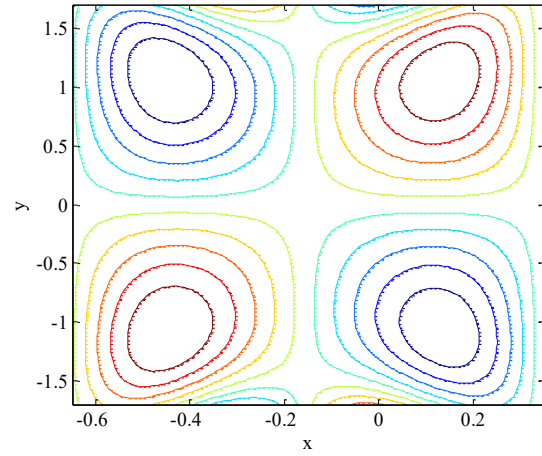


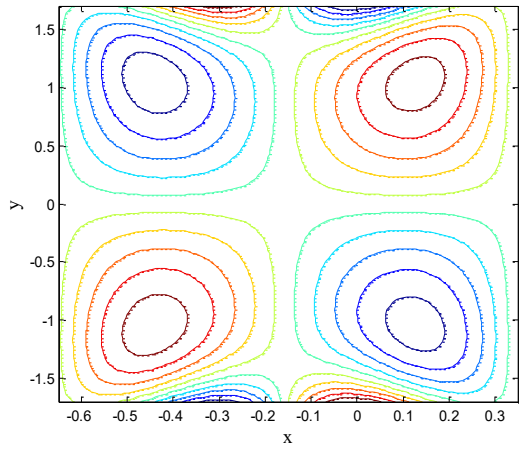
Fig. 38.  $Sh$  response for  $Rd$ .



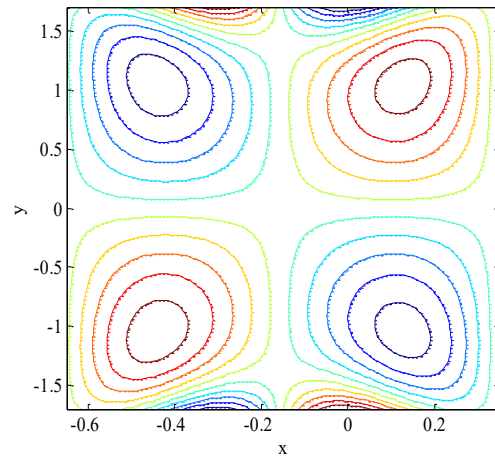
**Fig. 39.** Stream lines for  $A = 0$ .



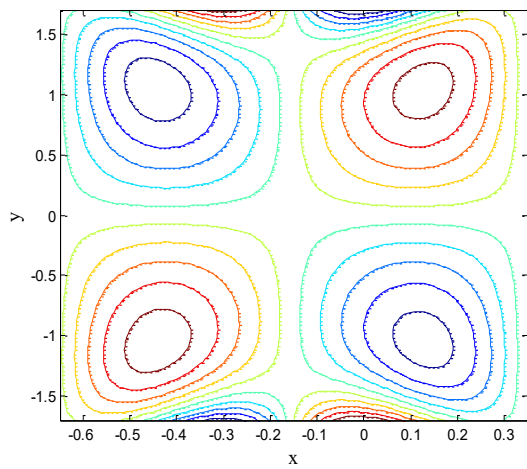
**Fig. 42.** Stream lines for  $B = 3$ .



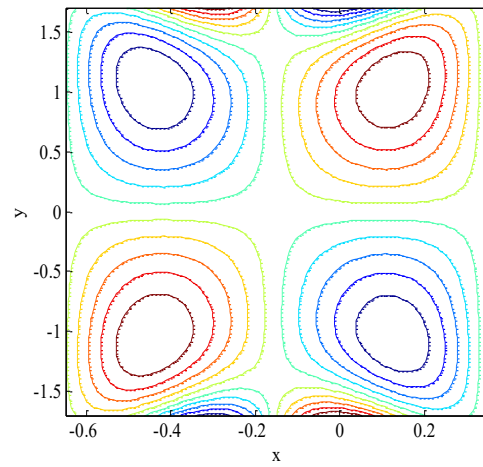
**Fig. 40.** Stream lines for  $A = 3$ .



**Fig. 43.** Stream lines for  $Da = 1$ .



**Fig. 41.** Stream lines for  $B = 1$ .



**Fig. 44.** Stream lines for  $Da = \infty$ .

## 5. Conclusions

In the current paper, the MHD motion of chemical reaction and radiation effects on peristaltic flow of Eyring-Powell fluid are analyzed through a porous medium in a complaint wall channel with slip conditions. The governing equations are modeled using Creeping flow approximations, and then the perturbation method is used for the approximate solution of different physical parameters of interest. The significant points of the current analysis are mentioned below:

1. The axial velocity enhances to the higher values of  $Da$ .
2. The temperature diminishes with increasing  $Rd$ .
3. The concentration increases with the larger values of  $\gamma$  and  $Rd$ .
4. If the Darcy number goes to infinity and the absence of chemical reaction parameter, radiation parameter are in good agreement.
5.  $Nu$  and  $Sh$  are found oscillatory.
6. The size of trapped bolus rises when  $Da$  enhances.
7. The size of trapped bolus diminishes as the Powell-Eyring parameter  $A$  increases, while it enhances as another Powell fluid parameter  $B$  increases.

## Appendix

$$N = \sqrt{\frac{\left(M^2 + \frac{1}{Da}\right)}{1+B}},$$

$$N_2 = \sqrt{Sc\gamma},$$

$$L = \frac{8\pi^3\epsilon}{(1+B)} \left( \frac{E_3}{2\pi} \sin(2\pi(x-t)) - (E_1 + E_2) \cos(2\pi(x-t)) \right),$$

$$L_1 = \frac{L}{\cosh(N\eta) + \beta_1(1+B)N \sinh(N\eta)},$$

$$L_2 = \frac{L_1}{N^3},$$

$$L_3 = -\frac{L}{N^2}, L_4 = \frac{3L_2^3 N^8}{4(1+B)}, L_5 = -\frac{L_2^3 N^8}{4(1+B)},$$

$$L_6 = \frac{L_4}{72 N^2}, L_7 = \frac{L_5}{2 N^3},$$

$$L_9 = -\frac{(Br(1+B)L_2^2 N^4)}{2(1+Pr Rd)}, L_{10} = \frac{L_9}{4N^2},$$

$$L_{11} = -\frac{L_9}{2},$$

$$B_1 = -L_{10} \cosh(2N\eta) - L_{11} \eta^2 - \beta_2 \left( \frac{2NL_{10} \sinh(2N\eta)}{+2L_{11}\eta} \right)$$

$$B_2 = -\frac{1}{\eta + \beta_2} \left( B_1 + L_{10} \cosh(2N\eta) + L_{11} \eta^2 + \beta_2 (2NL_{10} \sinh(2N\eta)) + 2L_{11}\eta \right),$$

$$L_{12} = -\frac{2Br(1+B)}{1+Pr Rd}, L_{13} = \frac{Br}{3(1+Pr Rd)},$$

$$L_{14} = \frac{L_{12}L_2A_7N^2}{8}, L_{15} = \frac{L_2L_8N^2L_{12}}{8},$$

$$L_{16} = \frac{9L_2N^2L_{12}L_6}{32}, L_{17} = \frac{L_{12}L_2NL_7}{8},$$

$$L_{18} = \frac{L_2L_7NL_{12}}{4}, L_{19} = \frac{L_2^4L_7N^8L_{13}}{4},$$

$$L_{20} = L_{15} - 4L_{16} - L_{17} + L_{18} - \left( \frac{L_{19}}{2N^2} \right),$$

$$L_{21} = L_{16} + \left( \frac{L_{19}}{32N^2} \right),$$

$$L_{22} = L_{17}N, L_{23} = \frac{3}{4}L_{19} - 2N^2L_{15} - 2N^2L_{18},$$

$$L_{24} = 2NL_{20} + L_{22}, L_{25} = -\frac{4N^2L_{10}ScSr}{4N^2 - N_2^2},$$

$$L_{26} = \frac{2ScSrL_{11}}{N_2^2},$$

$$B_4 = -\frac{1}{\eta + \beta_2} \left( L_{14} \sinh(2N\eta) + 2\beta_2NL_{14} \cosh(2N\eta) \right),$$

$$B_3 = - \left( \begin{array}{l} B_4\eta + L_{14} \sinh(2N\eta) + L_{20} \cosh(2N\eta) + \\ L_{21} \cosh(4N\eta) + L_{22}\eta \sinh(2N\eta) + L_{23}\eta^2 \\ + \beta_2 \left( \begin{array}{l} B_4 + 2NL_{14} \cosh(2N\eta) + L_{24} \sinh(2N\eta) \\ + 4NL_{21} \sinh(4N\eta) + 2NL_{22}\eta \cosh(2N\eta) \\ + 2L_{23}\eta \end{array} \right) \end{array} \right)$$

$$C_2 = 0,$$

$$C_1 = - \frac{1}{\cosh(N_2\eta) + \beta_3 N_2 \sinh(N_2\eta)} \left( \begin{array}{l} L_{25} \cosh(2N\eta) + L_{26} + \\ 2NL_{25}\beta_3 \sinh(2N\eta) \end{array} \right)$$

,

$$L_{27} = 2NL_{24} + 2NL_{22}, L_{28} = - \frac{4ScSrN^2 L_{14}}{4N^2 - N_2^2},$$

$$L_{29} = \frac{16ScSrN^3 L_{22}}{4N^2 - N_2^2} - \frac{ScSrL_{27}}{4N^2 - N_2^2},$$

$$L_{30} = - \frac{16ScSrN^2 L_{21}}{16N^2 - N_2^2}, L_{31} = - \frac{4ScSrN^2 L_{22}}{4N^2 - N_2^2},$$

$$L_{32} = \frac{2L_{33}ScSr}{N_2^2}, L_{33} = 2NL_{29} + L_{31},$$

$$C_4 = - \frac{1}{\sinh(N_2\eta) + \beta_3 N_2 \cosh(N_2\eta)} \left( \begin{array}{l} L_{28} \sinh(2N\eta) + \\ 2NL_{28}\beta_3 \cosh(2N\eta) \end{array} \right)$$

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